Research on Multibeam Bathymetry Coverage Width and Overlap Rate Based on Geometric Models

Xinshu Wang *, Xiang Li, Qijia Liu

College of Computer Science and Software Engineering, School of Hohai University, Changzhou, China, 213,200

* Corresponding Author Email: 18005259182@163.com

Abstract. Due to its efficient and accurate seabed mapping capabilities, multi-beam bathymetry systems are widely used in marine research and resource development and have become a hot spot in marine scientific research. In this study, the sine theorem, geometric relationship, and recursive relationship are used to construct the calculation model of coverage width and overlap rate of multi-beam bathymetry, and the influence of seabed slope is considered. Under the condition of varying slopes, further research is carried out, and the elliptical chord length is formed by using the conic section to solve the width, and a similar triangle is used to reduce the amount of double calculation. The analytic geometry and stereo geometry methods were combined to construct a calculation model of the relationship between beam coverage width and position. This research result provides important technical support for deep-sea surveying and mapping, injects new impetus into the development of marine science, promotes the scientific development and utilization of deep-sea resources, provides more accurate technical support for marine environmental protection, and promotes the sustainable development of the marine economy.

Keywords: Multi-beam sounding system, seabed mapping, recurrence relations, analytic geometry, solid-dimensional geometry.

1. Introduction

Seabed surveying and mapping is very important at present, and professional ocean surveying and mapping technology can bring great convenience to human production and life [1], and the multi-beam detection system has shown great application potential and significant advantages in the field of seabed surveying and mapping [2], and the multi-beam detection system can cooperate with other technologies to better complete some operational tasks [3]. In many places that cannot be observed with the naked eye, multi-beam detection systems can perform more accurate underwater detection [4]. By simultaneously emitting multiple beams and receiving their echoes, the resulting 3D data can be analyzed to visually reflect the underwater topography [5]. This characteristic makes it useful in the investigation and study of seabed topography, geomorphology, marine life distribution and resource exploration. Compared with the traditional single-beam system, it not only improves the problem of single detection technology, but also provides more basis for long-term detection, and has a large overall development space [6], which is an effective means to solve the problem of seabed detection technology [7], and also improves the accuracy and reliability of the data and expands the measurement range [8, 9], and the detection time of the multi-beam detection system is shorter, and the influence of underwater obstacles on the detection results can be greatly reduced [10]. Therefore, the multi-beam detection system plays an indispensable role in the fields of marine geology, marine ecology, and marine resource development, and provides strong technical support for in-depth understanding of the marine environment, protection of marine ecology, and development of marine resources.

In this study, the relevant practical conditions are set up and the plane geometry model is established. Using the trigonometric function and similarity relation in geometry, the depth D, covering width W, overlap rate η with the previous line, and slope α are derived. The relation equation between the opening Angle θ of multi-beam transducer. Then, the pre-set initial data is brought into the equation to solve, and the required research results are obtained. To make the research results more widely applied, a more in-depth mathematical model is established after further considering the

Angle β between the projection of the slope normal on the horizontal plane and the direction of the measured line. Considering that the sea floor is a tilted rectangle when the survey ship is fixed at a point on the sea surface, the intersection between the tilted sea floor plane and the cone is an ellipse. By studying the properties of the ellipse, the beam coverage width is calculated. First, the ellipse is placed in the plane rectangular coordinate system, the major axis and semi-major axis of the ellipse are determined, and then the beam coverage widths at different distances are calculated according to the similarity relation. In the problem-solving stage, using the established model and equation and the preset initial data, the parameters of seawater depth and coverage width in various cases are calculated by programming. The results of this study provide important theoretical support for understanding the characteristics of seabed topography and the working principle of the multi-beam sounding system.

2. Geometric Model and Coverage Analysis of Multibeam Bathymetry

Data in this study are derived from www.mcm.edu.cn.

Firstly, this study will try to construct a plane geometric model. Based on the trigonometric function in the geometric Figure and the similarity relationship, the relationship equation between seawater depth D, coverage width W, overlap rate η and slope α with the previous measurement line, and the opening angle θ of the multi-beam transducer can be obtained. Thus, the preset initial data is brought in to obtain the desired research results.

