

Research on the Integration and Application of CNN and MRF in Geotechnics

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Abstract. Stratigraphic uncertainty can result in serious construction accidents or extra construction costs when the geotechnical information is limited. The Markov random field (MRF), based on the stochastic theory, provides engineers with a useful but time-consuming solution to predict the unknown stratum and relevant parameters under some priori knowledge. A new integrated model of convolutional neural network (CNN) and MRF has been proved to greatly improve the computation efficiency compared with the mentioned conventional method while keeping a relatively good accuracy, but it has not been widely used yet. In the background of rapid development of artificial intelligence (AI) and big data technology, this paper aims to introduce the above-mentioned stochastic geological modeling method to stimulate engineers to explore possible data-driven solutions to geotechnical problems. This paper firstly introduces the progress of stratigraphic uncertainty analysis, and then analyses the principles and applications in geotechnic fields of MRF and CNN. Consequently, the integrated model and a case study are introduced to prove its feasibility. Finally, the evaluation and foresight of the model is given.

Keywords: Stratigraphic uncertainty; convolutional neural networks; Markov random field; stochastic geological modeling.

1. Introduction

Stratigraphic evolution is a complex natural behavior over long time scales [1-3]. Naturally, the stratum parameters and distribution show a complex spatial dependency, which means the geological properties of a site point may be influenced by surrounding points [4]. This dependency implies that geological data are not randomly distributed in space, but vary according to some pattern or laws, which however may be difficult to describe simply due to the complexity of geological processes [5, 6].

The limited construction cost constraints the acquisition and analysis of geological information. Consequently, limited investigations make it more difficult to adequately characterize stratigraphic uncertainties [7]. In the context of modern construction technology that is nearly mature, probability theory has become an important research direction to solve geotechnical spatial uncertainty problems. Uncertainty analysis aims to identify and quantify the uncertainty in the system model or input parameters on the output results [2, 8]. It focuses on the variability and potential errors due to the lack of precise information [2] and can be widely used in scientific research and engineering design [9]. Among them, random field (RF) shows unique advantages in dealing with complex spatial data with obvious spatial dependence such as stratigraphic uncertainty. RFs can predict the properties of uninvestigated regions based on the prior knowledge. By modelling the spatial distribution of geological parameters, RF is able to generate possible scenarios of subsurface conditions, which contributes to risk assessment and decision making. However, RF suffers from high computational complexity and computational cost, sensitivity to data quality and quantity, and poor model interpretability in practical applications [10]. Among them, Markov random field (MRF) is typical in geotechnical applications, so this paper will focus on MRF to discuss.

The machine learning (ML) provides a good paradigm for solving the analysis of geotechnical problems that rely on complex data, and can identify complex nonlinear relationships between the inputs and outputs of the data sets [10, 11]. In the past decades, machine learning has received increasing attention in solving problems of geotechnical data analysis [12-15].

An MRF can transform the image processing problem into a maximum a posteriori probabilistic inference problem. And the convolutional neural network (CNN) has a very significant advantage in large-scale and -amount image processing problems [12]. In recent years, many researchers have concurrently considered the spatial variability prediction capability of the MRF and the efficiency of the CNN for limited data to deal with geotechnical engineering problems. For example, Zhang et al. used a hybrid machine learning model to process the fluctuation of soil spatial variability generated in an MRF based on CPT [16]. As well as Wang proposed a method by treating random field as an “image” and using a metamodel based on a CNN and thus predict the spatial distribution of strata, reducing the computational complexity of random field-finite element model (RF-FEM) [17]. The combination of the MRF and the CNN is commonly used, but very few researches have specifically discussed the principle of the integrated use.

This paper aims to analyse the application of MRF to spatial uncertainty in geotechnical engineering problems, and to explain how a CNN can improve the efficiency of its analysis. The paper will firstly explain the mathematical principles of the MRF, and then explain why it can be applied to spatial uncertainty problems in geotechnical engineering, and then analyse the intrinsic computational logic of the CNN-MRF method, following with a simple case demo before finally discussing the challenges and prospects associated with the problem.

2. MRF Applied in Geotechnics

2.1. The Principle of the MRF

Fig. 1 illustrates a simple MRF model. The MRF is an undirected probabilistic graphical model based on Markov properties. Its fundamental feature is that a set of random variables (i.e., the random field as a whole) on a multidimensional space, where the state of any node is related only to the state of its connected nodes and is independent of the states of other nodes. By means of dependencies between local nodes, the MRF models the global or joint probability distribution of the whole system in order to compute and infer the probability properties of any part of the system.

