Synthesis of Ecosystem Impacts of Lampreys Sex Ratio Based on the Von Bertalanffy and Lotka-Volterra Models

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Abstract. This study focuses on investigating the impact of changing sex ratios within the lamprey population on the broader ecosystem. To achieve this objective, a Von Bertalanffy growth model is developed, considering resource availability, to analyze how alterations in the sex ratio of lampreys influence the ecosystem. Firstly, the growth model of lampreys is integrated with ecosystem components. Secondly, unknown parameters of the model are obtained by simulating a comparative lamprey dataset. Additionally, an improved system dynamics model and metacellular automata model are utilized to simulate the effects of lamprey sex ratios on both the inorganic environment and the biological community, respectively. Furthermore, the study conducts a comprehensive analysis of resource utilization and reproductive success trends under varying sex ratios within a region, achieving a relative error of 5.5% between calculated and actual resource utilization values. The findings demonstrate that changes in sex ratio significantly impact population size fluctuations and ecosystem stability, with a sex ratio close to 0.5 favoring population reproduction and ecosystem stability. The refined model facilitates the attainment of population equilibrium over time turnover for both species, reflecting the ecosystem's self-regulation ability. These results provide insights into species' adaptation strategies in response to resource availability and shed light on the broader ecosystem implications of sex ratio changes across different species. This research contributes to a better understanding of ecological dynamics and informs future analyses of ecosystem responses to species-specific demographic shifts.

Keywords: Von Bertalanffy, Lotka-Volterra, Cellular Automata, System Dynamics Model.

1. Introduction

When the number of lampreys leads to the change in their sex ratio, it will have a significant impact on the larger ecosystem [1]. Using the traditional Von Bertalanffy model to analyze the growth of lampreys under the influence of external environment, food and other factors will have a greater impact on the prediction error of sex ratio [2-4]. In this paper, the improved Von Bertalanffy model is used to analyze the growth of lampreys under the influence of external environment, food and other factors, and the gender decision function is introduced into the growth equation to obtain the differential equation of sex ratio and population [5]. At the same time, the Lotka-Volterra population dynamic model was established to analyze the influence of the sex ratio of lamprey on the biological community [6-8], and an improved system-based dynamics model was established to analyze the influence on the inorganic environment. Finally, the influence of gender ratio on the two types of environment is simulated by cellular automata [9-11].The results show that fluctuations in sex ratio affect ecosystems and that balancing sex ratio is essential for ecological balance.

2. Impact of changing sex ratios on ecosystems

2.1. The Relationship Model between Population Sex Ratio and Ecosystem

This paper requires exploring the impact on larger ecosystems when the population of lampreys can change their sex ratio. Therefore, our specific analysis of lampreys is as follows:

2.1.1 Lampreys individual growth model

Firstly, the paper introduced the influence function $R(G,Z,E)$ of environmental factors on the growth of lampreys. Although ideally $R\approx 1$, considering the influence of food quantity and living temperature on the growth of lampreys, the expression is updated as shown in Formula 1:

$$R\left(G,Z,E\right)=\frac{Z\left(t\right)}{1+e^{-b\left(G-G_{out}\right)}}e^{-c\left(E\left(t\right)-E_{out}\right)^{2}} \tag{1}$$

Among them, b represents the slope of the curve of the effect of food amount on the individual growth of lamprey.

Therefore, the improved Von Bertalanffy growth equation [1] this paper constructed for the lamprey is shown in Formula 2:

$$V\left(t\right)=V_{\infty}\cdot\left(1-e^{-k\left(t-t_{0}\right)}\right)R\left(G\left(t\right),Z\left(t\right),E\left(t\right)\right) \tag{2}$$

In which, $V(t)$ represents the length of the individual at the moment time t. k means the growth rate. $Z(t)$ stands for the amount of food in the environment at time t.