2.1. Model building

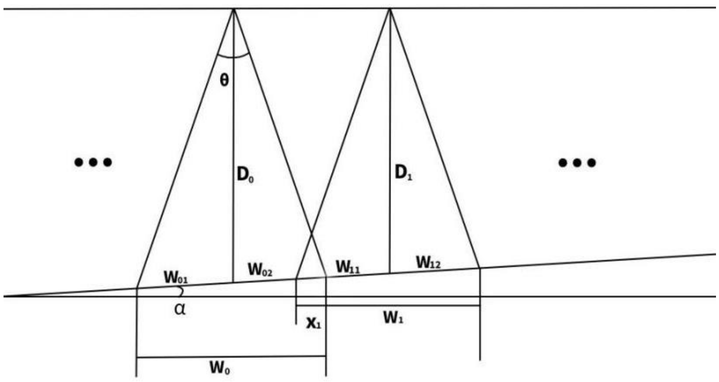

Figure 1. Two-dimensional case multi-beam detection model diagram

As shown in the Figure1, considering the two measuring lines with adjacent interval d and the geometric diagram of the emitted beam, the seawater depth is, and the coverage width is respectively, and the two vertical lines of the excessive beam transducer will divide the coverage width into, and the overlap width of the adjacent coverage area is $D_1\ W_0\ W_1\ W_{01}\ W_{02}\ W_{11}\ W_{12}\ x_1$.

According to the sine theorem, the equation can be listed as follows:

$$\begin{cases} \dfrac{\sin\dfrac{\theta}{2}}{W_{01}} = \dfrac{\sin(\dfrac{\pi}{2} - \dfrac{\theta}{2} - \alpha)}{D_0} \\ \dfrac{\sin\dfrac{\theta}{2}}{W_{02}} = \dfrac{\sin(\dfrac{\pi}{2} + \alpha - \dfrac{\theta}{2})}{D_0} \end{cases} \tag{1}$$

Get

$$\begin{cases} W_{01} = \dfrac{\sin\frac{\theta}{2}}{\cos(\alpha + \frac{\theta}{2})} D_0 \\ W_{02} = \dfrac{\sin\frac{\theta}{2}}{\cos(\alpha - \frac{\theta}{2})} D_0 \end{cases} \tag{2}$$

$$W_0 = \left(\frac{1}{\cos(\alpha + \frac{\theta}{2})} + \frac{1}{\cos(\alpha - \frac{\theta}{2})} \right) \sin\frac{\theta}{2} D_0 \tag{3}$$

Also, have:

$$W_n = \left(\frac{1}{\cos(\alpha + \frac{\theta}{2})} + \frac{1}{\cos(\alpha - \frac{\theta}{2})} \right) \sin\frac{\theta}{2} D_n \tag{4}$$

Let the distance between the line and the center point be x (x>0 indicates the line on the right of the center point, x<0 indicates the line on the left of the center point), the depth at the center point is, and D (x) indicates the seawater depth from x of the center point, then D_0:

$$D(d) = D_0 - x \tan\alpha \tag{5}$$

The overlap width of adjacent coverage areas can be obtained from geometric relationships x_1:

$$x_1 = W_{02} + W_{11} - d = \frac{\sin\frac{\theta}{2}}{\cos(\alpha - \frac{\theta}{2})} D_0 + \frac{\sin\frac{\theta}{2}}{\cos(\alpha + \frac{\theta}{2})} D_1 - d \tag{6}$$

Among:

$$D_1 = D_0 - d \tan\alpha \tag{7}$$

The n th band overlaps the width with the previous one x_n:

$$x_n = \frac{\sin\frac{\theta}{2}}{\cos(\alpha - \frac{\theta}{2})} D_{n-1} + \frac{\sin\frac{\theta}{2}}{\cos(\alpha + \frac{\theta}{2})} D_n - d \tag{8}$$

Among:

$$D_n = D_0 - nd \tan\alpha \tag{9}$$

The overlap rate of the n th line and the previous line is calculated below:

$$\eta_n = \frac{x_n}{W_{n-1}} = \frac{\dfrac{\sin\frac{\theta}{2}}{\cos(\alpha-\frac{\theta}{2})}D_{n-1} + \dfrac{\sin\frac{\theta}{2}}{\cos(\alpha+\frac{\theta}{2})}D_n - d}{\left(\dfrac{1}{\cos(\alpha+\frac{\theta}{2})} + \dfrac{1}{\cos(\alpha-\frac{\theta}{2})}\right)\sin\frac{\theta}{2}D_{n-1}} \tag{10}$$

As you can see, if the seabed terrain is flat, that is, there is no slope, then all are equal D_n

$$\eta = 1 - \frac{d}{W} \tag{11}$$

2.2. Data verification

To facilitate the introduction of the initial data into the calculation, we set the, $\theta = 120°$ $\alpha = 1.5°$ $D_0 = 70m$.