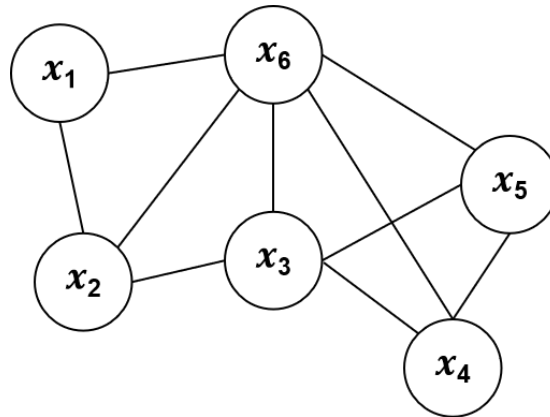


Fig 1. A simple MRF demo.

A nodes subset of an MRF, in which any two nodes are connected by edges, is called a “clique”. For example, in Fig 1, $\{x_1, x_2, x_6\}$ is a clique but $\{x_1, x_2, x_3, x_6\}$ not as a clique. For a specific MRF with n random variables $X = \{x_1, x_2, \dots, x_n\}$, let the set of all clusters be C , and the set of variables of any single cluster be, then the global joint probability distribution of the MRF can be expressed as follows:

$$P(X) = \frac{1}{Z} \prod_{Q \in C} \psi_Q X_Q \quad (1)$$

$$Z = \sum_X \prod_{Q \in C} \psi_Q X_Q \quad (2)$$

Where ψ_Q is the potential function of Q , which is a kind of prior information function and each clique has a uniquely determined potential function that corresponds to Q . Z is the normalization factor. Zhou [11] and Rue [18] have given very detailed explanations at the mathematical and theoretical level of MRFs, so some of the derivations and inferences will not be repeated here.

After the image information is mapped to the MRF, the image processing problem (image segmentation, denoising, etc.) is transformed into a maximum a posteriori probabilistic (MAP) inference problem by solving the joint probability distribution of the MRF, which is then transformed into an energy minimization problem by the Hammersley-Clifford theorem [19].

2.2. The Modeling of MRF in Geotechnics

2.2.1. Neighborhood system

The MRF discrete the target stratum cross section into mesh units to characterize the spatial relationships between them. Based on this property, the concept of neighborhood system is introduced in the MRF. Fig. 2 shows a local neighbourhood system N_i about i : for unit i , it has 8 neighbours j_1 to j_8 . Excepting i , the 8 neighbors form a neighbourhood system with i as a shared node. It is worth mentioning that Jiang [20] highlights the neighbourhood form can also have multiple types according to the particular location of i .

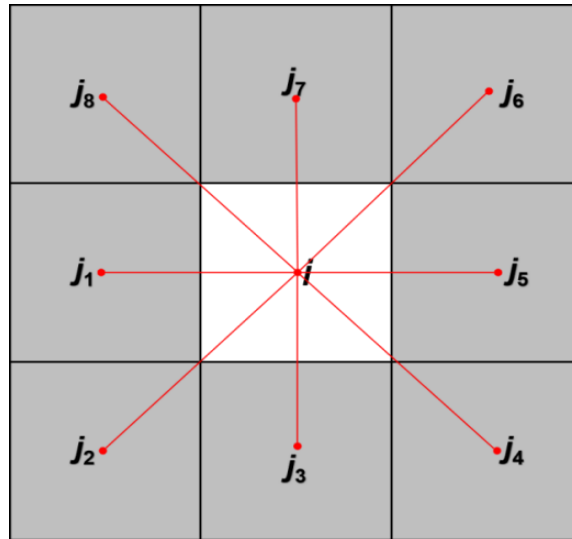


Fig 2. Local neighborhood system of element i .

Based on the random distribution properties and the definition mentioned above, the following properties of neighbour system can be obtained as equation (3) and (4):

$$P(X = x) = P(x) > 0, \forall x \in \mathfrak{R} \quad (3)$$

$$P(x_i | x_j, i \neq j) = P(x_i | x_j, j \in N_i) \quad (4)$$

Where x is the realization of X , and $\mathfrak{R}$ is the set of all possible realizations in this neighbourhood system. Equation (3) and (4) respectively imply that every possibility of a MRF is positive and every random variable can be only affected by the adjacent variable (s).

2.2.2. Gibbs distribution and Energy function

The joint probability of an MRF given by equations (1) and (2) is also applicable for any neighbourhood system in the MRF. Hammersley-Clifford theory establishes a reciprocal relationship between the MRF and the Gibbs distribution, transforming the probability distribution into the form of an easily computable exponential function. Equations (1) and (3) are modified as:

$$P(x) = Z^{-1} \exp \left(- \sum_{c \in \mathcal{C}} \frac{V_c(x)}{T} \right) \quad (5)$$

And the normalization factor Z in equation (2) is modified as:

$$Z = \sum_{x \in \mathfrak{R}} \exp \left(- \sum_{c \in \mathcal{C}} \frac{V_c(x)}{T} \right) \quad (6)$$

Where the energy function $U(x)$ is defined as the sum of all potential function $V_c(x)$ of all clique c , or in other words:

$$U(x) = \sum_{c \in \mathcal{C}} V_c(x) \quad (7)$$

Where T is a self-defined "temperature" function applied to control the value of $\frac{V_c(x)}{T}$ close to 1 [20, 21].

