Ecosystem Impacts of Changing Sex Ratios in Lampreys Based on Multi-Model Analysis

Zhebin Liang [*], Xiaofeng Li, Guangmeng He

College of Electronic Engineering and Automation, Guilin University of Electronic Technology
Guilin, China, 541004

* Corresponding Author Email: 18707895715@163.com

Abstract. Through constructing different models, comprehensive research has been conducted on the adaptability of Lampreys and the impact of sex ratio changes on their population and ecosystem. Firstly, an established population dynamics model based on Lotka-Volterra was used to simulate the predation relationships between lampreys and other organisms in the ecosystem. The model was employed to investigate the impact of sex ratio changes in lampreys on the population dynamics of other organisms in the ecosystem. Secondly, an established model was utilized to analyze the relationship between resource availability and sex ratio, further refining the population dynamics model. The population dynamics of this model were tested and compared under different adverse conditions with a model where the sex ratio remains constant. After that Introducing a new participant, aquatic plants, to the population dynamics model formed a simple food chain model. The eigenvalues of the Jacobian matrix were calculated to assess the stability of the food chain model. Further introducing a new participant, aquatic plants, into the population dynamics model formed a simple food chain model. The eigenvalues of the Jacobian matrix were calculated to assess the stability of the food chain model. Finally, the Entropy Weighting TOPSIS model was employed to evaluate the advantages and disadvantages of other species in the ecosystem under changes in the lamprey's sex ratio. The final results indicated 0.59 points for aquatic plants, 0 points for herbivores, and 1 point for parasites. Through the research using the above methods, it can be concluded that changes in the sex ratio of Lampreys have different impacts on various aspects of the ecosystem. For Lampreys themselves, changes in sex ratio favor the stability of their own population numbers. As for the producers, changes in Lamprey sex ratio provide advantageous conditions. Herbivorous animals involved in predation relationships will experience negative effects. Parasites engaged in competitive relationships will maintain dynamic equilibrium in their interrelationships.

Keywords: Lotka-Volterra, Jacobian Matrix, Cellular Automata, Entropy Weighting TOPSIS.

1. Introduction

Lampreys are one of the few remaining jawless vertebrates that are metamorphosed during development, belonging to “pests” and “parasites” that can cause devastating damage to the fish in their waters.

Lamprey has adaptive sex ratio variation with changes in the external environment, and this adaptive sex ratio variation will have a profound impact on the role of lamprey in the ecosystem and its interaction with other species.

The article has addressed the following four issues in light of the background discussed above.

(1) Predict changes in the sex ratio of lamprey populations and consider the impact of this species on larger ecosystems.

(2) Considering the adaptive sex ratio variation, the survival advantages and disadvantages of lamprey populations in the ecosystem were analyzed.

(3) In this paper, whether the change of the sex ratio of lamprey with the environment will lead to changes in other species in the ecosystem, thereby affecting the stability of the whole ecosystem.

(4) Based on the established model, analyze and discuss whether ecosystems with changes in the sex ratio of lamprey populations can provide advantages to other species in the ecosystem, such as parasites.

2. Materials and methods

2.1. Data

The research data in this article is sourced from some open-access websites related to lampreys, with the main data concerning gender ratio of lampreys. Analysis revealed the presence of some erroneous data in different ecological environments, leading to the preprocessing of certain data.

2.2. Method introduction

According to the requirements to design the following methods.

(1) Develop an ecological niche model that considers fluctuations in the sex ratio to simulate changes in Lamprey communities. The model is constructed based on the differential equations of the Lotka-Volterra [1-2] equation, taking into account how the lamprey population's ability to adjust its sex ratio according to resource availability impacts both its own population and the ecosystem.

(2) Explore the effects of population sex ratio changes caused by adaptive sex ratio variations on the reproduction rate of lamprey populations. It is suggested that the highest reproduction rate of lampreys may occur when the male-to-female ratio is 1:1. When the connection weights are positive, it implies an exciting state of the current link. Conversely, negative link coefficients suggest a state of suppression.

(3) Utilize this model to investigate the influence of resource availability on the sex ratio of lamprey populations. The findings illustrate that lampreys can adapt their sex ratio in response to resource availability, enabling them to cope with challenging conditions. Under harsh circumstances, more resilient males may be generated to uphold the overall population stability.

(4) Examine the impact of adaptive sex ratio variations on the stability of the ecosystem where lamprey populations are situated. By establishing a Lotka-Volterra-based food chain and introducing a Jacobian matrix, species interactions are simulated. The system's stability is determined through the eigenvalues of the Jacobian matrix [3-5].

(5) Employ the model to explore whether lamprey populations offer benefits to other ecosystem populations, such as parasites. Introduce a group of parasites into the food chain to construct a Lotka-Volterra-based food web, using cellular automata [6-7] to analyze species interactions.

