Localization of Underwater Submarines Based on a Two-Dimensional Trajectory Tracking Model

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Abstract. With the increasing development and utilization of marine resources, the demand for precise localization of underwater vehicles in complex marine environments has also been growing. This study constructs a multi-level positioning model for an underactuated underwater vehicle. In the horizontal direction, considering the influence of ocean currents, a two-dimensional trajectory tracking model is developed to describe the kinematics and dynamics characteristics of the underactuated underwater vehicle. The model considers different combinations of initial velocities and ocean current directions to obtain the displacement of the underwater vehicle at different times. In the vertical direction, a gravity-buoyancy model is established, and environmental factors such as seawater damping coefficient, salinity, and temperature are considered, introducing uncertainties. The model is then subjected to robustness verification. Experimental results demonstrate that the proposed positioning model exhibits good robustness and accuracy, providing effective prediction and control for the localization of submarine.

Keywords: Submarine Positioning, Two-dimensional Trajectory Tracking, Gravity-Buoyancy Model.

1. Introduction

With the continuous advancement of modern technology, people now have the means to explore every corner of the Earth. Safety is paramount in engineering endeavors. While striving to ensure the safety performance of submarines, potential hazards should not be overlooked. During underwater exploration, various potential faults may occur with submarines, such as communication failures, mechanical malfunctions, and electrical faults. In the event of such incidents, it is necessary to promptly determine the position of the submarine and implement a rescue plan to prevent or minimize casualties caused by malfunctions. Only by meeting these requirements can the safety of underwater exploration be maximized. Therefore, establishing an accurate submarine positioning system is crucial for initiating search and rescue operations.

In order to achieve accurate submarine positioning, marine scientists and engineers have been striving to find more effective positioning methods. Through literature review, it was found that previous research has achieved certain results in the field of submarine positioning. By considering factors such as navigation time and ocean current disturbances, a path search algorithm was proposed, and a path tracking control model was established to optimize the smoothness, obstacle avoidance, and ocean current issues of path planning [1-2]. By using reinforcement learning, deep learning, and other methods, a node position prediction model was proposed, a multi-AUV cooperative navigation algorithm was proposed, and the problem of node localization in underwater networks was studied [3-5]. By considering various factors such as the angle of fall, height of fall, and physical properties of objects falling into the water, the influence of the water environment on cylindrical objects was studied [6]. Additionally, by introducing two spherical coordinate transformations and the concept of direction index correction (EMO), the problem of three-dimensional trajectory tracking of torpedo-shaped underwater vehicles was studied [7]. Furthermore, indirect positioning of UUVs was achieved through ultra-short baseline measurements. This method uses a spatial position geometric model and proposes a new adaptive tracking control algorithm [8].

Despite significant progress in submarine positioning research, a comprehensive theoretical analysis of the various influencing factors in submarine positioning remains lacking. To address these issues, this paper will focus on analyzing the effects of factors such as ocean currents, salinity, density, temperature, and damping coefficients on submarine positioning, and propose corresponding solutions. This study utilizes seafloor information data obtained from the National Marine Data Center. It conducts research in both horizontal and vertical directions, integrating marine environmental factors to propose a positioning method for underwater submarines.

2. The basic fundamental of Submarine Positioning Mode

In this model, analysis is conducted separately in the horizontal and vertical directions to predict the movement of a Submarine by considering various factors that affect its motion. This approach enables the establishment of a model capable of predicting the Submarine's position.

2.1. Horizontal Direction

Ocean currents, large-scale water movements in the ocean, are influenced by factors such as wind, the Earth's rotation, temperature variations, and differences in salinity. In the underwater domain, these currents are a significant factor affecting the horizontal motion of submarines. While there are various forms of ocean currents, this paper simplifies the dynamics model of the submarine vehicle by assuming that ocean currents are constant and non-rotating [9].

In the investigation of the Submarine's horizontal motion, it is assumed that the Submarine experiences no oscillatory, pitch, or roll movements. The position information of the Submarine is expressed using the vector $n = [x, y, \phi]^T$, where x and y denote its positions on the horizontal coordinate axes, and ϕ represents the deviation angle. Additionally, the velocity information of the underwater vehicle is represented by the vector $\boldsymbol{v} = [u, v, r]^T$, where u denotes the search speed, v represents the lateral velocity, and r is the yaw angular rate. Consequently, the vector equation describing the underwater motion of the Submarine can be formulated as:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}, \tag{1}$$

$$\boldsymbol{R}(\psi) = \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Where the matrix $\boldsymbol{R}$ is the position transformation matrix, satisfying $\boldsymbol{R}^\top \boldsymbol{R} = \boldsymbol{R}\boldsymbol{R}^\top = \boldsymbol{I}$, and $\det(\boldsymbol{R}) = 1$.