For further reduction of calculations, Geman and Geman [22] came up with that introduce Markov Chain Monte Carlo (MCMC) method to gain realization set of x and MAP of $\mathfrak{R}$ on the basis of Gibbs distribution.

In addition, on the basis of Gibbs distribution, Geman and Geman [22] advised to apply MCMC method to generate realization set of x and MAP estimate values of $\mathfrak{R}$. In this way, the global probability distribution is reduced to be a local neighbourhood system. For the neighbourhood about unit i around unit j :

$$P(x_i | x_j, j \in N_i) = P(x_i | x_{N_i}) = Z_i^{-1} \exp \left(- \frac{\sum_{j \in N_i} V_c(x_i, x_j)}{T} \right), x_i \in \mathfrak{R} \quad (8)$$

$$Z_i = \sum_{x_i \in \mathfrak{R}} \exp \left(- \frac{\sum_{j \in N_i} V_c(x_i, x_j)}{T} \right) \quad (9)$$

$$V_c(x_i, x_j) = \begin{cases} 0, & (j \in N_i) \cap (x_i = x_j) \\ -\beta(i, j), & \text{else} \end{cases} \quad (10)$$

Where $\beta(i, j)$ is an a priori function for measuring the strength of the neighbourhood system space connectivity, and its magnitude depends on the a priori information from the designer. A larger value represents a smaller value of the neighbourhood space potential function and a more enhanced inter-unit correlation.

Fig. 3 is a model of the geometrical relationship between element i and its neighbour elements j . The mesh line circle is the standard geometric correlation, where θ is the intersection angle between the element i and element j (taking j_6 as an example). And the blue solid ellipse is the corresponding spatial correlation. The ellipse takes the minor axis as a unit length 1 and the major axis length a as a variable, which imply the normal and tangential correlation between element i and j respectively. And φ is also a variable. The length of $\rho(\theta)$ is the equivalent substitution of $\beta(i, j)$. It is obvious that the $\rho(\theta)$ will get different values following the rotation of ellipse, calculated as:

$$\beta(i, j) = \rho(\theta) = \frac{a}{\sqrt{\cos^2(\theta - \varphi) + a^2 \sin^2(\theta - \varphi)}} \quad (11)$$

Generally, designers try to control φ or a to measure the spatial correlation of elements in different scenarios [21]. The bigger $\rho(\theta)$ is, the stronger the spatial relationship in elements is, referring for more detailed explanation in Li [21] and Zhang [22].

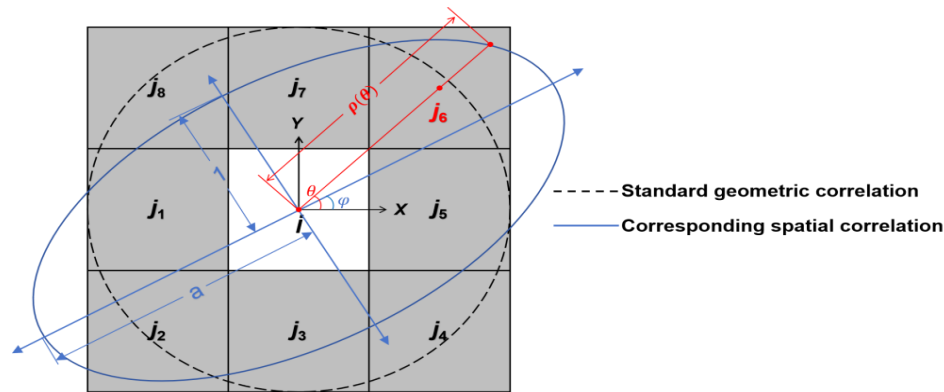


Fig 3. Local neighborhood system geometry properties.

3. Integrated Application of CNN and MRF

3.1. The Application of CNNs in Geotechnics

CNN is one of the products of the development of DL. As shown in Fig. 4, a basic component of CNN can be divided by: input layer, convolutional layer, pooling layer, activation layer, fully-connected layer, and output layer, and its basic operation flow has been elaborated by numerous textbook and paper studies, which is able to automatically learn the features required for image classification from the training image data, so as to improve the classification accuracy and efficiency without manual selection [11, 23].

