Effects Of Changes in Adaptive Sex Ratios on Biome Dynamics: Based on Lotka-Volterra Modeling

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Abstract. This study presents a dynamic model to elucidate the complex interactions between environmental factors and sex ratio changes in a population of sevengill eels. The model integrates a logistic function-based approach to elucidate changes in sex ratios as influenced by food availability. In addition, the Lotka-Volterra competition model incorporates sex ratios to reveal their effects on population dynamics. The simulation results highlight the dynamic relationship between sex ratios, prey abundance and population dynamics. Comparative analyses highlight the advantages and disadvantages of varying sex ratios in sevengill eel populations, revealing their competitive advantages as well as recovery dynamics.

Keywords: Lampreys, Sex ratio dynamics, Population ecology.

1. Introduction

Understanding the dynamics of biological populations is crucial in ecology, particularly in comprehending how species respond and adapt to their changing environments. Lampreys, a fascinating group of vertebrates, exhibit adaptive sex ratio changes in response to environmental cues, such as food availability. In this study, we delve into modeling these dynamic changes in lamprey populations, focusing on the interplay between sex ratio adjustments and ecological factors. Specifically, we develop and analyze models that incorporate gender proportion changes into population dynamics, shedding light on the intricate relationships between gender ratios, prey abundance, and competitive interactions within ecosystems [1-2].

2. Model of dynamic changes in biome

2.1. Sex ratio change model based on Logistic function.

Lampreys exhibit adaptive sex ratio changes in response to their environment, e.g., lampreys become males or females depending on their growth rate in the larval stage, and the growth rate of these larvae is influenced by food availability [3].

The sex ratio change model we developed considers only food availability in the environment when determining the factors that influence sex ratio.

Food availability refers to the relative ease with which an individual organism can obtain the resources needed for food in a given environment. Food availability is affected by the abundance and availability of food in the environment, as well as by the adaptations that individuals have made to obtain food.

Many factors affect food availability, with the amount of food being the most important. In order to simplify the model, here we only consider the effect of food quantity on sex ratio change [4].

2.1.1 The probability of larvae developing into different sexes varies as the amount of food supplied.

We first established the correspondence between the probability of larvae developing into different sexes and the amount of food, and we described this relationship with a Sigmoid function:

$$P = \frac{1}{1+e^{k\cdot Y}} + \frac{1}{2} \tag{1}$$

Where P is the probability that a lamprey larva becomes a male, and accordingly its prob- ability of becoming a female is 1 −P, k is a parameter, and Y is the amount of food (food here refers to the prey of the lamprey alone) [5-6].

The functional relationship is plotted in figure 1.

According to the actual situation and supported by the data, the more the food supply, the higher the probability that the lamprey larvae become males, as shown in the figure, and the function we defined reflects this relationship well: the Sigmoid function maps the amount of food, Y, to (0, 1), which means that the probability of the lamprey larvae becoming males varies with the food supply.

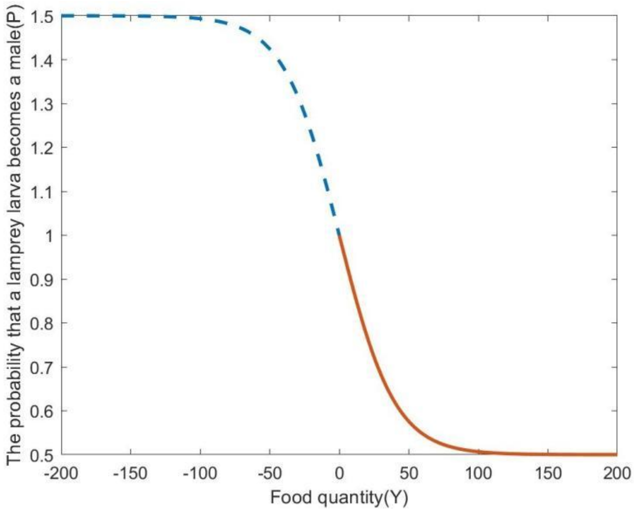

Figure 1: Probability of sex ratio change

2.1.2 The birth rate varies as the male sex ratio.

Next, we define the male sex ratio.

$$R_m = \frac{X_m}{X_m+X_f} = \frac{X_m}{X} \tag{2}$$

Among them, X_m Indicates the num$R_m = \frac{X_m}{X_m+X_f} = \frac{X_m}{X}$ber of male lampreys, X_f represents the number of females, X represents the total population of lampreys, and Rm represents the proportion of male individuals in the lamprey population at a certain moment.