(6) In summary, emphasis is placed on the influence of resource availability on the sex ratio and population size of lampreys. Alterations in the lamprey population's sex ratio may contribute to maintaining population stability. However, drastic fluctuations in the sex ratio of lampreys could potentially destabilize multiple populations within the ecosystem, triggering a chain reaction that impacts the overall ecosystem stability.

2.3. Notations

The symbols and their definitions are shown in Table 1.

Table 1: Notations Table

Notations	Definition
$N(t)$	Population size of lampreys at time t
$P(t)$	The population size of herbivorous animals at time t
r_P	The natural growth rate of herbivorous organisms that are preyed
α_P	Probability of predation in herbivorous organisms that are preyed
r_N	Reproductive rate of lampreys
d_N	Mortality of lampreys
b_N	Lampreys convert food into an efficiency for population growth
$M(t)$	Male ratio at time t
$F(t)$	Female ratio at time t
r_0	Base growth coefficient
$Q(t)$	The number of populations of aquatic plants at time t
r_Q	Growth rate of aquatic plants
α_Q	The probability that aquatic plants are preyed
b_P	The efficiency with which herbivores convert food into population growth
$O(t)$	Number of parasite populations at time t
r_O	Parasite growth rate
d_O	Parasite mortality
b_O	The efficiency with which parasites convert food into population growth
α_O	Probability of parasites preying on herbivorous plants
β_{NO}	The rate of competition between lampreys and parasites
β_{ON}	The rate of competition between lampreys and parasites

3. Results and analysis

The food web model generally refers to a concept in ecology that describes the food relationships between various organisms in an ecosystem. In this model, the various organisms are organized into a hierarchy that shows the food chain and food web between them. The model can involve interactions between a single species or multiple species, or it can take into account the dynamics of ecosystems.

3.1. A population dynamics model based on Lotka-Volterra

The Lotka-Volterra model is a classical model of population dynamics used to describe the interaction between predators and predators. The basic step of the model is shown in Figure 1:

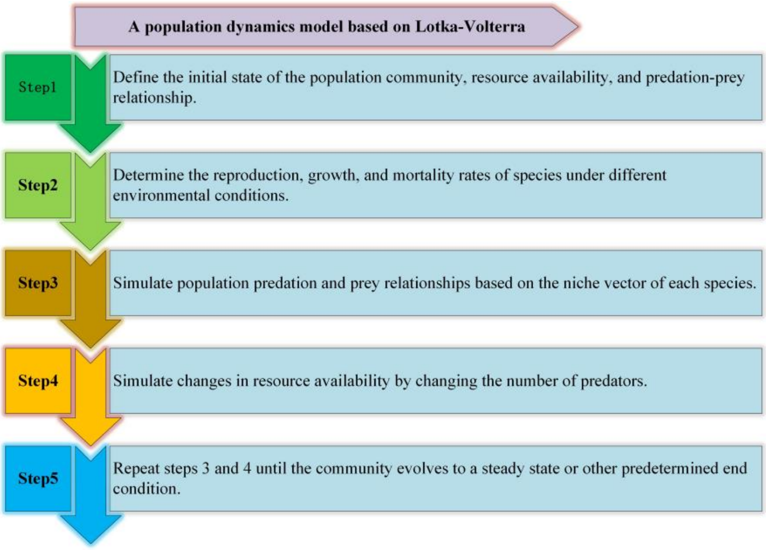


Figure 1: Steps of the population model

The process of population change is shown in Figure 2:

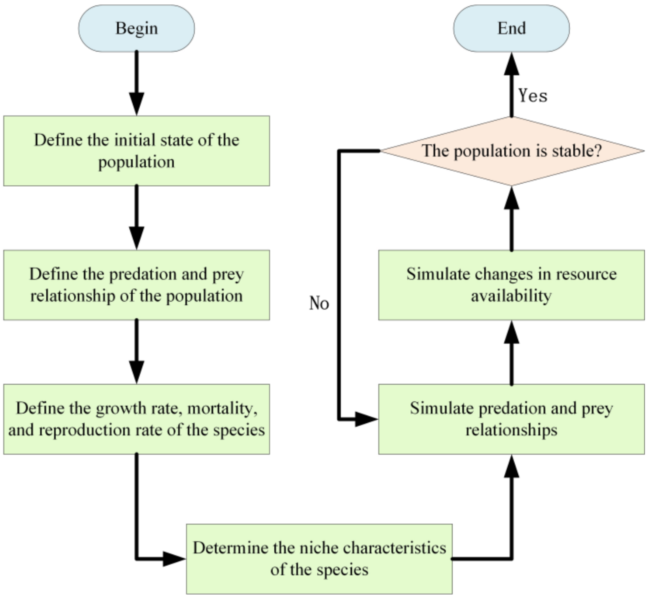


Figure 2: Population change process.