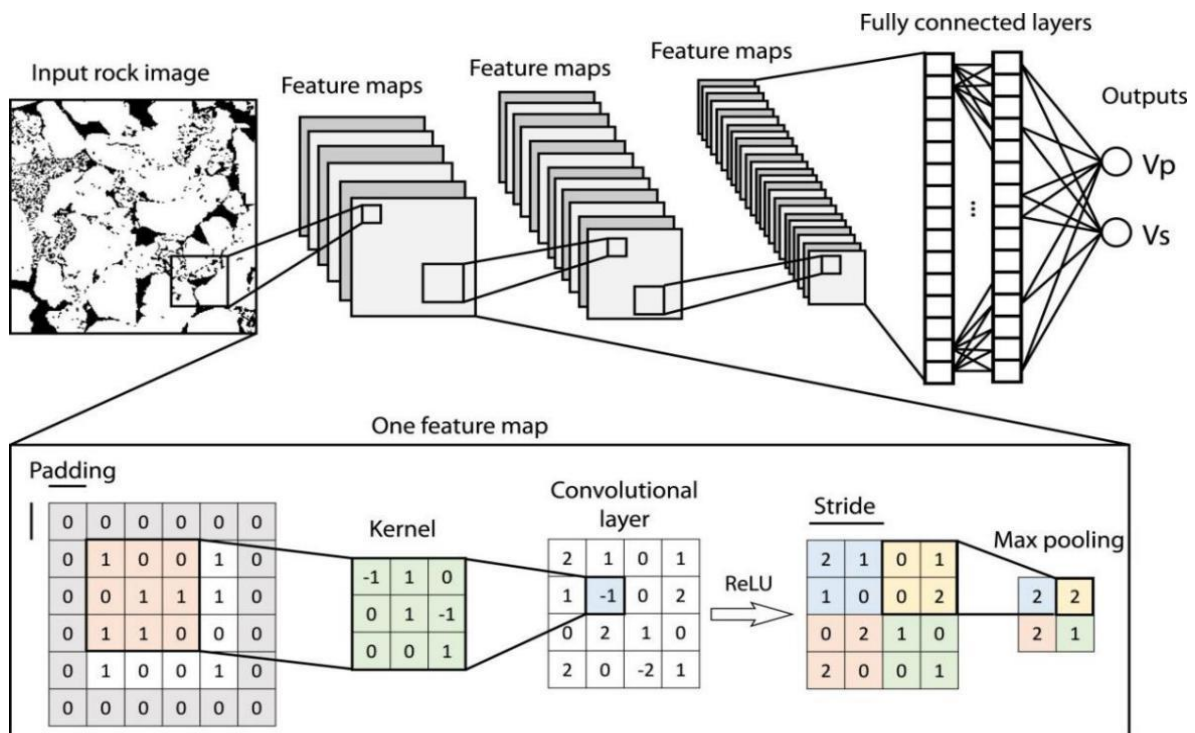


Fig 4. Schematic illustration of a convolutional neural network [24].

Following the rapid development of DL methods in computer vision and medical sciences, CNNs and their hybrid models have made many advances in geotechnical engineering. For example, CNNs are used for soil particle size classification, tunneling and lithology investigation soil properties inversion and characterisation, etc. [25-27].

Fig. 5 lists the percentage of CNN applications in geotechnical engineering from 2010 to 2023. It can be seen that this method has received more attention in the field of geotechnical engineering in recent years due to the increase in computational power and storage capacity.

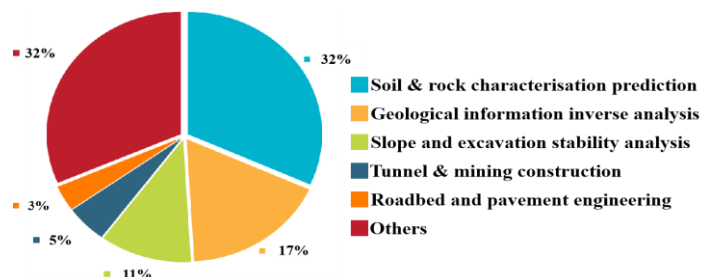


Fig 5. CNN application fields distribution (from web of science, 01/01/2010-31/12/2023).

3.2. Combination of CNNs and MRFs

3.2.1. Feasibility of integrated application

Fig. 6 allocates the publications and citations number from 2000 to 2023. Although CNNs were proposed very early in 1989, integral application of CNNs together with MRFs were very rare before 2015. After that, the co-use of CNNs and MRFs started to be noticed by researchers and produced amounts of cases in several fields. For example, Bao et al used an MRF model embedded with CNNs to segment objects in a video and demonstrated that the segmentation results were 25% more accurate than the previous typical model [28], followed by Zhang [29] and Zhao [30] applying this coupling to the field of high-precision remote sensing measurements for achieving high-resolution satellite images. Geng and Sun et al used this technique to distinguish super-pixel segmentation method to differentiate between road and other regions in dealing with highly complex road environments and obtained good robustness [31].