According to relevant literature, if the gender ratio is too high or too low, the success rate of reproduction will decrease due to the inability of males to find females or females to find males. Therefore, when giving a natural birth rate b_X, we consider using an adjusted birth rate B_X, which may be as follows:

$$B_X = a \cdot R_m^2 + b \cdot R_m + c \tag{3}$$

Among them, a, b, and c are all parameters. According to the birth rate pattern discussed above, when R_m=1/2, i.e. the male to female ratio is 1: 1, we have an adjusted birth rate B_X

reaching maximum value b_X; When Rm = 0 or Rm = 1, the population is either all females or all males, and the adjusted birth rate BX = 0. Based on these rules, we calculate reasonable parameters, and the function becomes:

$$B_X = 4b_X \cdot (-R_m^2 + R_m) \tag{4}$$

The relationship between the adjusted birth rate BX and the male sex ratio Rm is shown in the figure 2:

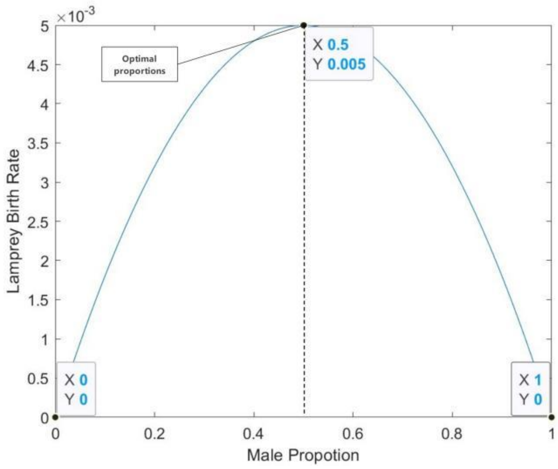

Figure 2: The quadratic function of birth rate

At this point, we have established a relationship between gender ratio changes and food supply. Given the gender ratio Rm and food quantity Y at a certain moment, the birth rate B_X is determined by the gender ratio, and the probability of larvae developing into males is determined by the amount of food supply. Therefore, the number of males and females also varies with the birth rate B_X and the probability of larvae developing into males P, which leads to changes in the sex ratio and further causes changes in the birth rate and larval development probability P, so we initially obtained a model that involves changes in gender ratio and food quantity [7-8].

2.2. A Lotka Volterra competition model combining gender proportional changes.

The Lotka Volterra model is a predator-prey model. We extended this model by incorporating factors of gender ratio changes and taking into account the competitive population of lampreys, establishing a multi species competition model.

In order to further investigate the impact of gender ratio changes on population dynamics of lampreys, we consider the prey quantity of lampreys as their food quantity. Therefore, we expect to use the Lotka Volterra predator-prey model.

Now let’s assume that there are two populations in this community: the lamprey (predator) and the prey of the lamprey, where the population of the lamprey is X and the prey population of the lamprey is denoted as Y. And in order to analyze the characteristics of gender ratio changes in the lamprey population more clearly and meticulously, we divided the lamprey population into male lamprey (population size Xm) and female lamprey (population size X_f).

