

Logistic and Lotka-Volterra based dynamic ecological modeling of Sea lamprey.

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Abstract. The parasitic sea lamprey contributed to the devastating collapse of native fish communities after invading the Great Lakes during the 1800s and early 1900s. Addressing this threat requires studying the environmental impacts of changes in sea lamprey populations. Sea lampreys become male or female depending on how quickly they grow during the larval stage, Sex affects the population status of Sea lampreys. Population status affects the environment. This paper introduces a comprehensive model, incorporating a resource consumption rate alongside Logistic growth and Lotka-Volterra competition frameworks, to elucidate the complex dynamics between Sea lampreys, their environment, and other species. Through this model, dubbed the Sea Lamprey Ecological Model, we deduce a significant increase in the male-to-female ratio under constrained environmental resources. Further analysis, integrating a sex ratio variable and human intervention factor into the Sea Lamprey Ecological Model, reveals a decrease in sex-quantity disparity, diminished competitive edge for counterparts, and heightened survival pressures for prey species. The paper culminates in a critical evaluation of the model's efficacy and areas for further research, highlighting its potential to inform sustainable management practices for mitigating the adverse biodiversity impacts of Sea lamprey invasion in the Great Lakes.

Keywords: Sea Lamprey, Lotka-Volterra Model, Logistic, Sex Ratio.

1. Introduction

Sexual differentiation in most species on Earth is usually determined by genetic mechanisms; however, the process of sex determination in some species may involve the interaction of genetic and environmental factors. The sex ratio of the sea lampreys is significantly influenced by external environmental conditions [1]. They mainly inhabit freshwater lakes, where they are a key component of the ecosystem. The rate of development of the larval stage of the life cycle is a key factor in sex determination, and this rate is constrained by nutrient availability. In nutrient-poor environments, larval growth rates slow down, resulting in a significant increase in the proportion of males, ranging from 56% to 78% [2].

The Logistic growth model, a classical model for describing the growth of populations of organisms under limited environmental conditions. The Lotka-Volterra model, the model of population dynamics describing a rise in the number of predators depending on the numbers of their prey is among the first attempts at mathematical clarification of the mechanisms controlling and enhancing species coexistence [3]. In 2021, competition among dolphins off the coast of Japan was analysed using the Lotka-Volterra model, and the analysis revealed that three dolphin species were in competition with each other, and that this competition may have a significant effect on their population dynamics [4]. In 2023, Bao et al. added a third species to the classic two-species Lotka-Volterra competition model and made any two species compete with each other and concluded that the system is an interaction system governed by both interspecific and intraspecific competition strengths [5].

Previous researchers have done less research on sea lamprey, in this paper based on the logistic growth model and the Lotka-Volterra competition model, with the introduction of a sex-ratio factor and a factor of human-controlled behaviour, this paper sheds lighter on the complex dynamics within the Sea lamprey's population, its habitat, and between other species.

2. Resource Consumption Rate Model

According to the method used by Kanaji Y et al. in their article, the competition coefficient is the ratio of the 0.75th power of the species' body weight, combined with the catch data in the study of Smith, C.D et al., the required data in our model were obtained and then the Sea lamprey population and its habitat were analyzed based on some reasonable assumptions.

For the purposes of this paper, the sea lamprey is considered to be in a predator position in the Great Lakes ecosystem in which it is found. According to the literature review, the more abundant the ecosystem resources, the higher the proportion of female sea lampreys. Therefore, when discussing the sex ratio, the amount of ecosystem resources is an important consideration. We assume that the amount of resource is and use to represent the rate of resource regeneration. Then we deduce that the rate of resource consumption should satisfy the following formula:

$$\frac{dR_e}{dt} = \gamma R_e - C_m N_m - C_f N_f \quad (1)$$

In formula (1), t represents time, $N_{m,f}$ represent the population of male and female sea lampreys respectively, and $C_{m,f}$ represent the consumption rate of male and female sea lampreys in their respective environment. As the energy required for the development of female and male gonads is higher, and the energy investment for producing oocytes is usually greater than that of sperm cells [8], so it can be considered to satisfy the formula as follows,

$$C_m < C_f \quad (2)$$

3. Male and female lamprey growth model

3.1. The establishment of model

Based on the above model analysis, it shows that females are in a relatively weak position in viability. When ecosystem resources are insufficient, such as habitat and food shortage, more larvae choose to become males when the gonads are differentiated, thus strengthening their competitiveness within the population.