After determining the motion of the Submarine, the influence of ocean currents on its movement is further considered. The vector indicating ocean current velocity is represented as $\boldsymbol{V}_c = [V_{cx}, V_{cy}, 0]^T$, where $V_{cx} = V_c \cos(\beta_c)$ and $V_{cy} = V_c \sin(\beta_c)$, with $\boldsymbol{V}_c$ being the absolute velocity magnitude of the ocean current, and β_c representing the direction of the ocean current velocity.

Furthermore, the Submarine's velocity relative to the ocean current, $\boldsymbol{V}_r = \boldsymbol{V}_t - \boldsymbol{V}_c$ (in vector form), can be obtained, with its direction given by $\phi_r = \phi + \beta_r$, where β_r is the relative offset angle, and its magnitude is $\beta_r = \arctan\left(\frac{v_r}{u_r}\right)$.

Having considered the effect of ocean currents on the Submarine, the kinematic properties of the Submarine's motion can be described using the relative velocity of the Submarine to the ocean current and the velocity vector of the ocean current acting on the Submarine. This is expressed component-wise as:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu} + \left[V_{cx}, V_{cy}, 0\right]^T, \tag{3}$$

$$\begin{cases} \dot{x} = u_r \cos\psi - v_r \sin\psi + V_{cx} \\ \dot{y} = u_r \sin\psi + v_r \cos\psi + V_{cy}, \\ \dot{\psi} = r \end{cases} \tag{4}$$

Simplify the dynamics model of the Submarine by excluding overly complex scenarios, considering only three symmetric planes and one neutral buoyancy plane. Additionally, disregard damping terms beyond the second order, focusing solely on first and second-order damping terms. The resulting dynamic equations for the Submarine are as follows:

$$\boldsymbol{M}\dot{v}_r + C(v_r)v_r + D(v_r)v_r = \tau, \tag{5}$$

Were

$$M = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & 0 & m_{33} \end{pmatrix}, \tag{6}$$

$$C(v_r) = \begin{pmatrix} 0 & 0 & -m_{22}v_r \\ 0 & 0 & m_{11}u_r \\ m_{22}v_r & -m_{11}u_r & 0 \end{pmatrix}, \tag{7}$$

$$D = \begin{pmatrix} d_{u_r} & 0 & 0 \\ 0 & d_{v_r} & 0 \\ 0 & 0 & d_r \end{pmatrix}, \tag{8}$$

M is the mass matrix of the Submarine, with m_{11} representing the mass in the surge direction, accounting for the influence of lateral forces (X_u). m_{22} corresponds to the mass in the sway direction, considering the effect of longitudinal forces (Y_v). m_{33} denotes the mass in the yaw direction, factoring in the influence of sway moment (N_r). D is the hydrodynamic damping matrix, were

$$d_{u_r} = X_u + X_{|u|u} \,|\, u_r \,|, \tag{9}$$

$$d_{v_r} = Y_v + Y_{|v|v} \,|\, v_r \,|, \tag{10}$$

$$d_r = N_r + N_{|r|r} \,|\, r \,|, \tag{11}$$

The matrix MM is the rigid-body mass and inertia, C is the Coriolis and centripetal matrix, and the vector $u = [u_s; 0; u_r]^T$ represents the external control input vector. The variable u_s denotes the control force for the Submarine's surge motion, and t_r represents its yaw motion control torque. These control forces and torques impact the Submarine's underwater motion. Since the underwater robot studied in this paper is underactuated in the sway direction, t_v is zero.

From the above equations, the component form of Equation (4) can be obtained:

$$\dot{u}_r = \frac{m_{22}}{m_{11}} v_r r - \frac{d_{u_r}}{m_{11}} u_r + \frac{\tau_u}{m_{11}}, \tag{12}$$

$$\dot{v}_r = -\frac{m_{11}}{m_{22}} u_r r - \frac{d_{v_r}}{m_{22}} u_r, \tag{13}$$

$$\dot{r} = -\frac{(m_{22} - m_{11})}{m_{33}} u_r v_r - \frac{d_r}{m_{33}} r + \frac{\tau_r}{m_{33}}, \tag{14}$$

Where $\dot{u}_r$ represents the rate of change of velocity in the horizontal (surge) direction for the Submarine. $\dot{v}_r$ Denotes the rate of change of velocity in the vertical direction. r represents the rate of change of rotational velocity, specifically the yaw rate.