First, formula (5) is used to calculate the seawater depth D_n at each line, and brings D_n into formula (4) and (10). Matlab programming to calculate the coverage width and the overlap rate with the previous line is shown in the table below.

Table 1. The calculation results

The distance of the line from the central point is /m	-800	-600	-400	-200	0	200	400	600	800
Water depth /m	90.95	85.71	80.47	75.241	70	64.76	59.53	54.29	49.05
Cover width /m	315.81	297.63	279.44	261.26	243.07	224.88	206.70	188.51	170.33
Overlap rate /% with the previous test line	——	33.64%	29.59%	25.00%	19.78%	13.78%	6.81%	-1.39%	-11.17%

The data in Table1 conform to the basic geometric relationship principle, and it can be verified that the established model is correct and effective.

3. Analysis of Coverage and Overlap under Variable Slope in Multibeam Bathymetry

The above study is based on the case of a single direction, without considering the line measurement scheme of other directions. To further consider the more realistic situation, that is, the β around the Angle of the direction of the slope normal direction and the Angle of the direction of the line). The preset condition here is, $\theta = 120°$ $\alpha = 1.5°$ $D_0 = 70m$.

3.1. Model building

When measuring a ship fixed on the sea, if the bottom is flat, namely no slope, different angles from the beam in the bottom for a positive circle, in a three-dimensional and multiple beam detection system to form a cone, now set the bottom for slope 1.5° rectangular floor, So as shown in Figure 2, the inclined bottom plane and conical into an intersection for the ellipse. When the measuring ship is in the same place with a different navigation direction, the beam cover width is the chord length of the ellipse over point A. Point A is the projection point of the measuring ship on the seabed slope, and M is the center point of the ellipse. To calculate the beam cover width, it is necessary to investigate the properties of this ellipse.

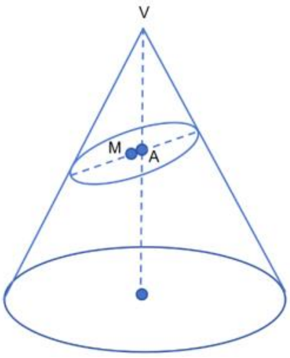

Figure 2. The cone is truncated as an ellipse

The ellipse is placed in the plane rectangular coordinate system in Figure3, where the x-axis direction is the normal direction of the seabed slope projected on the horizontal plane.

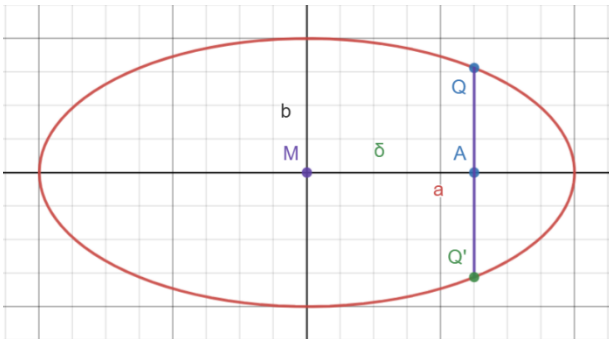

Figure 3. The center point of the ellipse is at the origin

It is easy to see the coastal bottom slope direction of the long axis of the ellipse, that is, the Angle between the long axis and the horizontal plane is α. Using the conclusion of the previous research, the left and right depth of D at any point along the slope direction is as follows:

$$\begin{cases} W_1 = \dfrac{\sin\dfrac{\theta}{2}}{\cos\left(\alpha + \dfrac{\theta}{2}\right)} D \\ W_2 = \dfrac{\sin\dfrac{\theta}{2}}{\cos\left(\alpha - \dfrac{\theta}{2}\right)} D \end{cases} \tag{12}$$

Then the semi-length axis a of the ellipse is as follows:

$$a = \frac{1}{2}(W_1 + W_2) \tag{13}$$

The distance of the cone projection point from the center of the ellipse is the distance between points A and M δ:

$$a = \frac{1}{2}(W_1 - W_2) \tag{14}$$

The cone vertex V and QQ' form an isosceles triangle, and the top angle of the cone is θ, so the length of the AQ is:

$$AQ = D\tan\frac{\theta}{2} \tag{15}$$

Thus the Q-point coordinates are $(\delta, AQ) = (\frac{1}{2}(W_1 - W_2), D\tan\frac{\theta}{2})$ Since the Q point is on the ellipse, let the trajectory equation of the ellipse be:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{16}$$