2.1.2 Lamprey Sex Determination Model

Since the probability of sex transformation of the lamprey is related to the availability of resources, the sex transformation probability function in the construction is improved by considering the influence of resource availability factors, and its expression is shown in Equation 3:

$$S\left(V,Z\right)=\frac{1}{1+e^{\gamma\left(V-V_{sex}+\frac{dZ}{dt}\right)}} \tag{3}$$

Whereto, $S(V,E)$ denotes the probability function of the sex transition of the lamprey and the length threshold of the sex transition. γ depicts the threshold of resource availability that affects sex transformation.

In summary, associating the above equations yields a sex transition model for the lamprey, as shown in Formula 4:

$$\begin{cases} \mathrm{V}\left(t\right)=V_{\infty}\cdot\left(1-e^{-k\left(t-t_{0}\right)}\right)R\left(V\left(t\right),Z\left(t\right),E\left(t\right)\right) \\ Gender\ as\ time\ t=\begin{cases} Male & if\ S\left(V\left(t\right),Z\left(t\right)\right)<S_{threshold} \\ Female & if\ S\left(V\left(t\right),Z\left(t\right)\right)\geq S_{threshold} \end{cases} \end{cases} \tag{4}$$

Of which $S_{threshold}$ is the resource availability threshold that affects gender transition.

2.1.3 Population Capacity Growth Model

In the natural environment, food resources are often limited. When the population capacity of this species is affected by the environmental carrying capacity, it will decline after reaching a specific value, showing an "S" distribution. Hence, the population growth of the lamprey is in line with the logistic growth model.

Since the logistic model assumes that the intrinsic growth rate is a linear function, whereas changes in environmental resources show non-linear changes in population size, we used the Gompertz model to modify and improve the initial model.

In the Gompertz model, the paper assume that the growth rate of the two sex species is proportional to the remaining resources at the moment, as shown in Formula 5:

$$\begin{cases} \dfrac{dM}{dt} = \chi_1 \tau_M \left(1 - \dfrac{S_M(t)}{T_M}\right) N_M \\ \dfrac{dF}{dt} = \chi_2 \tau_F \left(1 - \dfrac{S_F(t)}{T_F}\right) N_F \end{cases} \tag{5}$$

$S_M(t)$、$S_F(t)$ denote the consumption of food by the male and female populations, respectively; χ_1、χ_2 correspond to the respective positive coefficients. The respective energy consumed can be specified as Formula 6:

$$\begin{cases} S_M(t) = s_M(t) + c_1^M \dfrac{dM}{dt} \\ S_F(t) = s_F(t) + c_1^F \dfrac{dF}{dt} \end{cases} \tag{6}$$

To obtain Formula 8, simply replace Formula 6 into Formula 5 as follows:

$$\begin{cases} \dfrac{dM}{dt} = \chi_1 \tau_M \left(\dfrac{T_M - s_M(t) - c_1^M \dfrac{dM}{dt}}{T_M + \tau_M c_1^M M} \right) \\ \dfrac{dF}{dt} = \chi_2 \tau_F \left(\dfrac{T_F - s_F(t) - c_1^F \dfrac{dF}{dt}}{T_F + \tau_F c_1^F F} \right) \end{cases} \tag{7}$$

Therefore, this paper constructs a differential equation of the sex ratio and the corresponding population number, as shown in Formula 8:

$$\frac{dR}{dt} = \frac{M}{M+F} = \frac{C_M + e^t}{C_F + C_M + 2e^t} \tag{8}$$

R, is the ratio of male lampreys to the total population. C_M, C_F are constants.

2.1.4 Sex Ratio Affects Ecosystem Models

(1) The impact of biological communities

The traditional Lotka-Volterra model makes it difficult to simulate the natural survival environment accurately, so this paper introduces the concept of population competition on its basis. The human influence on the lamprey is portrayed as the mortality rate, divided into natural mortality rate and mortality rate influenced by human activities.

In order to portray the impacts of different sex ratios of lampreys on the ecosystem, this paper establishes the population dynamics model of Lotka-Volterra for female lampreys and male lampreys, respectively. As shown in Equation 9:

$$\begin{cases} \dfrac{dM}{dt} = \tau_M M - d_M M + b_M \alpha_M LM \\ \dfrac{dF}{dt} = \tau_F F - d_F F + b_F \alpha_F LF \\ \dfrac{dN}{dt} = \tau_N N - d_N (M+F) + b_n \alpha_n LN \end{cases} \tag{9}$$

τ_M、τ_F represent the respective natural growth rates of male and female lampreys, while N denotes the environment's maximum carrying capacity for the total lampreys population.