The above various cases demonstrate that it is prospective to couple CNNs and MRFs for joint use. When it comes to the hybrid model application in geotechnical stratigraphic uncertainty problems, Phoon (2023) cited a series of engineering examples to demonstrate the reliability and high efficiency of imaging geotechnical data with the MRF and the CNN as the typical tools, emphasising the importance of modelling and machine training when oriented to reduce the data volume at the meanwhile [11, 31].

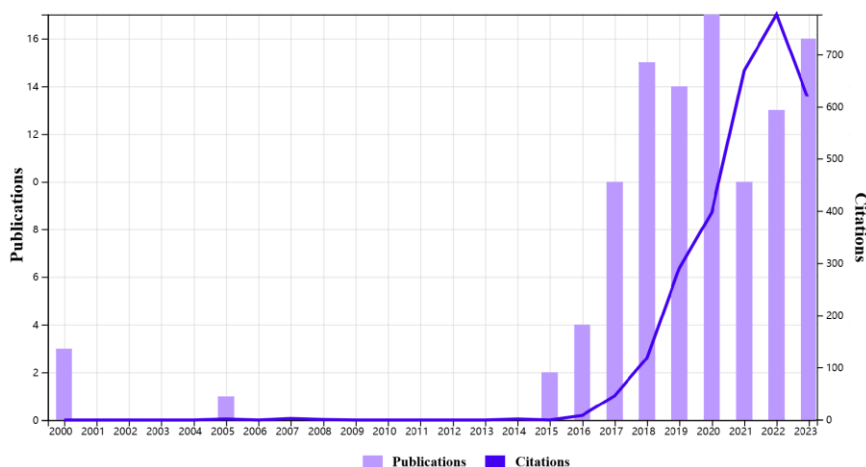


Fig 6. Publications and citations tendency of CNN combined with MRF (from web of science, 01/01/2000-31/12/2023).

Focusing on the geotechnical domain, the MRF mainly deals with stratigraphic identification with partially known information and response analysis due to the intrinsic variability of geotechnical systems [18, 20, 22, 25, 32]. Among them, the stratigraphic identification problem gives relatively high deterministic results due to the availability of known information. However, uncertainty-based response analysis requires repeated large number of random sampling realizations to obtain a possible value such as MAP, which exhibits a very high time cost due to the excessive computational effort in practical applications [21, 32].

In this regard, Wang and Goh invented a method seeing CNN as an alternative to Monte Carlo Simulation (MCS) to handle repeated simulations in finite elements more efficiently while ensuring the accuracy and reliability of the model, which has been widely referenced and applied in recent years [32-36]. Its specific thought process will be presented in the next section.

3.2.2. Methodology of integration

Fig. 7 shows a flowchart comparing the conventional and CNN based MRF for geotechnical response analysis. For the above conventional method, the modelling steps are as follows:

Step 1: mesh the target region and define the dimensions of unit cell, and apply the proposed neighborhood system equation (8-11)) in &2 to determine the spatial correlation parameters (θ , φ , β) of each system

Step 2: map the spatial model obtained in step 1 into a MRF, and traverse all grid cells of the random field, then iteratively calculate the a posteriori probability of each cell until the joint probability $P(\mathbf{x})$ or total energy $\sum U_x$ of the MRF converges, apply the last result as a global lithological probability realization and map it to the actual finite element model (FEM) or finite differential model (FDM).

Step 3: initialize the priori parameters of stratigraphic in FEM or FDM such as standard parameter value, coefficient of variation (COV), vertical and horizontal correlation lengths (CL_y , CL_x).

Step 4: apply the loading or displacements under the practical scenario to obtain a single stratigraphic parameter sample and a response.

Step 5: introduce MCS to stochastically repeat(simulate) step 4 N times to generate and record the response and parameter sample mentioned above, in which the Karhunen-Loève (K-L) expansion is applied to the data, and the lognormal distribution can be used to count the stochastic probability densities of the response and parameters in order to avoid negative parameters.

As for the MRF method coupled with the CNN, instead of performing a large number of MCS in step 5, only a smaller number of simulations are performed to generate multiple discrete grids with the corresponding response indexes (e.g. factor of safety) in the numerical model and put into a CNN for training. In this way, the MCS is used to generate a training and a test set to make the trained CNN predict the response coefficients. In reality, as shown in Fig 8, the analysis of discrete grids by CNN is not based on the pixel density values like conventional images, but the values corresponding to Gaussian integration points, also the number of color channels isn't three (red, green and blue), but the number of interested parameters of the stratum [33].

4. Case Study

4.1. Modelling Introduction

This section introduces an FEM about excavation and support proposed by Goh [36] as shown in Fig. 9 and refer to the modelling method some running results from Wu et al. [32] to compare the difference between the proposed conventional and hybrid MRF method. Detailly, through setting up the model in the commercial FEM software Optum G2 2023 based on the soil and structural parameters in Table 1, cohesion c and friction angle φ is determined as the target index of RF. Optum G2 2023 run the random model as the flow chart in Fig. 7 to generate a series of deflection responses of support structures.