Ultimately, to establish the relationship between position and time, it is necessary to integrate. The above dynamic equations. The integration process yields equations expressing the velocity components as functions of time.

(1) The direction along the x-axis:

$$v_{x1} = -[\frac{\ln(e^{c(\alpha-\beta)t}-1)}{c} + c(\alpha-\beta)t] + \beta t, \tag{15}$$

With $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2c}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2c}$.

$$v_{x2} = \frac{\ln cmt}{c} + \alpha t, \tag{16}$$

With $\alpha = -\frac{b}{2c}, \beta = -\sqrt{\frac{b^2}{4c^2}+\frac{a}{c}}$.

Where υ_{x1} or υ_{x2} represents the displacement of the submarine along the x-axis. $a = \frac{m_{22}}{m_{11}} v_r r + \frac{\tau_u}{m_{11}}$, $b = -\frac{X_u}{m_{11}}$, $c = -\frac{X_{|u|u}}{m_{11}}$. And when $b^2 > 4ac$ is considered, equation (15) is adopted, else if $b^2 < 4ac$ is considered equation (16) is adopted.

(2) The direction along the y-axis:

$$v_{y1} = \frac{\ln(e^{c(\alpha-\beta)t}-1)}{c} + c(\alpha-\beta)t + \beta t, \tag{17}$$

With $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2c}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2c}$.

$$v_{y2} = \frac{\ln(-cmt)}{c} + \alpha t, \tag{18}$$

With $\alpha = -\frac{b}{2c}, \beta = -\sqrt{\frac{b^2}{4c^2}+\frac{a}{c}}$.

Where υ_{y1} or υ_{y2} represents the displacement of the submarine along the y-axis. $a = \frac{m_{11}}{m_{22}} v_r r$, $b = -\frac{Y_v}{m_{22}}$, $c = -\frac{Y_{|v|v}}{m_{22}}$. And when $b^2 > 4ac$ is considered, equation (17) is adopted, else if $b^2 < 4ac$ is considered equation (18) is adopted.

(3) In the direction of rigid body rotation:

$$r_1 = \beta + \frac{\beta-\alpha}{e^{c(\alpha-\beta)t}-1}, \tag{19}$$

With $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2c}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2c}$.

$$r_2 = m\tan(cmt) + \alpha, \tag{20}$$

With $\alpha = -\frac{b}{2c}, m = \frac{\sqrt{4ac-b^2}}{2c}$.

Where υ_{r1} or υ_{r2} represents the angular velocity of the submarine's rotation. $a = -\frac{(m_{22}-m_{11})}{m_{33}} v_r u_r + \frac{\tau_u}{m_{33}}$, $b = -\frac{N_r}{m_{33}}$, $c = -\frac{N_{|r|r}}{m_{33}}$. And when $b^2 > 4ac$ is considered, equation (19) is adopted, else if $b^2 < 4ac$ is considered equation (20) is adopted.

For the force analysis of the submarine, the force analysis results are shown in Figure 1.

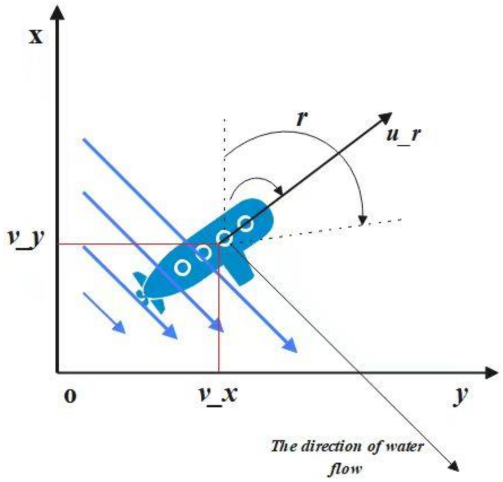


Figure 1. Conceptual diagram of submarine motion.

