Topology Optimization of Quadruped Robot Limbs based on Buckling Analysis

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Abstract. Quadruped robots, recognized for their stability and simplicity, are at the forefront of robotics research. A key to their functionality lies in the optimization of their limbs, which significantly influences their operational stability and efficiency. This paper utilizes buckling analysis with the Euler column formula to optimize the topology of limbs in order to improve the effectiveness. Through rigorous computation, this method made it possible to determine the ideal thigh-to-calf length ratios and articulation angles—two factors that are essential for optimizing the robot's load-bearing efficiency and operational stability. The optimal numerical range was then simulated using finite element analysis, providing the best structure based on yield stress and displacement magnitude. V-REP simulations confirmed the practical performance and adaptability in dynamic environments. This research found that the robot performs optimally when its thigh-to-calf length ratio is roughly 1.05 and its articulation angles are between 30° and 45°. The most effective limb designs showed that they could sustain loads up to twice their own while maintaining structural integrity. This progress in limb optimization not only represents a major improvement in the functionality and design of quadruped robots, but it also expands their range of applications, increasing their suitability for search and rescue operations.

Keywords: Quadruped Robot limbs, Topology structure optimization, Euler's Critical Load, Finite Element Analysis.

1. Introduction

Mobile robots are increasingly emerging as significant with the prosperity of technology. Mobile robots could currently be sorted out as tracked, wheeled, or footed robots [1]. In contrast to other types of mobile robots, footed robots, characterized by multi-limbed, multi-degree-of-freedom, and high fault-tolerant features, have grown into an intersection for research [2]. Especially quadruped robots, which are more in line with current industrial requirements because of the good stability, dynamic properties and the mechanism structural brevity [3]. According to prominent Japanese quadruped robot specialist and scholar Shigeo Hirose, quadruped robots are considered the optimal form of footed robots from various perspectives. The development of quadruped robots such as a robotic turtle ART4 and an autonomously performing several tasks robot ANYmal has been completed, playing a crucial role in a variety of industries, including industrial production and rescue operations [4, 5].

After decades of endeavor, quadruped robot mechanical limbs have made significant progress because of the simulation of animals. As a key component of quadruped robots, the design of the limbs determines the core application performance including walking speed and load capacity. Due to their varying degrees of freedom, quadruped robot limbs can be divided into several varieties: prismatic, articulated, and redundant articulated legs [6]. These groupings are used to adapt the robots to various terrains and each design has unique advantages. Experts and entrepreneurs are optimizing the structure of robot limbs through various approaches, focusing on compliant limbs, wheel-leg hybrid drive systems, and the development of 2 DOF articulated trunks [6-8].

Amongst all optimization approaches, topology optimization is the most extensively utilized [2]. With this technique, material distribution is mathematically optimized within a predetermined design region to improve system performance while satisfying certain requirements and boundary restrictions [9]. Researchers have concentrated on streamlining the limb structure and eliminating superfluous bulk in order to achieve lightweight and highly efficient limb designs [10]. To optimize the structure, a central slider-crank mechanism was added to the Wildcat [11]. Atlas, a robot with highly agile locomotion, integrate several channels and actuators into a single component [12]. Although most of the cases show satisfactory results in structural optimization [10]. There are limitation and complexity. Few topology optimization treatments by experts were performed on the simple limb structure itself.

The content of the article concentrates on applying buckling analysis for topology optimization the structure of quadruped robot limbs. Unlike traditional approaches that primarily focus on material minimization, the article systematically adjusts limb configurations for optimal limb design [9]. Kinematic modeling and finite element analysis are applied to test the effectiveness of design strategies in the subsequent section. The objective is to discover limb configurations that improve the stability and load-bearing capacity of quadruped robots. The proposed design provides a scientific foundation for further development of quadruped robots.

2. Overall Structure Design

Research reveal that naturally occurring quadrupeds—like horses and dogs—display exceptional stability in addition to higher load-bearing capacities. Integrating the biomechanical insights with the fundamentals of robotics engineering and topology optimization is useful [13].