(2) Effects of Inorganic Environment

This study examines the impact of changes in the lamprey population sex ratio on the inorganic environment of the ecosystem, using water resources as a representative case. A system dynamics model based on an improved framework is developed to analyze this impact. By exploring the internal relationships within the lamprey sex ratio, we comprehensively understand the organism's living environment. The system comprises four sub-models: water quality influence, water temperature influence, water flow influence, and pollutant influence.

The differential and LV determination equations in the system flow diagram are presented in Equation 10:

$$\begin{cases} y_j = f_A z_A + f_B z_B + f_C z_C \\ z_B(t) = z_B(t-dt) + z_B dt \\ y_s = f_r z_r + f_m z_m \\ z_r(t) = z_r(t-dt)(r_c - r_s)dt \\ y_z = z_t(r_t \alpha + 1) + z_h(r_h \beta + 1) \\ z_t(t) = z_t(t-dt) + z_t(t-dt) r_t \end{cases} \tag{10}$$

According to the LV model, it is necessary to determine the relationship between water quality and water temperature in the system differential equation, and the corresponding evolution with time has the following relationship:

$$\frac{dz_i}{dt} = r_i z_i - \frac{r_i}{m_i} z_i^2 - \frac{r_i d_i}{m_i} z_h z_t \tag{11}$$

Suppose it takes such a result $a_i = r_i, b_i = \frac{r_i}{m_i}, c_i = \frac{r_i d_i}{m_i}$, the above equation can be simplified to:

$$\frac{dz_i}{dt} = a_i z_i - b_i z_i^2 - c_i z_h z_t \tag{12}$$

In the formula, z_i、r_i is the population size and population growth of lamprey, d_i is the coefficient of interaction between them.

2.2. Simulation and Analysis

2.2.1 Improved System Dynamics Model

Given that differential equations belong to theoretical modeling, we still need to verify the accuracy and reasonableness of the results we seek. Therefore, we constructed a system dynamics model for the quantitative relationship between the ratio of females and males. Using the data related to the sex ratio of the lampreys populations in the past years and the above-mentioned influencing factors of the survival environment, we can get the effects on the inorganic environment under different sex ratios as shown in the Fig 1:

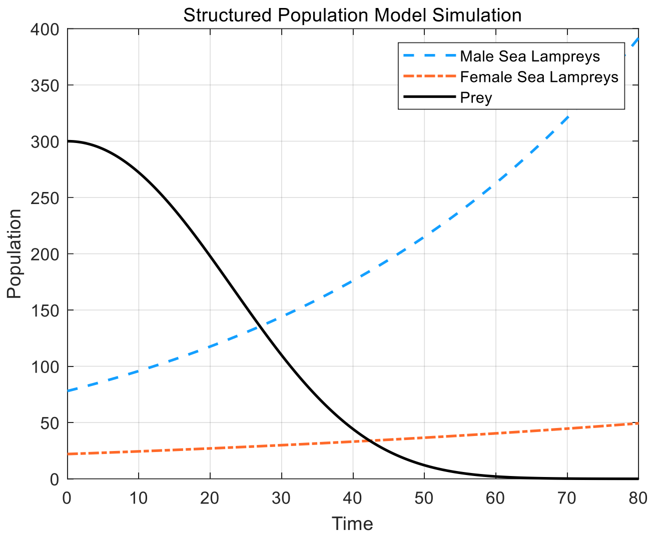

Figure 1. Simulation of structured population quantity model

The above figure verifies that the effects of changes in the sex ratio of lampreys on ecosystem dynamics, population size fluctuations, and ecosystem stability can be explored more precisely and that changes in the sex ratio between osprey and males may lead to different behaviors in terms of their predation and prey. If sex ratio changes are too drastic, they may lead to the instability of multiple populations in an ecosystem, making it difficult for them to maintain equilibrium. This may cause ecosystem-level instability, with knock-on effects on the interactions of other species, ultimately affecting the ecosystem as a whole.