2.2. Vertical Direction

During vertical movement, a submarine encounters variations in salinity, temperature, and pressure within the surrounding seawater, leading to changes in seawater density and, consequently, in the submarine's displacement volume. This discussion will address the effects of salinity, temperature, and pressure individually to ascertain the resultant change in buoyancy for the submarine and subsequently determine the magnitude of the buoyant force it experiences [10].

Considering temperature's influence on seawater density and submarine displacement volume while assuming other factors remain constant, we can derive the change in buoyancy during the submarine's descent process as follows:

$$\Delta B = B - B_0 = g(\Delta\rho_t V_0 - \rho_0 \Delta V_t - \Delta\rho_t \Delta V_t), \tag{21}$$

Here, ρ_0 is the initial seawater density, $\Delta\rho_t$ and ΔV_t represent the changes in seawater density and submarine displacement volume due to temperature variations.

Similarly, the depth of descent affects both seawater density and AUV displacement volume. The change in buoyancy due to the variation in descent pressure is given by:

$$\Delta B = B - B_0 = g(\Delta\rho_p V_0 - \rho_0 \Delta V_p - \Delta\rho_p \Delta V_p), \tag{22}$$

Here, $\Delta\rho_p$ and ΔV_p represent the changes in seawater density and submarine displacement volume due to pressure variations. The increase in buoyancy for the submarine is linearly related to the pressure change, and the greater the pressure, the greater the increase in buoyancy. In the descent process at greater depths, the buoyancy increase due to pressure is a significant factor.

Considering the influence of salinity on both seawater density and submarine displacement volume, under the assumption of constant other factors, the expression for the change in submarine buoyancy due to salinity variation is given by:

$$\Delta B = 709\Delta S g V_0, \tag{23}$$

Here, ΔS is the variation in seawater salinity. Therefore, the greater the salinity in the seawater, the greater the buoyancy experienced by the submarine.

Combining the impacts of the three aspects on buoyancy, the change in buoyancy with the depth of descent is given by:

$$\Delta B = g(\Delta\rho_t V_0 - \rho_0 \Delta V_t - \Delta\rho_t \Delta V_t) + 709\Delta S g V_0 + g(\Delta\rho_p V_0 - \rho_0 \Delta V_p - \Delta\rho_p \Delta V_p), \tag{24}$$

Conducting force analysis on the submarine, the net external force in the z-axis direction is given by:

$$F_z = g(\Delta\rho_t V_0 - \rho_0 \Delta V_t - \Delta\rho_t \Delta V_t) + 709\Delta S g V_0 + g(\Delta\rho_p V_0 - \rho_0 \Delta V_p - \Delta\rho_p \Delta V_p) - G + \rho_0 g V_0, \quad (25)$$

The acceleration of the submarine is given by:

$$a = \frac{F_z}{m}, \quad (26)$$

Integrating the acceleration yields the velocity-time equation:

$$v(t) = \int a(t)dt + z_0, \quad (27)$$

Here, z_0 is the initial vertical displacement. Further integration provides the relationship between vertical displacement and time:

$$z(t) = \int a(t)dt + z_0 v, \quad (28)$$

Here, $z_0 v$ is the initial vertical velocity. Considering the influence of underwater terrain, when the submarine is in motion and the net external force is downward, reaching the seafloor results in the submarine coming to a halt. In other words, the underwater terrain acts as a boundary constraint on the submarine's motion; otherwise, the submarine would continue to be subject to various forces.

3. Results

3.1. Sensitivity Analysis of Varied Initial Velocities

When the submarine encounters a malfunction, it loses engine power but retains a certain initial velocity. Besides the influences of ocean currents and resistance, the submarine's velocity undergoes changes. The initial velocity is a crucial factor affecting the submarine's movement distance. Since communication is disrupted after the malfunction occurs, the exact initial velocity is often unknown. In such cases, we assume the Submarine moves in the originally intended direction with a certain initial velocity. Assuming other influences on the submarine remain constant, we vary the magnitude of the initial velocity, simulate its trajectory, and obtain the distance projected on a single axis in the coordinate system, as illustrated in Figure 2.

From the motion curves in the figure, it is evident that in the simulated results, after a malfunction occurs, the slopes of the distance traveled curves vary significantly. The submarine covers considerably different distances over a certain period, indicating that the initial velocity of the submarine has a substantial impact on its motion.