2.1. Structure of the Robot

The quadruped robot adopts a simpler 12-degree-of-freedom design, which is relatively easy to control and analysis [13]. The front and rear limbs of this quadruped robot should be symmetrically identical to enhance the robot's locomotion and balance. The simplified structure topology of the quadruped robot design is shown below in Figure 1.

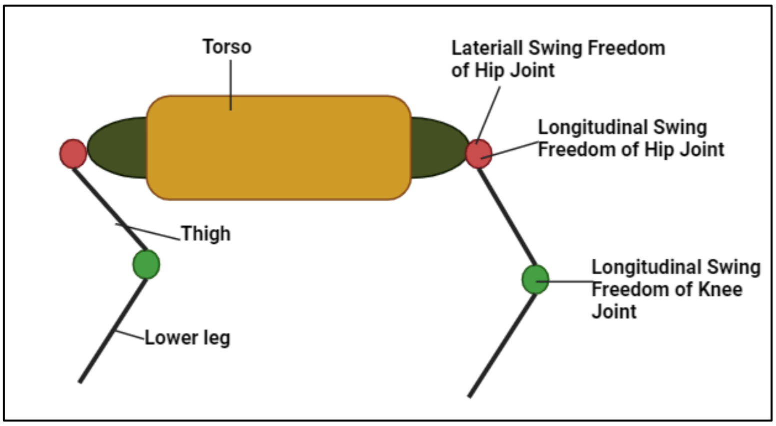

Fig. 1 Structure topology of the design (Photo credited: Original)

This quadruped robot has articulated limb with rotary joints, which has good bionic characteristics [6]. Moreover, the topology of the articulated limb is well suited for subsequent dynamics analysis and buckling optimization of the limb configuration.

2.2. Performance Requirements

A variety of technical specifications and limitations were found to be necessary for the quadruped robot's limb optimization to be successful:

1) The optimal limb design and allocation is used to optimize the quadrupedal robot's load capacity and work efficiency. The quadruped robot in this paper seeks to be of practical use in transporting materials while being lightweight.

2) Achieving the ideal contact angle between the quadruped robot's lower limbs and the ground, as well as observing an increase in the robot's overall efficiency and capacity to transport loads.

3) To minimize Euler's critical load and the buckling of the quadruped limb unit in the same circumstances.

2.3. Limb Design

The limb section of a quadruped robot is the most important component and must be made to be both robust and lightweight. To satisfy the 2.2 performance requirements as well as the ensuing dynamic analysis, the quadruped robot's limb assembly shown in Figure 2 should be articulated and divided into two simple parts.

The single limb assembly consists of different specific parts of the limb unit: hip joint, knee joint, thigh and lower leg. The axial force is transmitted by a bearing during the forward movement of the thigh hip joint. Additionally, a basic mechanical structure connects the knee joint, making the force analysis that follows easier to comprehend. To distribute the force, the ground and the bottom of the leg are rounded.

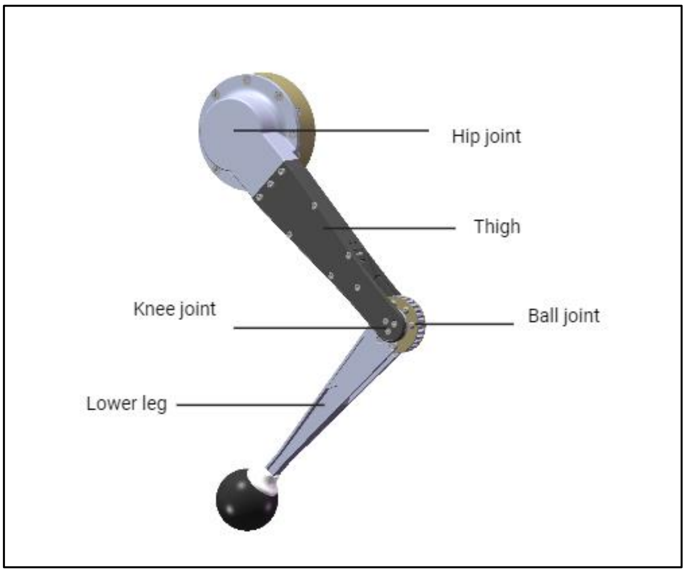

Fig. 2 Single limb assembly (Photo credited: Original)

The whole structure of the leg limb unit based on an animate skeleton is simplified into the left-hand side model in Figure 3. The connection point between a hip joint actuator and a thigh component is shown in Part A, while Part B represents an actuator connecting the thigh part and the calf part. Point D serves as the point of contact between the quadruped robot and the ground.