3.2. A dynamic model of the lamprey population in Lotka-Volterra

As a predator, the population of lamprey is affected by birth rate, mortality rate, and reproduction rate. The predators are herbivorous organisms, and their populations are affected by the natural growth rate and predation rate. The Lamprey population dynamics model based on Lotka-Volterra is as follows:

$$\frac{dN}{dt} = r_N N(t) - d_N N(t) + b_N \alpha_P P(t) N(t) \tag{1}$$

$$\frac{dP}{dt} = r_P P(t) - \alpha_P P(t) N(t) \tag{2}$$

$N(t)$ is the population of lampreys at time t.

$P(t)$ is the population of herbivores at time t.

r_P is the natural growth rate of herbivores.

α_P is the probability that herbivores are preyed.

r_N is the reproduction rate of lampreys.

d_N is the mortality rate of lampreys.

b_N is the efficiency with which lampreys convert food into population growth.

Lampreys have no natural predators except humans, and as omnivores, lampreys are only used as a diet for a very small percentage of the population, so there is no need to put humans directly into the Lotka-Volterra model.

3.3. Relationship between the reproduction rate and sex ratio of lamprey

The reproduction rate r_N of lampreys is related to the proportion of males and females:

$$r_N = f(M,F) \tag{3}$$

$M(t)$ and $F(t)$ are the proportions of males and females at time t, respectively, and it can be seen that:

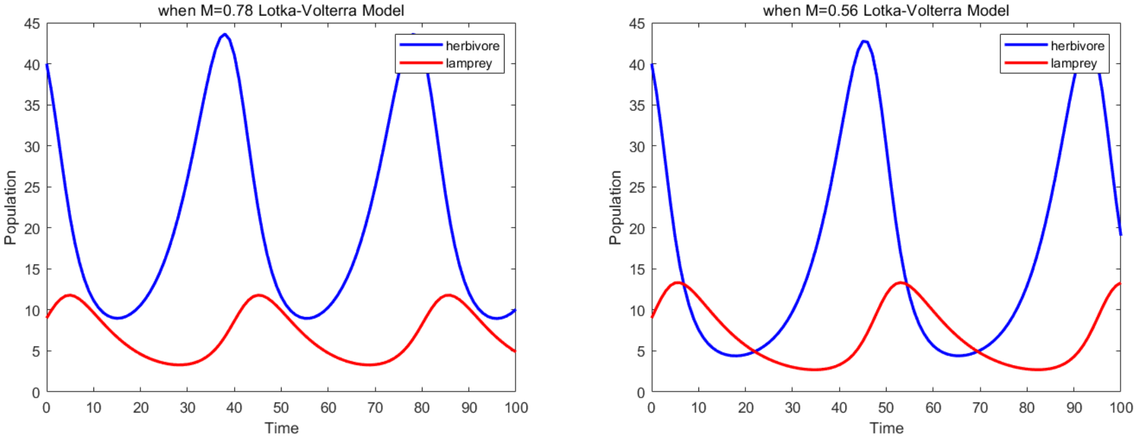
$$F(t) = 1 - M(t) \tag{4}$$

When the male-to-female ratio is 1:1, the reproductive rate is the highest, and there are:

$$r_N = r_0 M(t) F(t) \tag{5}$$

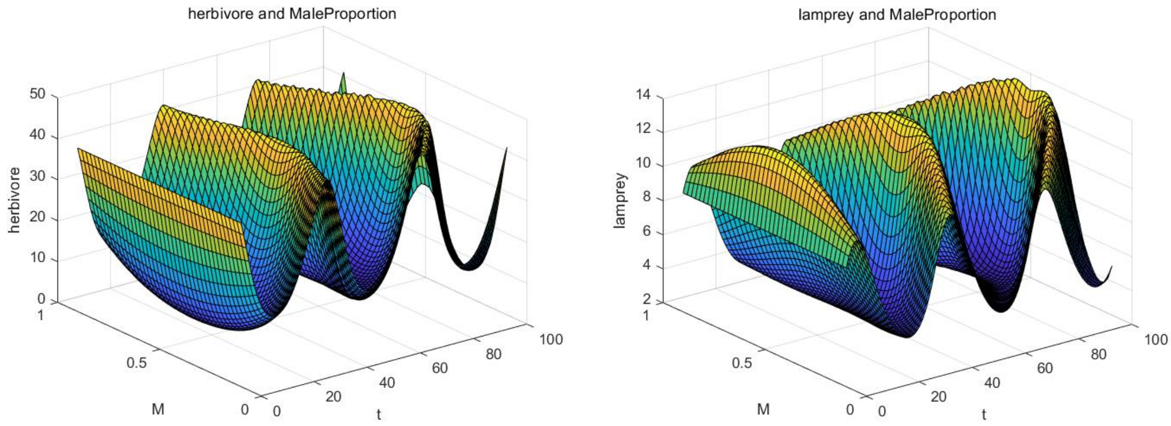
where r_0 is the growth coefficient.