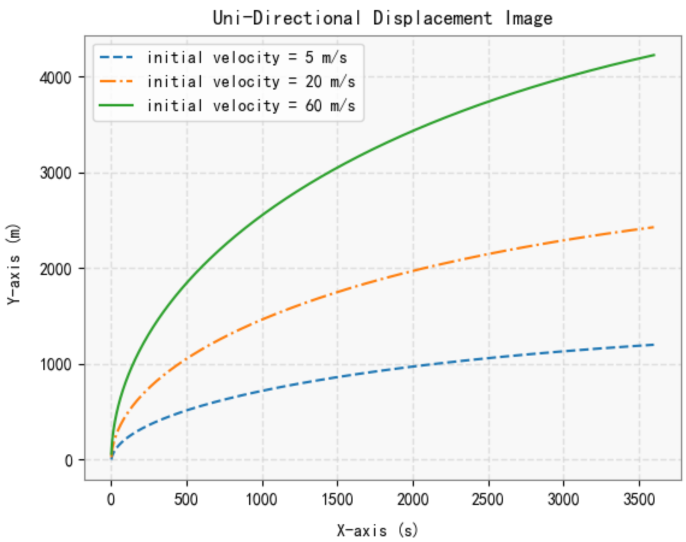


Figure 2: Sensitivity Analysis Graph for Initial Velocity Variation

3.2. Sensitivity Analysis of Damping Coefficients

Damping coefficients are crucial parameters describing the magnitude of resistance experienced by the submarine in the ocean. After losing power, the primary source of external force acting on the submarine is the force exerted by seawater, making damping coefficients critical. Here, simulations are conducted to analyze how changes in second-order linear damping coefficients and hydrodynamic rotational damping coefficients affect submarine motion, focusing on their sensitivity.

In this section, simulations were conducted to analyze the sensitivity of submarine motion to changes in damping coefficients. The simulations were performed separately for different magnitudes of second-order linear damping coefficients and hydrodynamic rotational damping coefficients, while other factors were kept constant.

The analysis began by studying the sensitivity arising from second-order damping coefficients. The initial velocity was set to 5 m/s, and other factors remained constant. The researchers sequentially increased the second-order linear damping, obtaining simulated submarine motion trajectory projections on a single axis of the coordinate system, as depicted in Figure 3.