Other complex mechanical structures are simplified in the topology model for subsequent modeling and analysis. $l_1$and l_2 represent the length of thighs and calves respectively. Angles θ_1 and θ_2 represent the angle where the thigh and calf bend and the angle between the calf and the ground when the force inflicting, respectively.

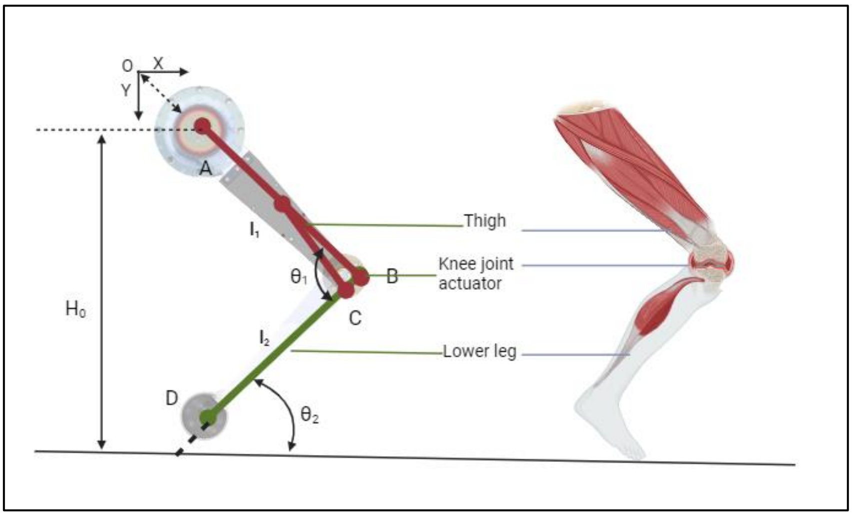

Fig. 3 Simplified model of the limb unit based on a skeleton (Photo credited: Original)

3. Topological Optimization of Structural Parts

The MIT Cheetah is a series of agile, high-speed robotic quadrupeds designed for dynamic movements and modularity [14]. I have taken the aforementioned open-source quadruped robot as the basis for my research and made improvements upon the limb structure, which was shown in the Figure 4.

The design method for topology optimization includes a standardized workflow [7]. Initially the design challenge for the robot's limbs must be defined. This includes appropriate loading conditions, force and buckling analysis, and multiple design objectives. Subsequently, each iteration of the 3D design domain is constructed using finite element analysis (FEA) to compute limb deformations. Based on FEA results, sensitivity analysis directs repeated changes via an update mechanism, which ends with the simulation of the optimized design. The following image shows a preliminary version of the quadruped robot model.

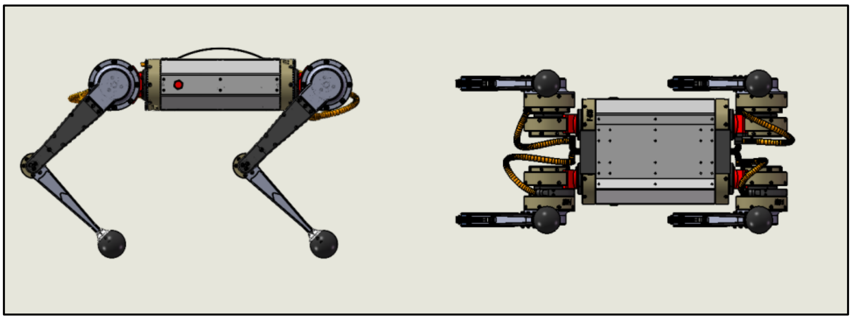

Fig. 4 The simplified structure of the quadruped robot (Photo credited: Original)

3.1. Formulation of the Buckling Analysis for Quadruped Robot Limb

In this research, we have utilized the fundamental knowledge of engineering mechanics to analyze the force and momentum on each limb. Figure 5 illustrated that the simplified model of the limb unit can be regarded as articulated legs topology. Under this circumstance, the quadruped robot limbs could be analogized to the column buckling to calculate and analyze.