The simulated changes between the model and the Lotka-Volterra-based population dynamics model of lamprey is shown in Figure 3 and Figure 4:



(a) M = 0.78 (b) M = 0.56

Figure 3: Lotka-Volterra model of the lamprey and herbivore



(a) herbivore (b) lamprey

Figure 4: Lotka-Volterra model of the lamprey and herbivore with male ratio

From the above change description diagram, it can be found that when the sex ratio of lamprey changes, it will lead to the extreme difference in the population of other species, which may break the food chain, reduce biodiversity, and affect the survival of other populations, which may have a negative impact on the stability of the ecosystem. And we can find that when the proportion of lamprey males increases, the population fluctuation will be more frequent, and the population fluctuation of its own population will also trigger the fluctuation of the number of predators, which will have a chain reaction and affect the entire ecosystem.

3.4. The relationship between resource availability and sex ratio

Lampreys can reach about 78% of the population in males in environments with low food availability. In environments where food is more readily available, males are observed in about 56% of the population.

$$M(t) = g(P(t)) \tag{6}$$

$$F(t) = 1 - M(t) \tag{7}$$

$M(t)$ is the proportion of males and $F(t)$ is the proportion of females, considering that the proportion of males increases with the decrease in food supply, and the principle of preference for linearity.

$$M(t) = \beta_0 P(t) + \beta_1 \tag{8}$$

At this point, the dynamic model can be modified to:

$$\frac{dN}{dt} = r_0(\beta_0 P(t) + \beta_1)(1 - \beta_0 P(t) - \beta_1)N(t) - d_N N(t) + b_N \alpha_P P(t)N(t) \tag{9}$$

$$\frac{dP}{dt} = r_P P(t) - \alpha_P P(t)N(t) \tag{10}$$

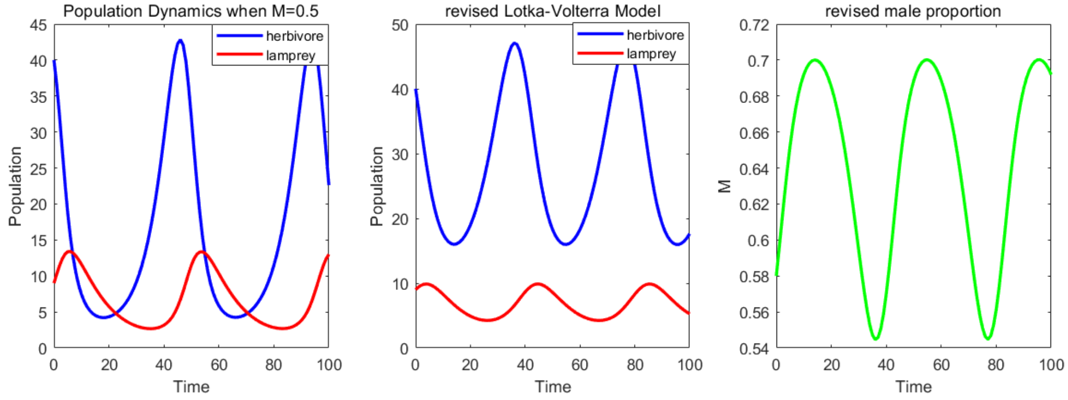


Figure 5: Comparison of population dynamics between species with constant sex ratio and lamprey

Taking the same initial conditions, the model before and after correction is solved:

Test for stability under harsh conditions1 (significant reduction in food supply)

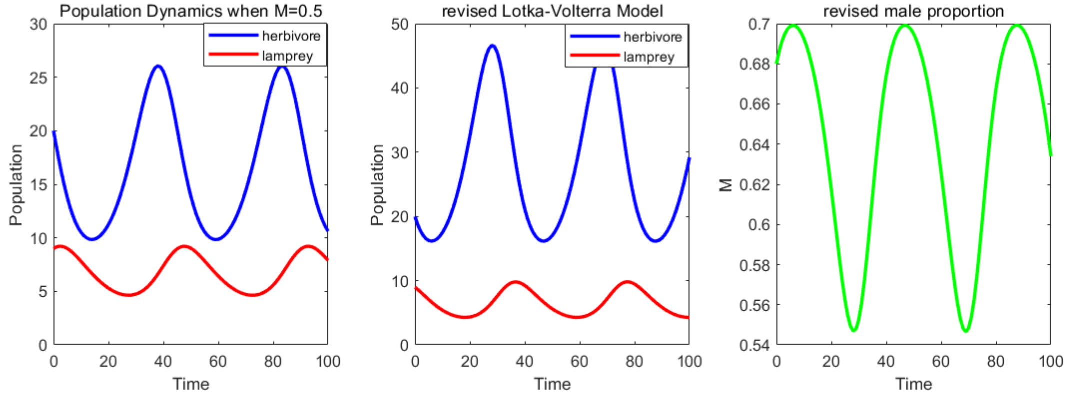


Figure 6: Population dynamics comparison under harsh condition 1

Test stability under harsh conditions2 (increased initial number of lampreys)