Under our model assumption, the predation rate of male lampreys is slightly higher than that of females. This assumption is based on our finding that when food resources are scarce, the proportion of males will be higher, and we have reason to believe that males have a higher predation rate, which is beneficial for their population survival [9].

$$\frac{dX}{dt} = (B_X - d_X)\cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f = B_X \cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f - d_X \cdot X$$

$$\frac{dY}{dt} = r_Y \cdot Y - \gamma \cdot \left(\lambda_f \cdot Y \cdot X_f + \lambda_m \cdot Y \cdot X_m\right) \tag{5}$$

Where d_X is the natural mortality rate of the lamprey population. In this predator-prey model, we use the difference between birth rate and mortality rate ($B_X - d_X$) as the inherent growth rate of the lamprey population, while λm is the predation rate of male populations towards prey, λ_f is the predation rate of the female population towards the prey, r_Y is the inherent growth rate of

the prey, and the relationship coefficient between the predation rate of the prey and the predation rate of the predator is γ, where γ is greater than 1.

To consider gender changes, we established population dynamics equations for both male and female populations. Since gender change is determined by the probability of larvae becoming different genders, we have determined the following equation based on the number of newly added lampreys per unit time and the probability of gender transformation:

$$\frac{dX_m}{dt} = P\left(B_X \cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f\right) - d_X \cdot X_m$$

$$\frac{dX_f}{dt} = (1 - P) \cdot \left(B_X \cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f\right) - d_X \cdot X_f \tag{6}$$

Furthermore, in order to comprehensively consider the impact on the ecosystem, we also added competitors of lampreys (collectively referred to as other predators) and considered the dynamic changes of these three populations in the biological community. The final system

dynamics equation is as follows:

Among them, the predation effects of other predators on prey have been added, where μ is the predation rate of other predators on prey, d_Z is the natural reduction rate of other predators.

Through this model, we can simulate and comprehensively analyze changes in gender ratios and dynamic changes in populations in ecosystems.

$$\frac{dX}{dt} = \frac{dX_m}{dt} + \frac{dX_f}{dt}$$

$$\frac{dX_m}{dt} = P \cdot \left(B_X \cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f\right) - d_X \cdot X_m$$

$$\frac{dX_f}{dt} = (1 - P) \cdot \left(B_X \cdot X + \lambda_m \cdot Y \cdot X_m + \lambda_f \cdot Y \cdot X_f\right) - d_X \cdot X_f \tag{7}$$

$$\frac{dZ}{dt} = d_Z \cdot Z + \mu \cdot Y \cdot Z$$

$$\frac{dY}{dt} = r_Y \cdot Y - \gamma \cdot \left(\lambda_f \cdot Y \cdot X_f + \lambda_m \cdot Y \cdot X_m + \mu \cdot Y \cdot Z\right)$$

2.3. Analysis of the impact on ecosystems from the perspective of system dynamics

In order to obtain the simulation results of the above model, we use the fourth order Runge- Kutta method to solve the equation, which is an important iterative method for solving nonlinear ordinary differential equations, and its high accuracy is very suitable for solving the model [10].

2.3.1 Dynamic changes in gender ratio and prey quantity

We obtained the dynamic changes in the gender ratio of males R_m and food quantity over time as follows figure 3:

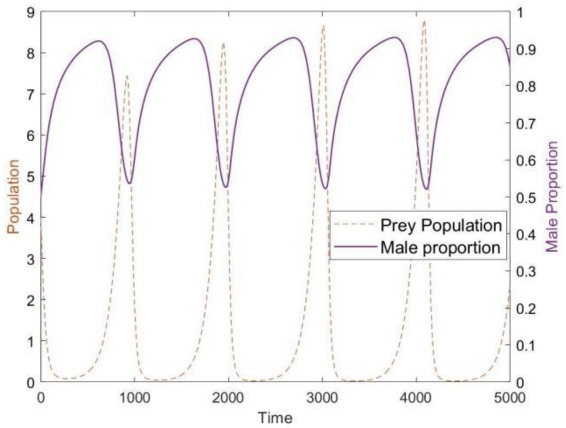

Figure 3: Dynamic changes in gender ratio and prey quantity

From the graph, we can see that when the amount of food is abundant, the proportion of male gender is relatively low, basically between 0.5 and 0.6; When there is a shortage of food, the male sex ratio is relatively high, basically between 0.8 and 0.92, which is consistent with the actual change in the sex ratio of lampreys with changes in food availability.