Long compression columns would collapse due to buckling before the column's yield strength is reached, as the idea of buckling illustrates [15]. Therefore, we aim to increase the load-bearing capacity by the column buckling analysis.

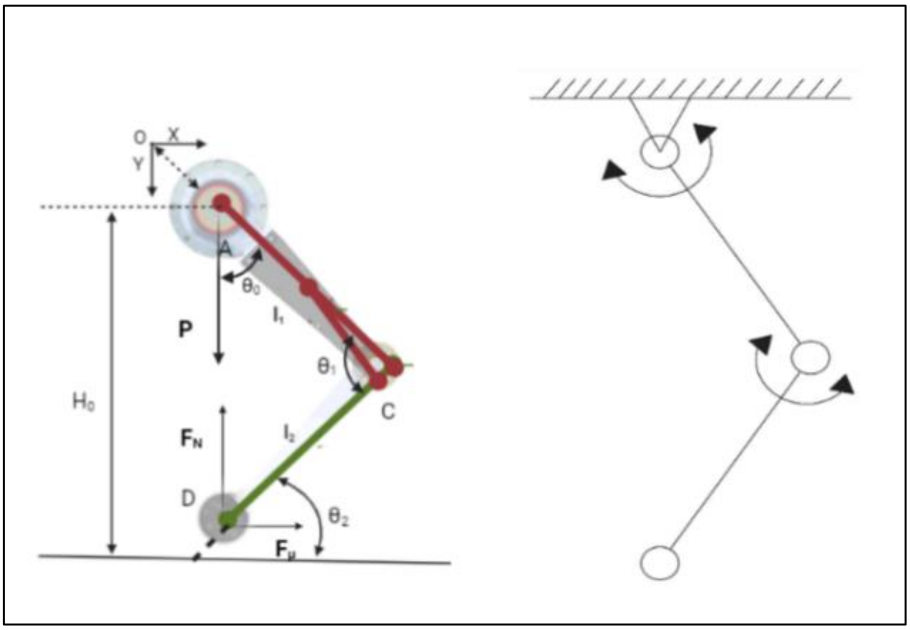

Fig. 5 Force diagram for the quadruped robot contacting the ground and articulated legs topology (Photo credited: Original)

The Euler column formula can be used to check if a long column that has a load applied along its center direction has buckled:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(K\frac{L}{r})^2} \tag{1}$$

Where σ_{cr} and P_{cr} represents the critical stress and critical force respectively. L and r represent the unsupported length limbs and the radius of gyration of the cross section respectively. E represents the elastic modulu. K represents the effective length factor, which is affected by the end conditions of the column [16].

Based on the Figure 5, the end condition of the thigh is analogized to the Pinned-Pinned condition and the calf part should be analogized to Pinned-Free. Therefore, the recommended effective length factor is 1 and 2.1 respectively, from the knowledge of engineering mechanics [16]. After substituting K into equation, the critical stresses are calculated as:

$$Thigh\colon\ P_{cr} = \frac{\pi^2 EI}{{l_1}^2} \tag{2}$$

$$Calf\colon\ P_{cr} = \frac{\pi^2 EI}{4.41 * {l_2}^2} \tag{3}$$

Accordingly, based on the equation given and the information on Figure 5, the design problem and topology optimization restrictions can be mathematically formulated as:

$$\begin{cases} Psin\theta_0 = \frac{\pi^2 EI}{{l_1}^2} \\ Pcos\theta_2 = \frac{\pi^2 EI}{4.41 * {l_2}^2} \\ l_1 cos\theta_2 + l_2 sin\theta_0 = H_0 \end{cases} \tag{4}$$

3.2. Calculation and Analysis

Given the foundational analysis presented in *Section 3.1*, where the buckling analysis for quadruped robot limbs was formulated, the intricate relationships between the lengths of the limbs (l_1, l_2), articulation angles (θ_0, θ_2), and their collective impact on the load capacity could be figured out.