This model consists of 560 finite units with 6 nodes for each. And the support system is linear elastic diaphragm wall (D-wall), with the elastic modulus is $2 \times 10^6 \text{ kN}$ and struts, in which the bending stiffness is $2.5 \times 10^6 \text{ kN} \cdot \text{m}^2$ and the compressive stiffness is $3 \times 10^6 \text{ kN}$. The MRF parameters refers to Table 2 applying the MCS. And according to the modeling experience from [33] the research of Wang and Goh, 3000 and 1000 are taken as the numbers of MCS and K-L method mentioned in &3.2.1 respectively [33]. As for CNN method, the paper runs 500 times in MCS to generate samples and take 200 of them as the training set and the others as the testing set. Finally, the

two mentioned methods respectively generate a series maximum deflection of wall. The fitting results will be recorded and the running time would be the evaluation index at the meantime.

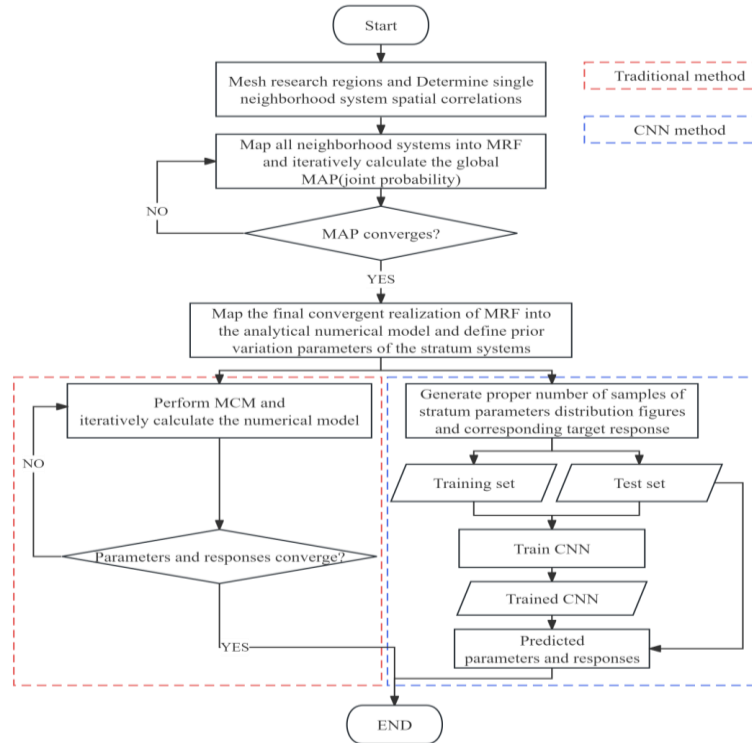


Fig 7. Traditional and hybrid MRF method.

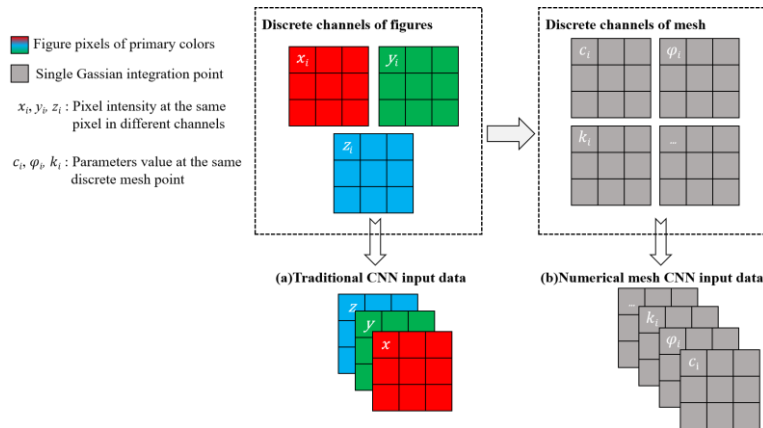


Fig 8. CNN input data patterns [33].

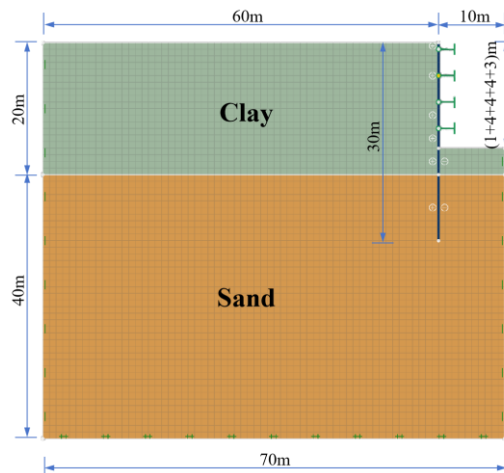


Fig 9. Target FEM [36].

Table 1. Soil model parameters.