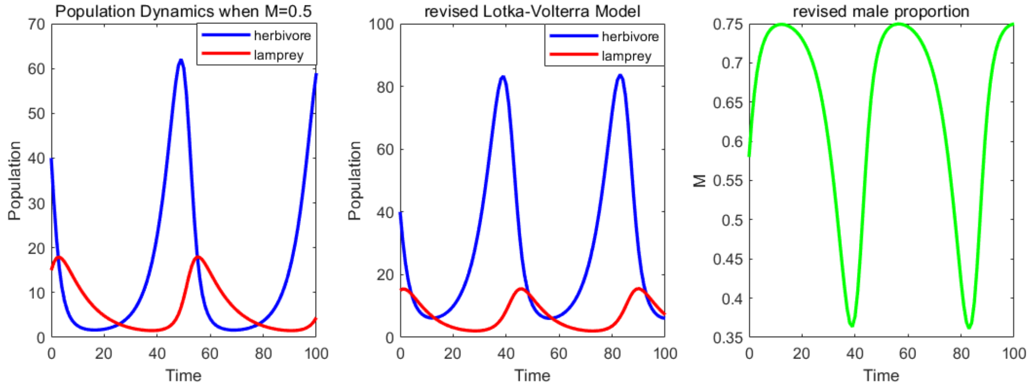


Figure 7: Population dynamics comparison under harsh condition 2

We observe the strengths and weaknesses of the lamprey population by comparing the changes in population size over time under the condition of variable and immutable sex ratios. From the analysis of Figure 5,6,7, it becomes apparent that the advantage of the lamprey population is that it can adapt to harsh conditions by adjusting the sex ratio, and the population fluctuation is always small under harsh conditions, such as more males in harsh environments to prey on and protect territory, and the male-to-female ratio is close to 1:1 under superior conditions, focusing on breeding and expanding the population. The disadvantage is that it affects the stability of other species, such as the predated herbivores we studied, whose populations fluctuate greatly, which in turn affects other members of the ecosystem and destabilizes their populations.

3.5. Simplified Food Chain Model

In order to study the effect of changes in the sex ratio of lampreys on ecosystem stability, we introduced aquatic plant populations into the above model. To joint creation of a simplified food chain model:

$$\frac{dN}{dt}=r_N N(t)-d_N N(t)+b_N\alpha_P P(t)N(t) \tag{11}$$

$$\frac{dP}{dt}=r_P P(t)-\alpha_P P(t)N(t)+b_P\alpha_Q Q(t)P(t) \tag{12}$$

$$\frac{dQ}{dt}=r_Q P(t)-\alpha_Q Q(t)P(t) \tag{13}$$

Thereinto:

$Q(t)$ is the population of aquatic plants at time t.

r_Q is the growth rate of aquatic plants.

α_Q is the probability of predation by aquatic plants.

b_P is the efficiency with which herbivores convert food into population growth.

We determine the stability of the system by finding the Jacobian matrix of the food chain model.

For lampreys:

$$\begin{cases}\dfrac{\partial}{\partial N}(r_N N(t)-d_N N(t)+b_N\alpha_P P(t)N(t))=r_N-d_N+b_N\alpha_P P(t)\\ \dfrac{\partial}{\partial P}(r_N N(t)-d_N N(t)+b_N\alpha_P P(t)N(t))=b_N\alpha_P N(t)\\ \dfrac{\partial}{\partial Q}(r_N N(t)-d_N N(t)+b_N\alpha_P P(t)N(t))=0\end{cases} \tag{14}$$

For prey herbivores:

$$\begin{cases}\dfrac{\partial}{\partial N}(r_P P(t)-\alpha_P P(t)N(t)+b_P\alpha_Q Q(t)P(t))=-\alpha_P P(t)\\ \dfrac{\partial}{\partial P}(r_P P(t)-\alpha_P P(t)N(t)+b_P\alpha_Q Q(t)P(t))=r_P-\alpha_P N(t)+b_P\alpha_Q Q(t)\\ \dfrac{\partial}{\partial Q}(r_P P(t)-\alpha_P P(t)N(t)+b_P\alpha_Q Q(t)P(t))=b_P\alpha_Q P(t)\end{cases} \tag{15}$$