Firstly, male and female lampreys were treated as two separate populations. We then introduce the Logistic growth model to describe the male and female growth models respectively. This article derives:

$$\frac{dN}{dt} = rN \quad (3)$$

In the above formula, N is the population number, r is a fixed growth rate. But this model assumes that the growth of lamp fish populations is not subject to any restrictions, therefore an environmental variable K was introduced to modify this model to enable it to describe the situation where lamprey population growth is limited by environmental capacity, yielding a logarithmic growth model as follows:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) \quad (4)$$

N / K : population density, the portion of population to environmental capacity. As the population density increases to 1, the population growth rate decreases to 0, the population density continues to increase to exceed 1, and the population growth rate becomes negative. In fact, females and males are competitive within the population, so this article introduces the Lotka-Volterra competition model, and formula (4) is modified to reflect the competition among the sexes, and the male growth model is obtained as follows:

$$\frac{dN_m}{dt} = r_m N_m \left(1 - \frac{N_m + \alpha_{fm} N_f}{K} \right) \quad (5)$$

Similarly, the female growth model:

$$\frac{dN_f}{dt} = r_f N_f \left(1 - \frac{N_f + \alpha_{mf} N_m}{K} \right) \quad (6)$$

$r_{m,f}$ represent the growth rate of male and female sea lampreys respectively, describing the growth rate of sea lamprey population over time under ideal conditions (i. e., sufficient resources, no other limiting factors), which will be affected by many factors such as the food supply, disease and predation pressure. In this paper, its growth rate is regarded as consistent to simplify the model:

$$r_f = r_m = r \quad (7)$$

α is the ratio describing the competitive pressure between males and females. For example, α_{fm} acts as a negative feedback term in the male growth model, representing the degree of female inhibition of male growth. In this model, females survive consume more energy and survive less, so the following equation can be obtained:

$$\alpha_{fm} < \alpha_{mf} \quad (8)$$

Formula (8) indicates that female inhibition of male growth is weaker than male to female growth suppression.

Furthermore, the article considers that the amounts of available resources can also have an impact on the population growth. When there are fewer resources, the growth of the lampreys will be significant. Further the following equation is obtained:

$$\begin{cases} \frac{dN_f}{dt} = r N_f \left(1 - \frac{N_f + \alpha_{mf} N_m}{K} \right) P N_f R_e \\ \frac{dN_m}{dt} = r N_m \left(1 - \frac{N_m + \alpha_{fm} N_f}{K} \right) P N_m R_e \end{cases} \quad (9)$$

P : The resource consumption efficiency of sea lampreys

R_e : The amounts of resources available to the ecosystem

By solving the above differential equations, the growth rates for males and females can be found separately.

3.2. Model checking and visual analytics.

Based on the resource consumption rate model and the male and female lamprey growth models described above, this article conducts simulations to solve the models. The initial number of bars for males and females is set to 56 and 44, respectively, totaling 100. In the case of insufficient environmental resources, the simulation predicts the change trend of total population and the sex ratio in 40 units time, as shown in the figure below:

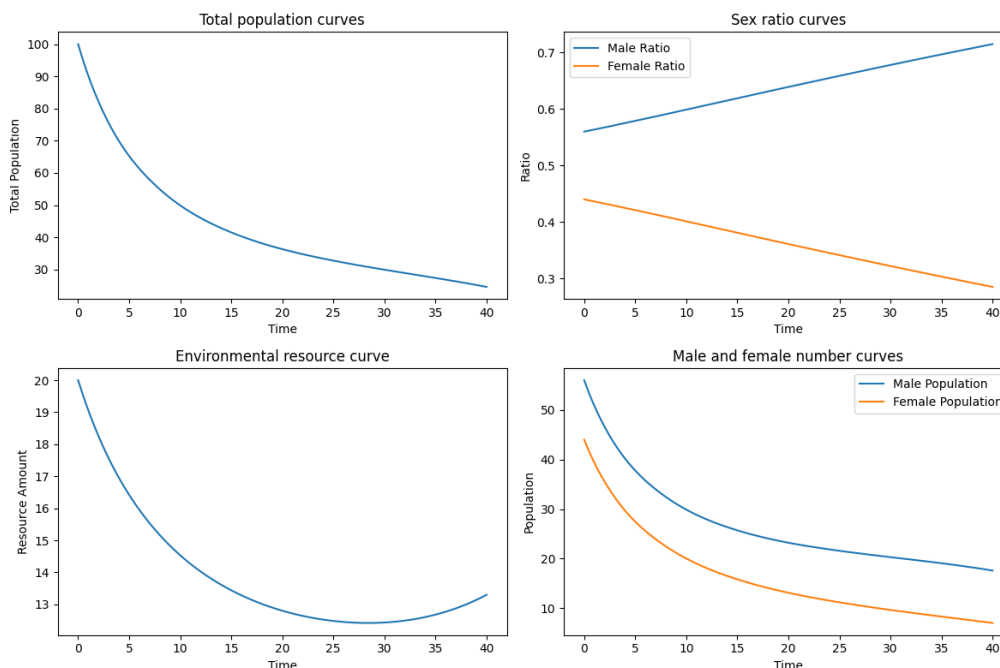


Figure 1: Simulation curve of population size and sex ratio change

According to the Figure 1, we can draw the following conclusions, when the total environmental resources are insufficient, but the number of sea lamprey is large, the number of sea lampreys will gradually decrease, and the rate of decline will slow down. In the case of inadequate environmental resources, the proportion of males increases significantly. It is acceptable that the total environmental resources will recover around the 28th time node. Both male and female numbers tend to decline, but they tend more slowly for males.

To further characterize the relationship between the sex ratio and the number of sexes, having created the following figure:

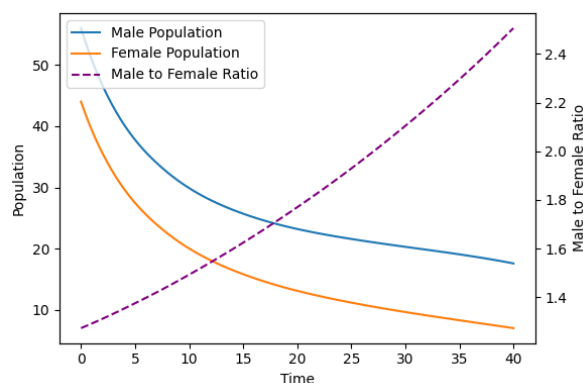


Figure 2: Number and sex ratio of hermaphroditic populations

According to the Figure 2, when the amounts of environmental resources is insufficient and the population size declines significantly, the ratio of male to female population will increase significantly, from 1.2 initially to 2.5. This is generally consistent with the actual situation and is considered acceptable.

3.3. Model extension

According to the Lotka-Volterra model, this article has constructed the differential equations of population changes for the species of the sea lampreys, competitors and predators.

The population model of the sea lamprey is as follows:

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha_{21} N_2}{K_1} \right) \delta - \beta_1 H_1 N_1 \quad (10)$$

δ shows the sex ratio in sea lampreys, values equal to the number of females divided by the number of males. The closer the sex ratio is to 1:1, the greater the value of δ , and the faster the number of sea lampreys increases.

The population model of competitors is as follows:

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \alpha_{12} N_1}{K_2} \right) - \beta_2 H_2 N_2 \quad (11)$$

$\beta_{1,2}$ in formula (10) and formula (11) respectively represent the influence coefficient of human on the control behavior of sea lamprey and competitors, and $H_{1,2}$ respectively represent the influence of control behavior. The control behavior can be regarded as the prey behavior of the predator to a certain extent.

The population model of prey is as follows:

$$\frac{dN_3}{dt} = r_3 N_3 - \varepsilon_1 N_1 N_3 - \varepsilon_2 N_2 N_3 \quad (12)$$

$\varepsilon_{1,2}$ respectively show the capture rate of prey by sea lampreys and competitors, then $\varepsilon_1 N_1 N_3$ and $\varepsilon_2 N_2 N_3$ reflect the effect of sea lampreys and competitors on the change of prey number, respectively.