It can be concluded from the previous formula that:

$$a = \frac{1}{2}(W_1 + W_2) \tag{17}$$

Taking the Q point coordinates into the elliptic equation:

$$b = \frac{D\tan\frac{\theta}{2}}{\sqrt{1 - \frac{(W_1 - W_2)^2}{(W_1 + W_2)^2}}} \tag{18}$$

It can be concluded that the elliptic equation is:

$$\frac{x^2}{\left(\frac{1}{2}(W_1 + W_2)\right)^2} + \frac{y^2}{\frac{(D\tan\frac{\theta}{2})^2}{1 - \frac{(W_1 - W_2)^2}{(W_1 + W_2)^2}}} = 1 \tag{19}$$

Pan the ellipse on the coordinate axis to take point A as the coordinate origin, as shown in Figure4:

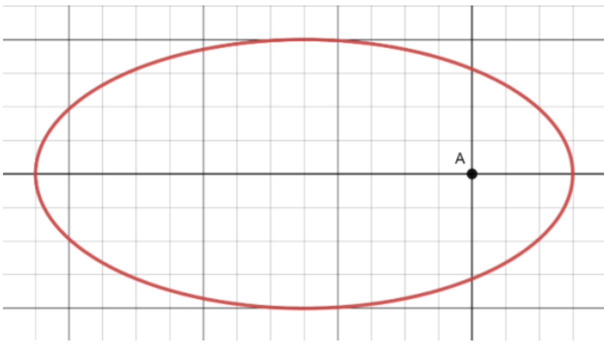

Figure 4. The ellipse after the translation

The elliptic equation after the translation is:

$$\frac{(x + \delta)^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{20}$$

As shown in Figure 5, When the measuring ship is in the same position and the navigation direction is different, the Angle β between the line direction and the normal direction of the seabed slope changes, which will cause the Angle φ between the string and x-axis of the ellipse over A point.

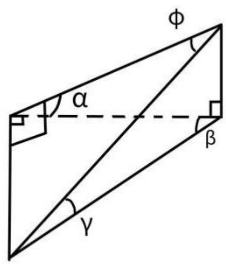

Figure 5. Survey line direction and normal direction of the submarine slope

From geometric relationships:

$$\tan\phi = \cos\alpha\tan\beta \tag{21}$$

$$\tan\gamma = \cos\beta\tan\alpha \tag{22}$$

The beam formation plane is perpendicular to the course of the measuring ship, so the angle between the string on the ellipse and the x-axis is φ:

$$\varphi = \arctan(\cos\alpha\tan(\beta + \frac{\pi}{2})) = \arctan(-\cos\alpha\cot\beta) \tag{23}$$

Bring the translated ellipse into the polar coordinate system to obtain:

$$\frac{\rho^2\cos^2\varphi + 2\rho\delta\cos\varphi + \delta^2}{a^2} + \frac{\rho^2\sin^2\varphi}{b^2} = 1 \tag{24}$$

Simplicity can be concluded:

$$(b^2\cos^2\varphi + a^2\sin^2\varphi)\rho^2 + 2\delta b^2\cos\varphi\rho + b^2\delta^2 - a^2b^2 = 0 \tag{25}$$

By Vader's theorem:

$$\begin{cases}\rho_1 + \rho_2 = \dfrac{-28\mathrm{b}^2\cos\varphi}{\mathrm{b}^2\cos^2\varphi + \mathrm{a}^2\sin^2\varphi} \\ \rho_1\rho_2 = \dfrac{\mathrm{b}^2\delta^2 - \mathrm{a}^2\mathrm{b}^2}{\mathrm{b}^2\cos^2\varphi + \mathrm{a}^2\sin^2\varphi}\end{cases} \tag{26}$$