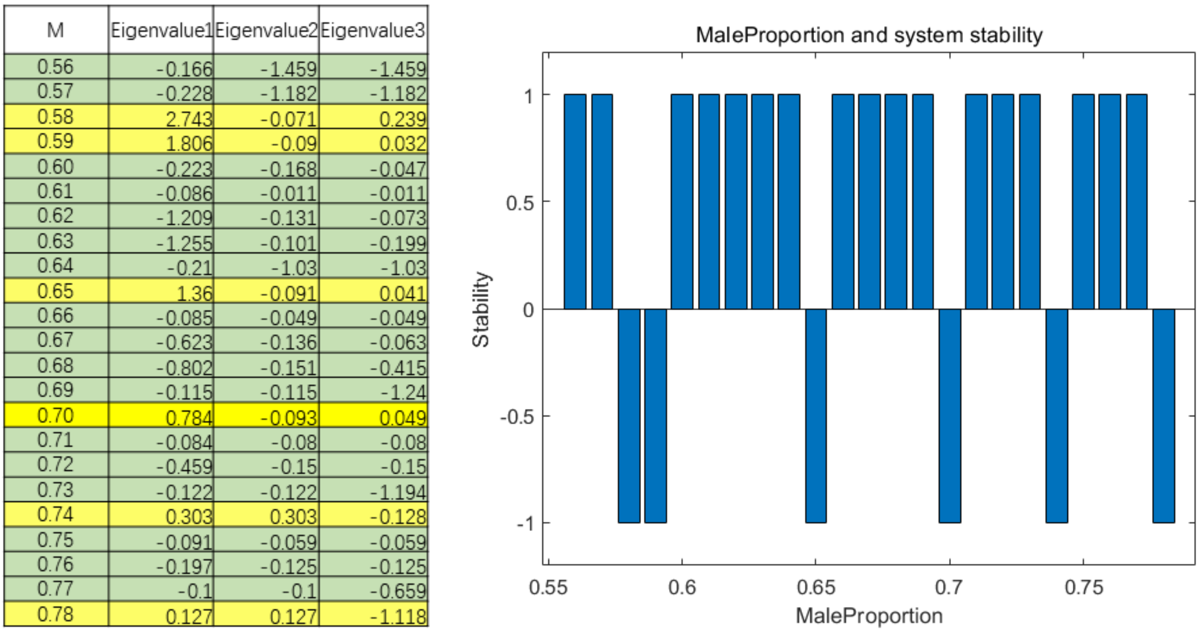
For aquatic plants:

$$\begin{cases}\dfrac{\partial}{\partial N}(r_Q P(t)-\alpha_Q Q(t)P(t))=0\\ \dfrac{\partial}{\partial P}(r_Q P(t)-\alpha_Q Q(t)P(t))=r_Q-\alpha_Q Q(t)\\ \dfrac{\partial}{\partial Q}(r_Q P(t)-\alpha_Q Q(t)P(t))=-\alpha_Q P(t)\end{cases} \tag{16}$$

Available food chain models Jacobian matrices:

$$J=\begin{bmatrix} r_N-d_N+b_N\alpha_P P(t) & b_N\alpha_P N(t) & 0\\ -\alpha_P P(t) & r_P-\alpha_P N(t)+b_P\alpha_Q Q(t) & b_P\alpha_Q P(t)\\ 0 & r_Q-\alpha_Q Q(t) & -\alpha_Q P(t)\end{bmatrix} \tag{17}$$

Then, the eigenvalues of the Jacobian matrix are calculated, and the eigenvalues are used to determine whether the system is stable. The result is shown in Figure 8:



M	Eigenvalue1	Eigenvalue2	Eigenvalue3
0.56	-0.166	-1.459	-1.459
0.57	-0.228	-1.182	-1.182
0.58	2.743	-0.071	0.239
0.59	1.806	-0.09	0.032
0.60	-0.223	-0.168	-0.047
0.61	-0.086	-0.011	-0.011
0.62	-1.209	-0.131	-0.073
0.63	-1.255	-0.101	-0.199
0.64	-0.21	-1.03	-1.03
0.65	1.36	-0.091	0.041
0.66	-0.085	-0.049	-0.049
0.67	-0.623	-0.136	-0.063
0.68	-0.802	-0.151	-0.415
0.69	-0.115	-0.115	-1.24
0.70	0.784	-0.093	0.049
0.71	-0.084	-0.08	-0.08
0.72	-0.459	-0.15	-0.15
0.73	-0.122	-0.122	-1.194
0.74	0.303	0.303	-0.128
0.75	-0.091	-0.059	-0.059
0.76	-0.197	-0.125	-0.125
0.77	-0.1	-0.1	-0.659
0.78	0.127	0.127	-1.118

(a) Male Proportion and system stability (b) Eigenvalue chart

Figure 8: Eigenvalues and stability of Jacobian matrices based on the Lotka-Volterra food chain model.