3.4. Results

Combining the three-differential formula (10)(11)(12), the article simulates and analyze the sea lampreys, competitor and prey. The initial values of the sea lampreys and the competitor are set to 100, and the initial value of the prey is set to 200. By changing the values of the competitor and prey, by simulating the relationship between the three populations over time when the sex ratios of sea lampreys are different, the results of the simulation are shown below:

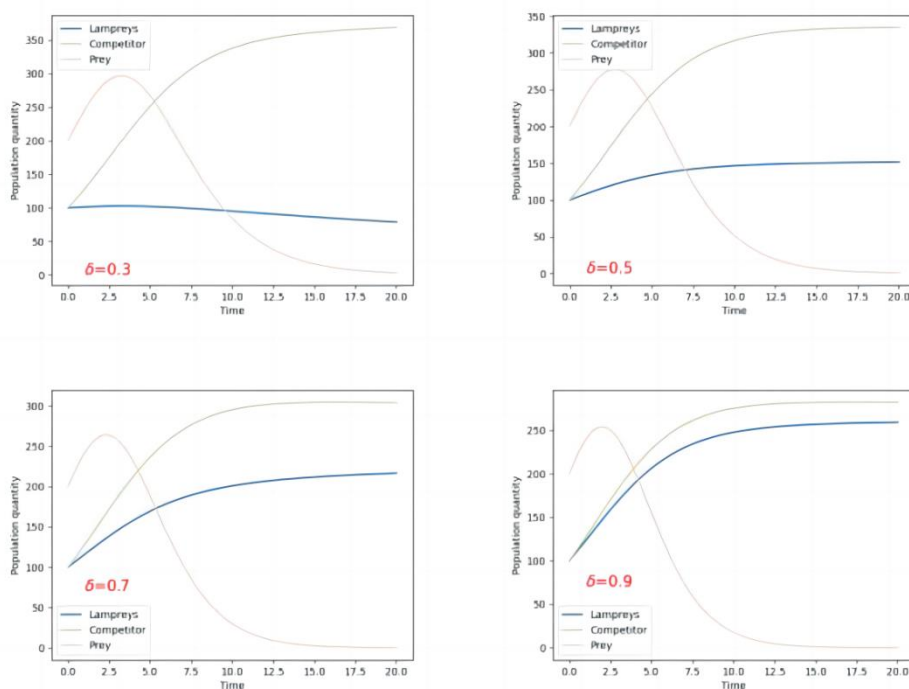


Figure 3: Changes in population size of various groups when the sex ratio of sea lampreys changed.

Analyzing Figure 3 the article can get the following conclusions: As the sex ratio of sea lampreys approaches 1:1 with increasing size, the population will grow faster in the presence of relatively abundant prey. This is consistent with our prediction that the closer to 1, the higher the reproductive rate of the population. As getting closer to 1, the peak in prey numbers decreases and occurs earlier, also dropping to 0 earlier. This is also consistent with our prediction that as the sea lamprey population gets larger, there will be more predators feeding on prey. With enlargement, the population of competitors grows less in the same amount of time, as shown in Figure 4. The article hypothesizes that this is due to the change in the sex ratio, which leads to an increase in the population size of sea lampreys, and it should be reasonable to assume that under conditions of limited food resources, the competitors' advantage over sea lampreys becomes progressively less significant.

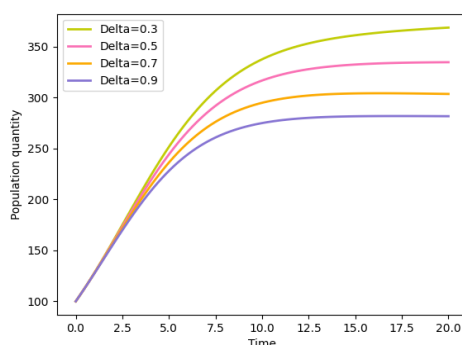


Figure 4: Competitor population size in relation to δ

4. Conclusions

This paper introduces the Sea Lamprey Ecological Model, which combines resource consumption rate with Logistic growth and Lotka-Volterra competition frameworks. The model aims at understanding the complex relationships between Sea lampreys, their environment, and other species. Analysis of the model suggests that limited environmental resources lead to an increase in the male-to-female ratio. Integration of a sex ratio variable and human intervention factor further reveals a decrease in sex-quantity disparity, resulting in diminished competitive advantages for competitors and increased survival pressures for prey species. Finally, the paper evaluates the model's effectiveness and identifies areas for further research, emphasizing its potential to inform sustainable management practices aimed at mitigating the negative biodiversity impacts caused by Sea lamprey invasion in the Great Lakes.

Currently, the model employed in this article is not yet comprehensive enough. It assumes a homogeneous environment and may not accurately depict ecosystems with intricate spatial structures. Consequently, in future research, it is hoped to incorporate more complex biological mechanisms such as the age structure, genetic factors, behavioral strategies, among others, while also considering spatial heterogeneity. This will enable the model to better align with real-world environments.

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