Material	Clay	Sand
Type	Mohr-Coulomb	Hardening soil
Cohesion c/kPa	30	0
Friction Angle $\varphi/^\circ$	20	32
Dilation Angle $\psi/^\circ$	-	2
Young's Modulus E/MPa	9	-
Standard reference secant stiffness $E_{50}^{\text{ref}}/\text{MPa}$	-	4
Oedometer reference secant stiffness $E_{\text{oed}}^{\text{ref}}/\text{MPa}$	-	40
Ultimate reference secant stiffness $E_{\text{ult}}^{\text{ref}}/\text{MPa}$	-	120
Stress-level dependency power m	-	0.5

Table 2. Design of coefficients of variations [32].

Coefficient	Mean value	COV	Distribution	CL_y	CL_x
Cohesion c/kPa	30	30%	Lognormal	2.5	25
Friction Angle $\varphi/^\circ$	20	10%	Lognormal	2.5	25

4.2. Modelling Results

Fig. 10 shows a portion of the wall deflection values from the CNN method against the values from the MRF-FEM(MCS) method and Fig. 11 is intended to illustrate the difference between the CNN-predicted the MCS-predicted values. In fact, all the deflection values are in the range of 60-85 mm, but due to the number of discrete points outside of the 65-78 mm range is very small and not typical for comparison, they are not discussed in this section.

The blue solid dots in Fig. 10 are from the training set obtained directly from MCS, while the orange hollow dots are the testing set. According to the regression analysis (black line in Fig 10 and x axis in Fig. 11), the correlation coefficient of the testing set is $R^2 = 0.88$, which can be considered to have strong correlation with the training set. And by analysing Fig. 11, it was concluded that the maximum absolute residual value between training set and regression value is 3mm, generally an allowable displacement in construction. The result proves the high feasibility of the CNN method.

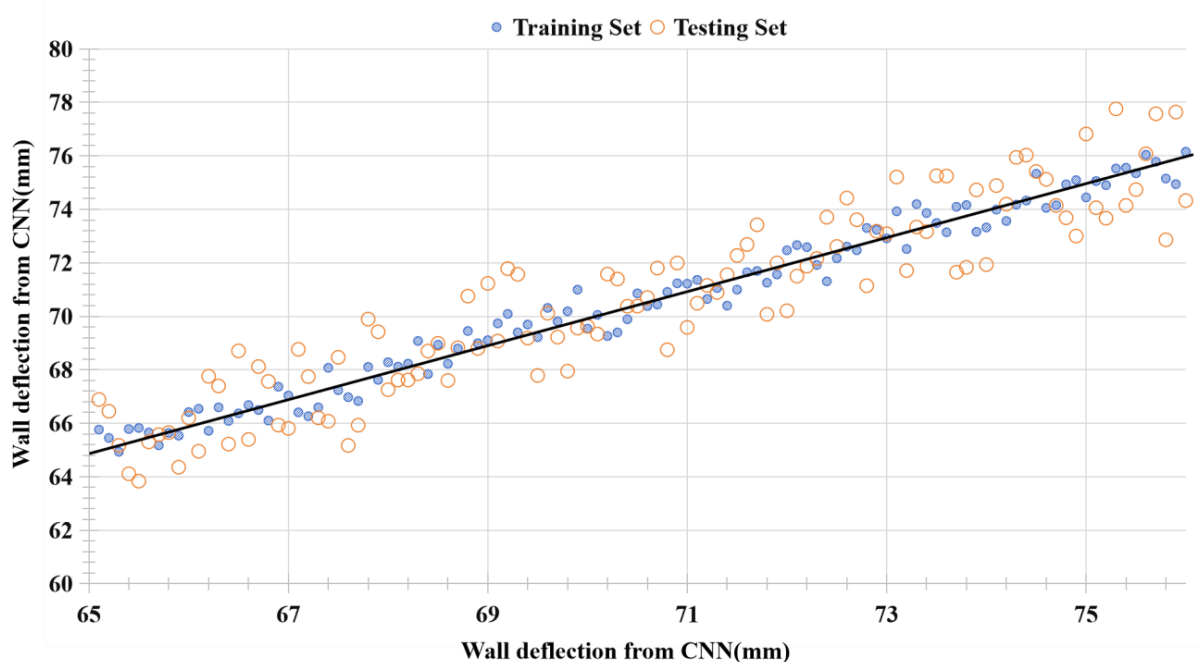


Fig 10. Regression analysis result.