In a subgraph of Figure 8, we calculate the corresponding real part of the eigenvalue of the Jacobian matrix in the process of changing the male proportion from 0.56 to 0.78, when the real part of the three eigenvalues is less than 0, it is stable, and marked with green annotation, and any of the three eigenvalue real parts is greater than 0, which is considered unstable and marked with yellow. In the b subplot of Figure 10, a value of 1 is stable, and a value of -1 is unstable.

From Figures 8, we can see that when the proportion of males in the lamprey population increases, the stability of the ecosystem decreases. As the environment becomes harsh, the number of males in the lamprey population increases, and its competitiveness becomes stronger, and its ability to invade other species becomes stronger, which makes other species face greater pressure to survive in the already harsh environment.

3.6. Simplified Food Web Model

Parasites were introduced into the food chain model to form a simplified food web model.

As shown in figure 9:

$$\frac{dN}{dt} = r_N N(t) - d_N N(t) + b_N \alpha_P P(t) N(t) - \beta_{NO} O(t) N(t) \tag{18}$$

$$\frac{dO}{dt} = r_O O(t) - d_O O(t) + b_O \alpha_O P(t) O(t) - \beta_{ON} O(t) N(t) \tag{19}$$

$$\frac{dP}{dt} = r_P P(t) - \alpha_P P(t) N(t) - \alpha_O P(t) O(t) + b_P \alpha_Q Q(t) P(t) \tag{20}$$

$$\frac{dQ}{dt} = r_Q P(t) - \alpha_Q Q(t) P(t) \tag{21}$$

$O(t)$ is the number of parasite populations at time t

r_O is the parasite growth rate.

d_O is parasite mortality.

b_O is the efficiency with which the parasite converts food into population growth.

α_O is the probability that the parasite will prey on herbivorous plants.

β_{NO} is the competition rate of lampreys for parasites.

β_{ON} is the rate of competition between parasites and lampreys.

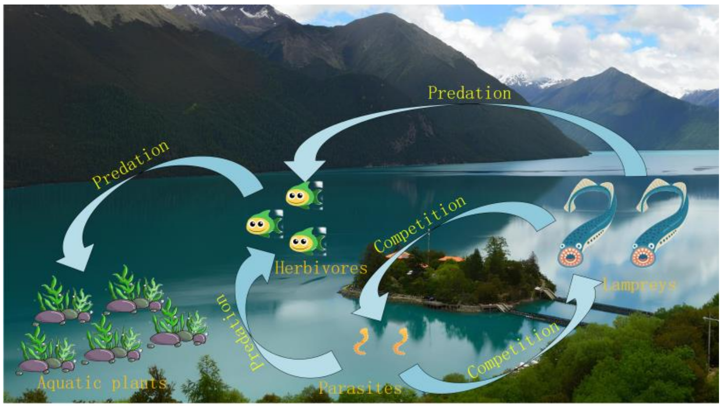
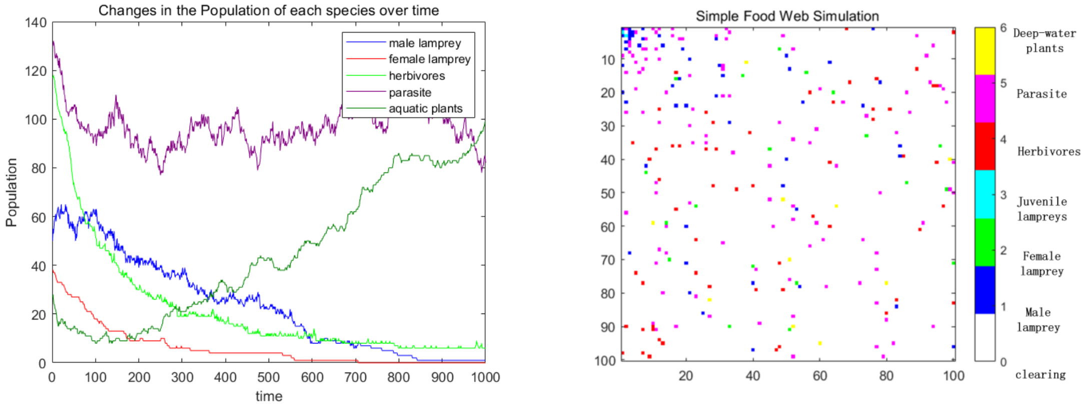


Figure 9: Simulating the biosphere.

A plot of the number of various populations over time and a simulation of a food web meta cellular automaton at a given point in time are shown in Figure 10.