The initial measurement width for the different angles is, namely, the width at the central point of the sea area: W_0

$$W_0 = |p_1 - p_2| = \sqrt{(p_1 + p_2)^2 - 4p_1p_2} \tag{27}$$

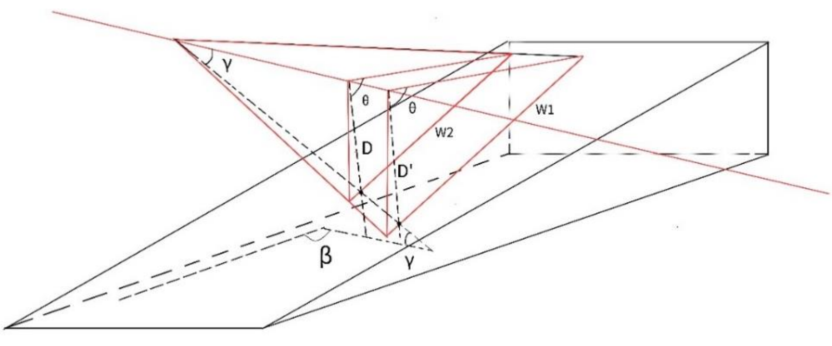

Figure 6. Measurement width of measuring vessel with different angles

As shown in Figure 6, set the Angle between the projection on the slope and the horizontal plane to be γ, and the measuring line and the slope intersect with a point at a very distance. Consider the two measurements on the measuring line, the distance between them is d, and the seawater depth of the location is D and D' respectively, then:

$$D' = D + d\tan\gamma \tag{28}$$

By similar relationship:

$$\frac{D'}{D} = \frac{W'}{W} \tag{29}$$

The conclusion of Equation (8) is available:

$$W' = \frac{D + d\tan\alpha\cos\beta}{D} W \tag{30}$$

3.2. Data verification

To facilitate the introduction of the initial data into the calculation, we set the $\theta = 120°$ $\alpha = 1.5°$ $D_0 = 70m$.

Firstly, (23) and (27) are used to calculate the beam coverage width in each direction at the center point of the sea area, and then a similar relationship in the same direction is used to calculate the beam coverage width at different distances in (30).

The results of this problem being calculated using Matlab programming are shown in the table below.

Table 2. Table of beam cover width at different distances and angles of line direction

		Measure the distance / nautical mile of the ship from the center point of the sea area							
		0	0.3	0.6	0.9	1.2	1,5	1.8	2.1
Line direction clip angle / °	0	415.69	466.09	516.49	566.89	617.29	667.69	718.09	768.48
	45	416.19	451.87	487.55	523.23	558.91	594.59	630.27	665.95
	90	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	135	416.19	380.51	344.83	309.15	273.47	237.79	202.11	166.43
	180	415.69	365.29	314.89	264.50	214.10	163.70	113.30	62.90
	225	416.19	380.51	344.83	309.15	273.47	237.79	202.11	166.43
	270	416.69	416.69	416.69	416.69	416.69	416.69	416.69	416.69
	315	416.19	451.87	487.55	523.23	558.91	594.59	630.27	665.95

The data in Table2 conforms to the basic principle of geometric relations, which can verify that the further established model is still correct and effective.

4. Conclusion

This research mainly focuses on the application of a multi-beam sounding system in a seabed topography survey. We understand the basic working principle of the single-beam sounder and compare it with the multi-beam sounder system, thus highlighting the advantages of the multi-beam sounder system in the seabed topographic survey, including automatic mapping, digital recording, high precision, high speed, and wide coverage, etc. A series of problems of seabed topography measurement by a multi-beam sounding system are studied in depth. A model is constructed and the coverage width of multi-beam sounding and the overlap rate between adjacent strips are calculated under the set Angle of submarine slope and projection. We use the equations based on the sine theorem, the Angle relation, and the length relation in the geometry, and then use the recursive relation to solve. To make the model more universal, we expanded the conditions of the research object. Based on the previous conditions, we considered the survey line scheme in all directions. Under this scheme, the slope of each measurement point changes with the distance of the survey ship from the central point of the sea area and the Angle between the direction of the survey line, which increases the difficulty of solving the problem. It is found that the chord length of the ellipse formed by the cone cut by the slope is the desired width, and this string passes the projection of the cone vertex on the ellipse, and the triangular similarity of the beam and the inclined plane in the same direction is used to reduce the amount of repeated calculation. The mathematical model of the relationship between the beam coverage width and the position is constructed by comprehensive use of analytic geometry and solid geometry methods, and the required results can be solved. For example, when the distance of the measuring ship from the center point of the sea is 0.6 nautical miles and the Angle of the measurement line direction is 45°, the beam coverage width is 487.5517meters.

This study provides a research idea and framework for the application of Marine mapping-related fields. The feasibility of this method is proved by experiments, and it provides theoretical support and guidance for the practical application of a multi-beam sounding system in a seabed topography survey.

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