The analysis is based on the idea that optimizing P, which improves the robot's stability, requires reaching equilibrium in force distribution. The quadruped robot is made of conventional aluminum alloy 7075-T6, which is suitable for the robot manufacture and operational dynamics [17]. Therefore, the Young's modulus of the material and the specific moment of inertia can be obtained, and H_0 can also be known according to the actual situation of the quadruped robot.

The calculation shows an optimized limb length configuration where l_1 is proportionally longer than l_2 and the ratio is usually between 1.05 and 1.2.

The articulation angles, $\theta_0$and θ_2, are considered to play a significant role in this optimization, with an ideal range likely falling between 30° and 45°. This range is posited to balance structural integrity with dynamic mobility effectively.

The Figure 6 below shows the critical force as a function of column length for the aluminum alloy 7075-T6, which adopts a pinned-free end condition with the calf as the background. When the calf length is 15 inches (38 cm), calculations reveal that the Factor of Safety on stress and on applied force exceeds 5, indicating a very safe quadruped robotic device.

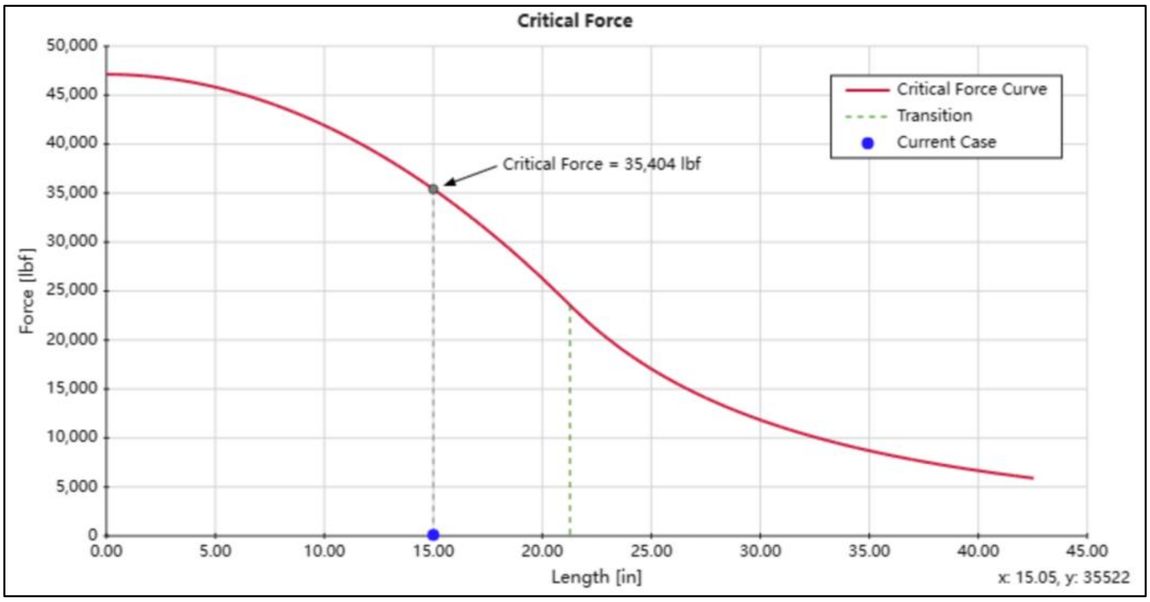

Fig. 6 Critical force plot (Photo credited: Original)

4. Simulation based on Finite Element Analysis and MATLAB

Based on the calculation and topological optimization of the quadruped robot limbs, the appropriate length configuration is confirmed. The length of thigh is proportionally longer than lower leg and the ratio is usually between 1.05 and 1.2. These theoretical insights and hypothesized configurations set the stage for empirical validation in this chapter, which will engage finite element analysis (FEA) and simulation to test the proposed models against real-world operational stresses [18].