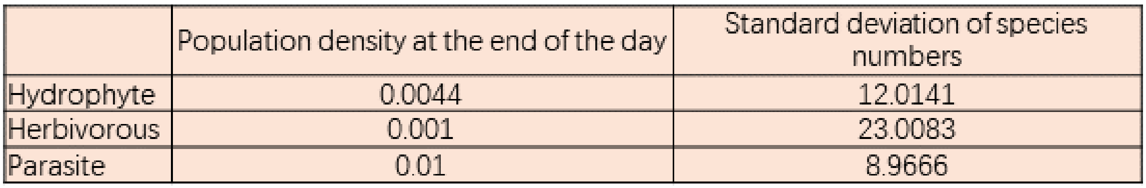


(a) Changes in population numbers (b) Cellular automata simulated at some point

Figure 10: Cellular automaton simulation

Cellular automaton (CA) is a discrete spatial and temporal computational model that can be used to simulate food web models.

There are many indicators to evaluate the strengths and weaknesses of species, and here we select the population density and the standard deviation of species number at the final moment. As shown in figure 11.



	Population density at the end of the day	Standard deviation of species numbers
Hydrophyte	0.0044	12.0141
Herbivorous	0.001	23.0083
Parasite	0.01	8.9666

Figure 11: Charts of evaluation indicators

The traditional TOPSIS model [8-10] scores each scheme based on the distance between the scheme and the positive ideal scheme and the negative ideal scheme, but the weight matrix is artificially defined, which is highly subjective, and the entropy weight method is an objective weighting method, which determines the index weight according to the impact of the relative degree of change on the whole by calculating the information entropy of the index.

The steps are as follows:

a) Data positiveization.

In terms of population density, we want it to be as large as possible in the carrying range of the environment.

In terms of species stability, the smaller the change in the number of species, the better, and the standard deviation of the number of species is positive.

$$X_i = \max(X) - X_i \tag{22}$$

b) Normalization of forward matrices.

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \tag{23}$$

c) Calculate the probability matrix.

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}} \tag{24}$$

d) Calculate the information entropy of each metric.

$$e_j = -\frac{1}{\ln(n)} * \sum_{i=1}^{n} \left(p_{ij} \ln(p_{ij})\right) \tag{25}$$

e) Calculate the information utility value.

$$d_j = 1 - e_j \tag{26}$$

f) Calculate entropy weights.

$$w_j = \frac{d_j}{\sum_{j}^{m} d_j} \tag{27}$$

g) Calculate the distance to the positive and negative ideal solutions.

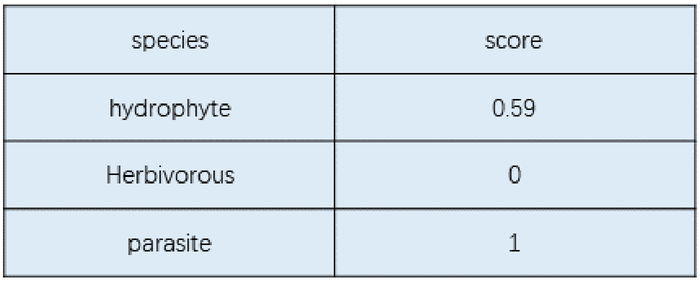
$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j * (z_{j\max} - z_{ij})^2} \tag{28}$$

$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j * (z_{j\max} - z_{ij})^2} \tag{29}$$

h) Score.

$$f_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{30}$$

The result is as shown in figure 12:



species	score
hydrophyte	0.59
Herbivorous	0
parasite	1

Figure 12: Species topsis score scale.

When the score exceeds 0.5, it means that an ecosystem with a changing sex ratio of lampreys' population can provide an advantage to the species in the ecosystem. According to the topsis score table, an ecosystem with a changing sex ratio of lampreys' population can provide an advantage to aquatic plants and parasites, but it is obviously not friendly to herbivores.

4. Conclusions

Based on the above methods, the following conclusions were drawn from the multi-model analysis of the effects of lamprey sex ratio changes on ecosystems.

(1) Biological populations with adaptive sex ratio variation can lead to extreme population differences in other species, which may break the food chain, reduce biodiversity, and affect the survival of other populations, which may have a negative impact on the stability of ecosystems.

(2) When the availability of resources is low, biological populations with adaptive sex ratio variation can improve the stability of their own populations by changing their own sex ratio, but it will affect the stability of other populations, making the population of other populations fluctuate greatly.

(3) Biological populations with adaptive sex ratio variation can create advantages by changing their sex ratio to better adapt to the harsh environment, but it will increase the survival pressure of other populations and disrupt the balance of the ecosystem.

(4) While populations of organisms with the ability to adapt to the variation of the sex ratio give an advantage to the ecosystem, they can also give advantages to other species.

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