2.3.2 Dynamic relationship between multiple groups

We obtained the population dynamics of lampreys, competitor populations, and prey as follows:

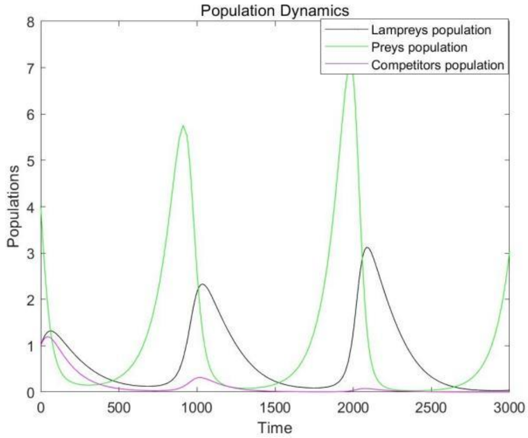

Figure 4: Dynamic changes of three populations

In the initial parameters we set in the figure 4, the predation rates of other predators are- tween those of male and female lampreys, indicating that the predation rates of other predators are similar to those of the lamprey population. In the initial state, the numbers of the two populations are consistent.

As shown in the figure, considering changes in gender ratio, the lamprey population wins in competition with other predators. In our hypothetical model, the two exhibit periodic changes, which is consistent with the classical predator-prey model; But at the same time, the overall growth trend of lampreys is on the rise, while the number of preys is slowly decreasing.

We further subdivide the population of lampreys into males and females and analyze their relationship with the number of preys:

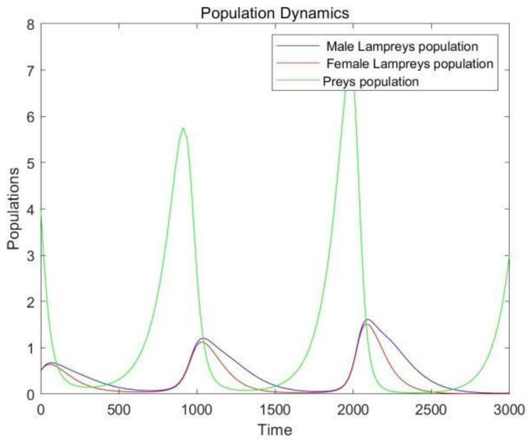

Figure 5: Dynamic changes in female and male populations

As shown in the figure 5, due to changes in gender ratio, the changes in the male and female populations of lampreys are not completely consistent. Except for their respective predatory roles, they both dynamically change with the number of praises, and the gender ratio also changes. This is due to the impact of prey quantity on gender ratio changes.

According to the analysis of the solution results, considering changes in gender ratio, the lamprey can dynamically adjust its gender ratio when the number of prey changes, which in turn affects the number of prey and also affects inter species competition.

2.4. Further research on the advantages and disadvantages of the population based on comparative methods.

Based on the model of the first question, we compared the gender ratio changes in the lam- prey population using a comparative method and made changes in the number of competing populations in two scenarios.

In the model of the first question, we discussed the model where there is a gender ratio change in the lamprey population. If we fix the gender ratio to 1:1, we obtain a model with- out any gender ratio change. We analyze the changes in the population caused by these two situations based on changes in the number of competitors, as shown in the figure below:

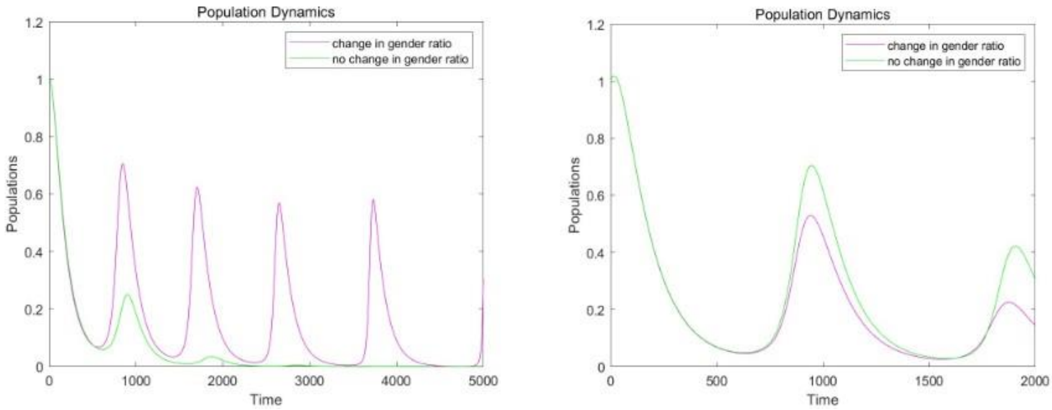

Figure 6: Comparison with and without changes in gender ratio

Advantage: As shown in the figure 6, the results indicate that the variable sex ratio of lam- preys gives them an advantage in population competition, accelerating the extinction rate of their competitors (other predators).

Analysis of the reasons: Based on the actual background and simulation results, we believe that when there is a change in gender ratio, the population of lampreys is more likely to develop into males in environments with less food, so the population can obtain a larger predation rate as a whole, thereby increasing its competitiveness against competitors, and ultimately benefiting the lamprey's population in a dominant position in population competition.

Since our Lotka-Volterra model, which combines gender ratio changes, assumes that food quantity is described as prey quantity, we now assume that the maximum environmental carrying capacity of the lamprey population under sufficient food is K. We use a logistic growth model to simulate the changes in lamprey population quantity after different shocks (different initial sex ratios) and compare the time to recover to steady state.

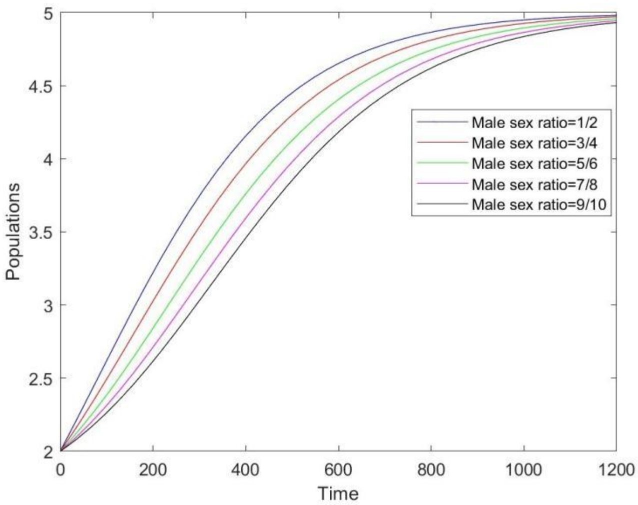

Figure 7: Comparison with and without changes in gender ratio

Disadvantage: As shown in the figure 7, in the case of sufficient food, if the prey quantity changes due to external shocks, it leads to a change in the initial sex ratio. The time it takes for the lamprey population to recover to a stable state varies under different sex ratios. Specifically, the larger the initial male ratio, the longer it takes to recover to a stable state.

Analysis of the reasons: Based on the actual background and simulation results, we believe that when the gender ratio is variable, the adaptability of different gender ratios to the environment is different, and their variable ability further intensifies the fluctuation degree of their interaction with prey quantity. This dynamic change requires longer time and resources for the population to reach balance.

3. Conclusions

In this study, we have explored dynamic changes in lamprey populations, considering the interplay between sex ratio adjustments and ecological factors. Through the development of models that integrate gender proportion changes into population dynamics, we have gained valuable insights into how lampreys adapt to varying environmental conditions and interact with other species within ecosystems. Our findings highlight the significance of considering sex ratio dynamics in understanding population dynamics and ecological interactions. Furthermore, our comparative analysis reveals both advantages and disadvantages associated with variable sex ratios in lamprey populations, emphasizing the importance of considering gender dynamics in ecological studies and conservation efforts. Moving forward, these models provide a foundation for further research on the intricate dynamics of biological populations and their implications for ecosystem functioning and stability.

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