4.1. Finite Element Analysis Simulation

The quadruped robot's initial iteration had a length ratio of 1.22 due to its 210mm thigh and 171mm calf lengths (Figure 2). Consequently, in subsequent optimization phases, we continuously refined the proportion between the lengths of the thighs and calves. Through finite element analysis, we sought to identify the optimal ratio to achieve the best transportation ratio for buckling analysis [19].

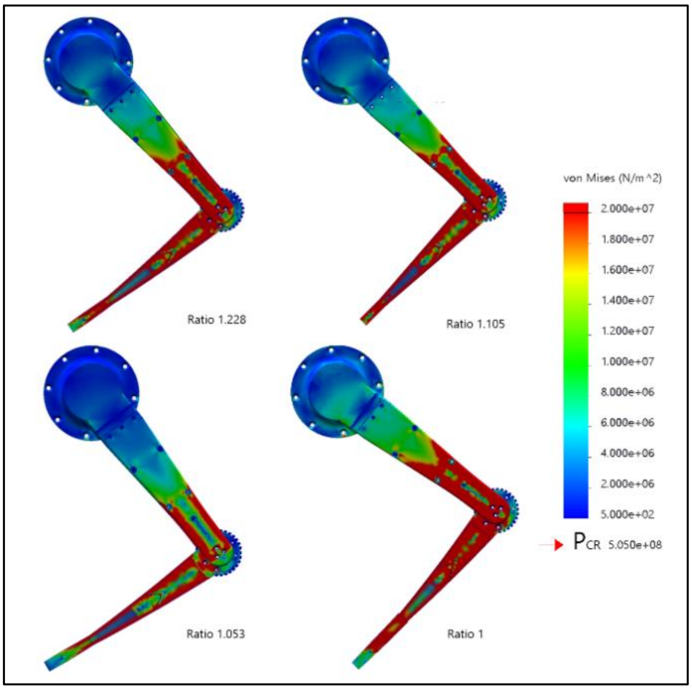

Fig. 7 Finite element analysis of the yield force (Photo credited: Original)

A finite element study of the limbs of quadruped robots is shown in the Figure 7. To ensure that the ground contact surface remains fixed while the limbs bend, constraints were applied to the force points located at the bottom of the analysis. The quadruped robot's load and friction were simulated in all four photographs by applying 500N of downward pressure perpendicular to the ground and 300N of force perpendicular to the direction of the calf beam. The force angle between the lower leg and the ground is fixed to 40° within the optimal range. The images' colors show the force's magnitude, changing from blue to red to symbolize rising force. This helps to graphically represent the topological optimization outcomes of various scenarios.

Four representative sets of thigh-to-leg ratios were selected from a series of possible ratios, and their significant impact on critical force was analyzed under identical environmental conditions. The respective data for each scenario are as follows:

$$\begin{cases} ①: Thigh: 210mm \quad Calf: 171mm \quad Ratio: 1.228 \quad yield\ force_{max}: 405922\ {}^{N}/_{m^2} \\ ②: Thigh: 210mm \quad Calf: 190mm \quad Ratio: 1.105 \quad yield\ force_{max}: 390334\ {}^{N}/_{m^2} \\ ③: Thigh: 200mm \quad Calf: 190mm \quad Ratio: 1.053 \quad yield\ force_{max}: 380334\ {}^{N}/_{m^2} \\ ④: Thigh: 190mm \quad Calf: 190mm \quad Ratio: 1.000 \quad yield\ force_{max}: 395372\ {}^{N}/_{m^2} \end{cases}$$

Based on the aforementioned four sets of data and FEA plots, it is evident that when the ratio between the thigh and calf approaches around 1.05, the yield stress is minimized, indicating the maximum load that can be borne on quadruped robots. The following Figure 8 illustrates another set of finite element analyses concerning buckling or displacement.