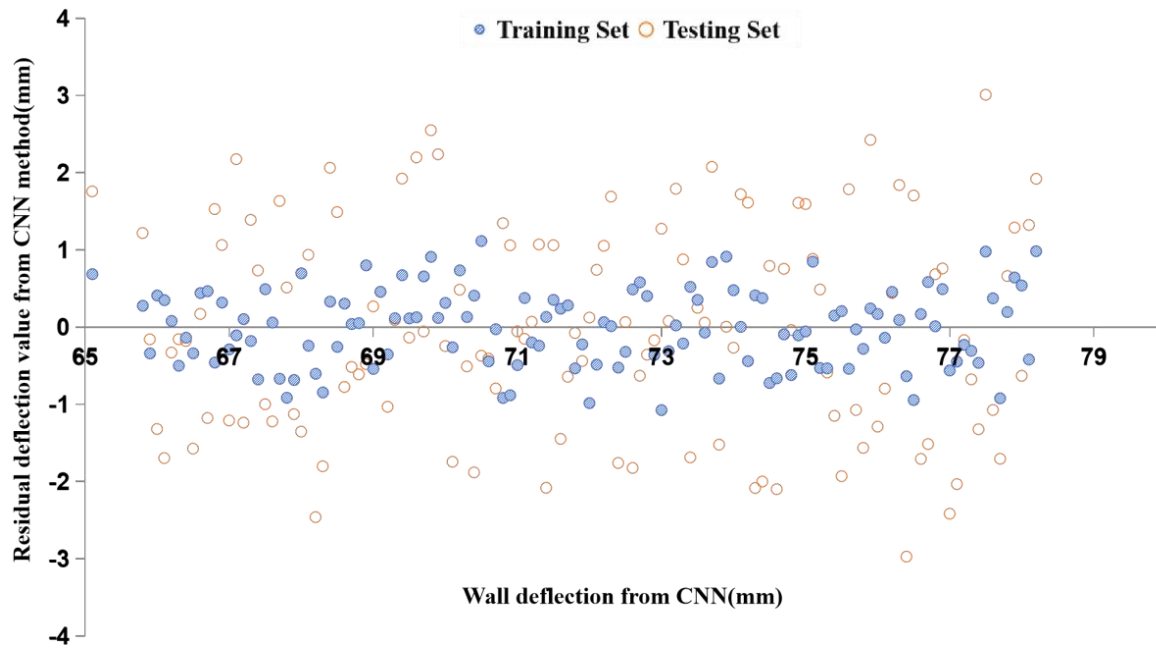


Fig 11. Residual value analysis result.

During the practical running period, a CPU of 13th Gen Intel(R) Core (TM) i9-13900HX was used as the trainer for the user-defined CNN and to run the MRF-FEM. Based on the standard of generating 500 samples, the result showed the CNN modeling time (including the training time) is around 4 hours, which is a rather different result from the performance of Wu et al.'s research [32]. This difference mainly from the detailed modeling methods, such as the random sampling methods, mesh types and so on. Besides, the framework of the CNN and the hardware may not be in the same. Therefore, the results seem to be acceptable. In addition, the performance of the pure MRF-FEM method gave a good matching running time with the existing experiments, more than 48 hours. Although it is difficult to quantify and only the final results are evaluated, the CNN method proposed in this paper greatly improves the efficiency of traditional methods and matches the previous literature [32, 33].

5. Conclusion

Considering the construction needs and the wide-spread use of AI, this paper introduced a new-developed CNN-MRF stochastic geological model applied to improve the computing efficiency when dealing with the stratigraphic uncertainty problems.

As an interdisciplinary technology, the paper respectively introduced the mechanisms of MRF and CNN, coming up with a series of practical construction cases to stress the feasibility of the two single methods. Anchored in the similar data structures, the mesh blocks in FEM can be transferred into a pattern as a “figure” with “channels” and “pixel intensity” according to the numbers and magnitudes of the interested parameters, this is also the inner reason why CNN can be integrated into (M)RF-FEM. The most outstanding function of CNN in MRF-FEM is reducing the requirement of random samples generation from MCS or other methods, which modifies the lengthy and jumbled random sampling into data learning. A brief case referred to existing models has proved the feasibility of the hybrid model.

From the analysis and the results of the simulation, it can be summarized that: Pure random sampling and CNN both rely on the performance of computer. However, CNN has the potential to parallel dispose the data, which means the efficiency improvement can be “dimensional” and “exponential”. Besides, different framework and data scale of models will greatly influence the computation velocity and the quality of generation. This is a common problem of simulation modeling but very apparent here especially in time. The basic random sampling is still necessary at the

beginning, since a group of minimal training and testing samples are necessary. However, how to balance the number of initial samples and the training time is still an unknown topic.

Stress that the proposed CNN-MRF method is just one of the forms of any data-driven method. For example, CNN can be replaced by XGBoost, MRF can be taken placed by conditional random field. It seems to have a bright future for such DL-hybrid models. However, the introduction of DL in such large-scale data model may lead to some other problems, such as the clutter of hyper parameters, “dimensional curse” or the difficulties on setting up a proper hybrid framework. However, there is no doubt that the combination of data-driven model and AI is more and more popular, it’s encouraged to explore more in this field.

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