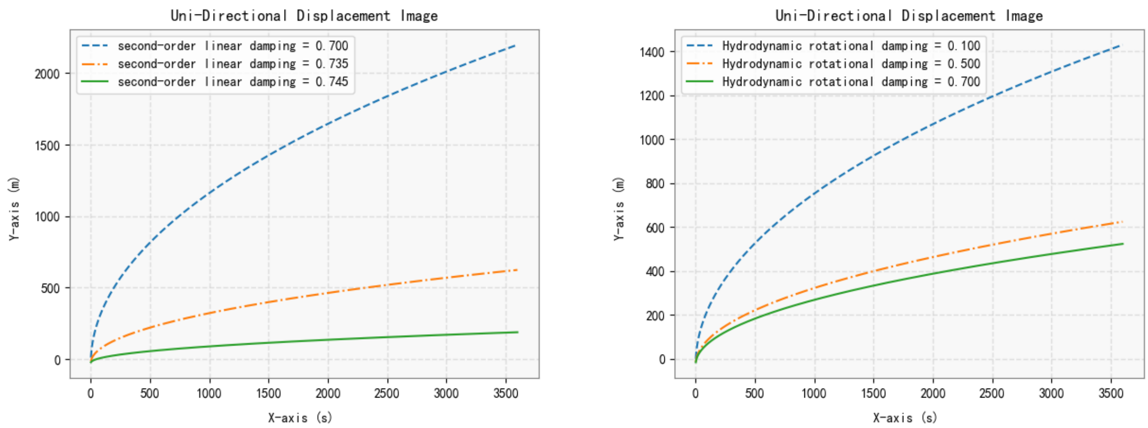


Figure 3. Uni-Directional Displacement

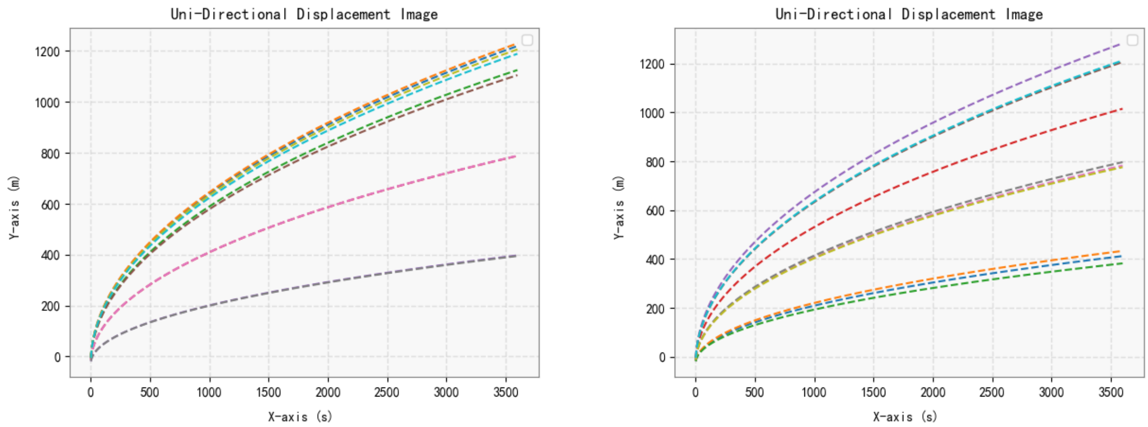
As the second-order linear damping coefficient increases, the resistance acting on the underwater vehicle intensifies. Analysis of the distance-traveled curves indicates a consistent trend: as the second-order linear damping coefficients successively increase to 0.700, 0.735, and 0.745, the travel distance of the underwater vehicle gradually decreases. This finding suggests that the second-order linear damping coefficient has a significant impact on the travel distance of the underwater vehicle.

Subsequently, the influence of hydrodynamic rotational damping coefficients on the motion of an underwater vehicle is investigated. With the initial velocity kept constant at 5 m/s and other factors unchanged, the specific effect of hydrodynamic rotational damping coefficients is analyzed. By varying their values and simulating the motion of the underwater vehicle, the simulated trajectory of the underwater vehicle's travel distance projection on a single axis in the coordinate system is obtained, as illustrated in Figure 3.

As the hydrodynamic rotational damping coefficient varies from 0.100 to 0.700, the speed of the submarine's motion gradually decreases, and its travel distance diminishes correspondingly with the increasing damping coefficient. This indicates that larger values of the hydrodynamic rotational damping coefficient have a more pronounced impact on the submarine's motion. Moreover, the influence resulting from the coefficient's increase from 0.500 to 0.700 is not as significant as the change observed from 0.100 to 0.500. This suggests that once the damping coefficient reaches a certain level, its effect on the travel distance becomes less pronounced compared to the initial variations.

3.3. Comprehensive Simulation of Multiple Random Factors

To explore the combined stochastic effects of various factors on the magnitude of submarine unidirectional displacement, randomness was introduced to the aforementioned studied factors (initial velocity, second-order linear damping, and second-order rotational damping). Each factor was perturbed by random fluctuations of ±10% and ±20% around their typical values to simulate real-world conditions in the Ionian Sea as accurately as possible. Ten sets of experiments were conducted with random variations, and simulated results for the motion trajectory's unidirectional projection on the coordinate axis were obtained, as illustrated in Figure 4.



a) Impact of 10% Random Factor Fluctuations b) Impact of 20% Random Factor Fluctuations

Figure 4. Comprehensive Simulation of Multiple Random Factors.

The diverse perturbations lead to varying displacement distances for the underwater vehicle. Following a malfunction, the submarine achieves its farthest distance of 1252 meters at 3500 seconds, while the closest distance is only 381 meters, resulting in a significant difference of 871 meters. Therefore, the perturbations have a pronounced impact on the submarine's movement.

Through the aforementioned experiments, it has been identified that the initial velocity, linear damping, and rotational damping in the Ionian Sea significantly influence the trajectory of the underwater vehicle.

4. Conclusions

To more accurately study the dynamics and positioning of submarines in complex underwater environments, a multi-level positioning model for an underactuated underwater vehicle was established by considering factors such as ocean currents, salinity, temperature, and damping coefficients. This model, which simplifies the dynamics model and models the effects of ocean currents on submarine motion from both horizontal and vertical directions, also investigates and models the effects of first and second-order damping terms, salinity, temperature, and pressure on submarine buoyancy and displacement. Simulation and sensitivity analysis of the model have verified its effectiveness, ensuring both accuracy and robustness. The results demonstrate that the submarine positioning model exhibits good adaptability and high accuracy in positioning under various transformations of underwater environments.

In future research, the submarine positioning model will be further strengthened and improved for potential application in practical submarine operations.

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