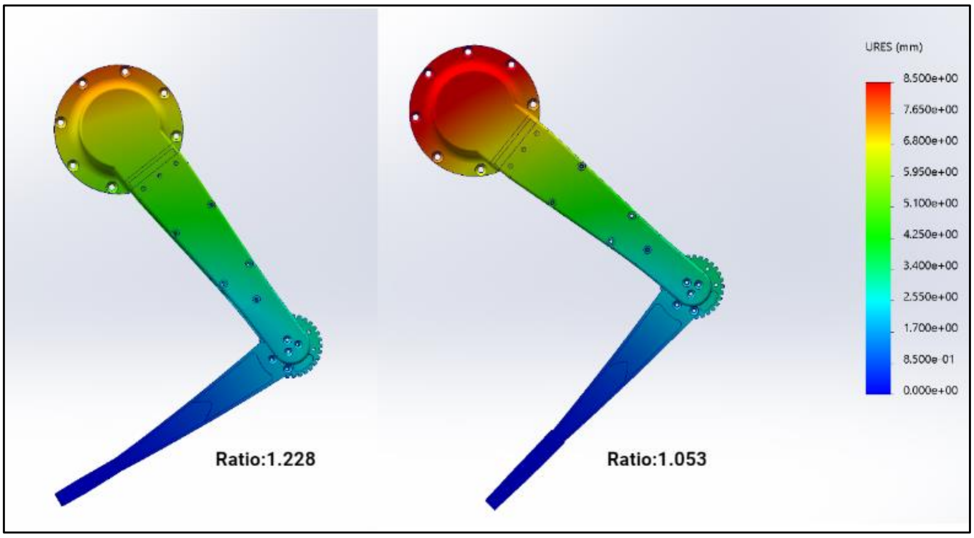

Fig. 8 Finite element analysis of the displacement (Photo credited: Original)

Force applied to the limb causes natural displacement and bending. The variations in color reveal significant displacement changes when comparing the best value with a ratio of 1.053 to the remaining values with a ratio of 1.228. Under pressure, the latter displays 0.145mm of displacement while the former merely displays 0.091mm. This is more evidence that a ratio of about 1.05. seems appropriate.

4.2. Analogue Simulation

The best topology optimization for the quadruped robot's limb section was further deduced using the previously mentioned calculations and organization. To validate the effectiveness of the optimized topology for the quadruped robot limbs, derived from the finite element buckling analysis, we utilized the Virtual Robot Experimentation Platform (V-REP). This environment is ideal for testing the physical properties of robot designs due to its robust physics engine [20]. The optimized limb structures were imported into V-REP as mesh models, with material properties assigned in accordance with those used in the finite element analysis [21].

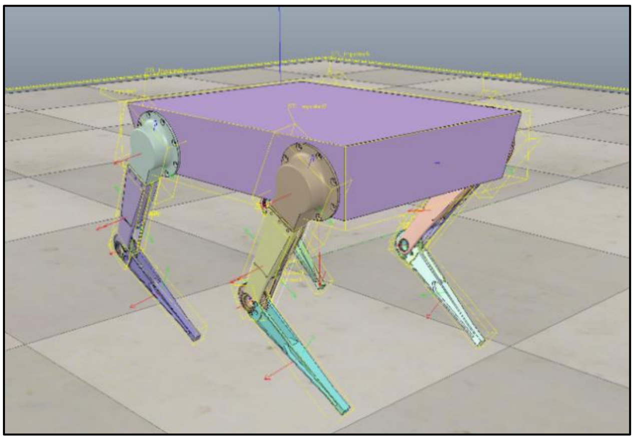

Fig. 9 Quadruped simplified model in V-REP application (Photo credited: Original)

Figure 9 demonstrates how the quadruped robot's limbs bend while it is under load. The definitive results from the V-REP simulations showed that there was no appreciable loss of structural integrity in the improved limbs' ability to withstand the applied loads. The fact that the robot can lift objects up to 80 kg, or twice its own weight, is proof positive that the topological optimization improved the structural performance [22].

Moreover, the limbs' ability to withstand such loads implies that the robot may be able to function in settings like search and rescue and logistics, which call for a high degree of strength and endurance [23].

This extended conclusion establishes the framework for the subsequent stages of the development process and links the simulation results to possible real-world applications.

5. Results and Discussion

Buckling analysis is used in this research to perform topological optimization of the quadruped robot's limb structure. Because structural beams and robotic limbs share similar load-bearing properties, comparing the two types of structures in buckling analysis makes conceptual sense. This parallel becomes especially relevant when one considers that the limbs must withstand dynamic stresses when the robot is in motion in addition to the robot's weight. Traditional beam theory may not properly account for extra elements like torque and lateral forces introduced by the complexity of robotic motion [24].

Due to calculating limitations for *Section 3.1*, the topological optimization procedure in this work required creative approaches to streamline the issue without sacrificing the accuracy of the outcomes. Through deliberate reduction of the model's complexity, we were able to concentrate on critical factors that impact the structural efficiency of the limbs. Finally, the proper results were got in *Section 3.2*. The topological optimization conducted in this paper is distinguished by its integration of buckling analysis to inform the design process. Unlike other optimizations, the emphasis is on material reduction [9]. This novel approach allowed for a more nuanced understanding of how the limbs would perform under various loading conditions, leading to designs that are both lightweight and capable of withstanding significant forces.

Finite Element Analysis and V-REP simulations complement each other in evaluating robotic limb designs. FEA offers in-depth insights into stress distributions under load, crucial for identifying potential failure points. In contrast, V-REP simulations assess dynamic performance, incorporating real-time physics to evaluate how limbs react to operational conditions. The synergy of these methods provides a holistic view of design viability, combining detailed material behavior analysis with practical, motion-based evaluation.

For quadruped robots and real quadrupedal animals, the relative lengths of the thighs and calves may differ due to their biological structures and modes of locomotion. In quadruped robots, it's common for the thighs to be longer than the calves in design. This is typically aimed at enhancing the stability and efficiency of movement. Longer thighs provide larger strides and better support, enabling the robot to move more stably and flexibly across various terrains. For example, developed by Boston Dynamics, robots like BigDog and AlphaDog exemplify this design approach [25].

On the other hand, in many quadruped animals such as dogs, the calves tend to be longer than the thighs. This structural configuration contributes to improving the animals' running speed and jumping ability. Longer calves provide greater leverage, allowing animals to propel their bodies more effectively. Additionally, the longer calves reduce the energy expenditure during running, as the extended leverage allows animals to utilize momentum more efficiently. Hence, while the proportions of thigh-to-calf lengths differ between robots and real organisms, they are both adapted to suit their specific modes of movement and biological needs.

6. Conclusion

The paper emphasizes the crucial application of the Euler column formula to improve load-bearing capacity and offers a novel method for topology optimization of quadruped robot limbs using buckling analysis. Through the incorporation of this fundamental engineering concept, the research develops limb configurations that are optimized for enhanced efficiency. Using finite element analysis and kinematic modeling, the research determines optimal limb designs that improve the performance of the robot while simultaneously greatly increasing its load capacity. Thigh-to-calf length ratio (usually 1.05–1.2) and articulation angle (30°–45°) are the parameters that best describe this design. Additional validation using V-REP simulations proved that these designs are feasible in practice. The improved limbs are able to lift weights up to twice their own while still retaining structural integrity. The results of this study have important implications. The Euler column formula-supported optimized limb constructions hold the potential to increase the robots' utility in fields like logistics and rescue operations. Furthermore, the robot's macrostructure is improved by this topology optimization technique, which makes it more appropriate for challenging tasks in demanding environments. The existing finite element modeling and robot limb buckling analyses are based on ideal conditions, which might not apply in any challenging environments. Future research should look into fields including soft robotics or adaptive systems to improve the biomimetic efficacy of robot limbs and make them more similar to their biological counterparts.

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