2.2.2 Cellular Automata(CA) Model

Meanwhile, since the ABM model is mainly used to simulate the decision-making process of individuals, the parallel computing nature of the CA model makes it more efficient to simulate and compute on large-scale systems, especially when simulating at scale. Therefore, we chose to use the CA model to simulate the relationship between the population size of the seven-gill eel and the population size of other substances when the sex is changed, and the flowchart of the algorithm is shown in Fig 2.

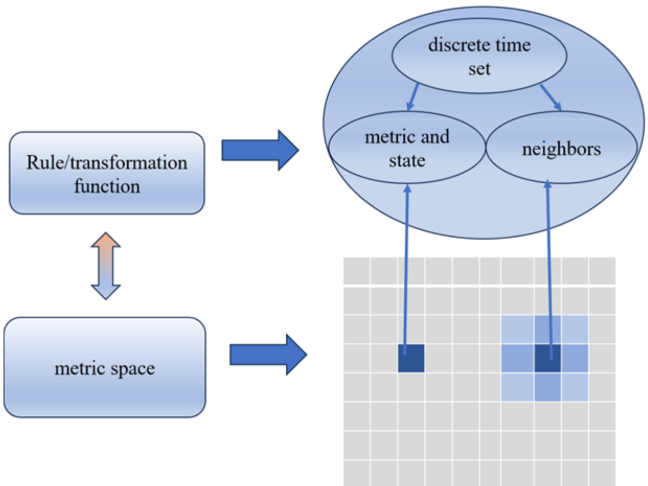

Figure 2. Working principle of cellular automata

Through this algorithm, we determined our initial values and other parameters by consulting much literature for better simulation, and the simulation results are shown in Fig 3:

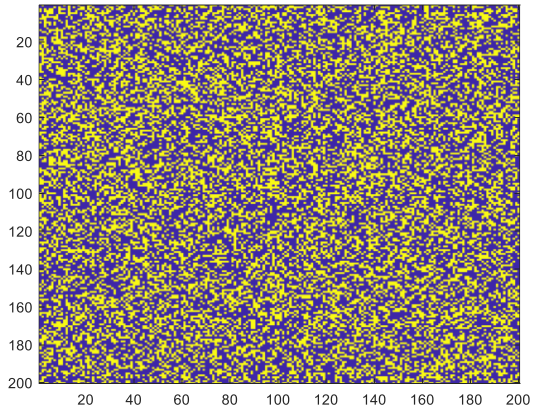

Figure 3. Cellular automata simulation process diagram

The above figure 3 can verify this: each pixel on the screen represents a lamprey, and lampreys have two states: life and death. The original rule is that a living lamprey will die if there are more than three living neighbors around it; a dead lamprey will be reborn only if there are just two living neighbors; at this time, the birth and death of the lamprey apt to be disordered, thus forming the chaotic scene shown in the figure.

2.3. The solutions for the question

2.3.1 Sex Ratio-based Resource Utilization and Reproductive Success Rate

Plotting via matlab simulation yields the following results to be displayed in Fig 4:

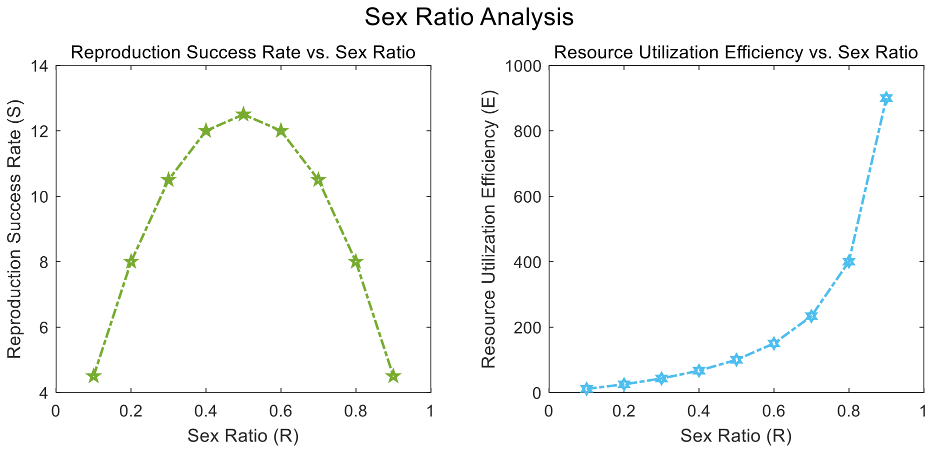

Figure 4. Sex ratio analysis

From the Fig 4, we can see that as the resource utilization rate increases, the sex ratio of males and females becomes closer and closer, and when the sex ratio is around 0.8, the resource utilization rate reaches the maximum, which is conducive to the reproduction of the population, but at the same time there will be a situation that when the resource utilization rate is too high, leading to over reproduction, the resource is insufficient to influence the upper trophic level and the next trophic level, which breaks the ecosystem; when the sex ratio of males and females is close to 0.5, the success of reproduction is highest. The reproductive success is highest when the sex ratio of males to females is close to 0.5, so when the sex ratios are close the number of offspring may not necessarily be high, and so the cycle repeats itself to maintain equilibrium.

2.3.2 Lampreys' Impact on Trophic Levels

Plotting via matlab simulation yields the following results to be displayed in Fig 5:

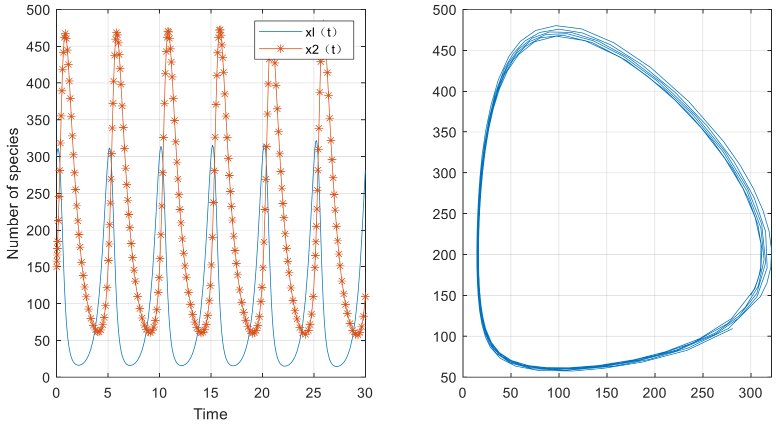

Figure 5. Lampreys and upper trophic level relationship

As can be seen, lampreys and the upper trophic level are mutually restricted. When the upper trophic level gradually increases, lampreys gradually decrease. When the upper trophic level increases to a certain amount, intense intra-species competition leads to a decrease in the number of lampreys, resulting in a decrease in the number of lampreys' natural enemies, which leads to an increase in the number of lampreys.

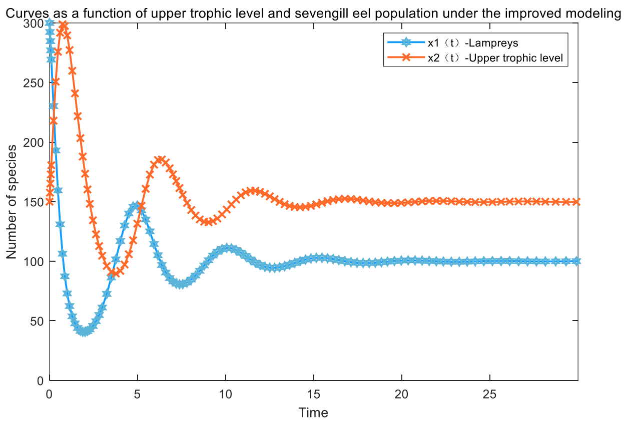

Figure 6. Trophic curve and lamprey increase with improved model

According to the Fig 6, after the improvement of the model, the time turnover between the two species will be carried out in the first period of the number of cycles; after the ecosystem regulation process, the number of both transformations in the mid-late slowly approaching leveling off, until finally reached a relative balance to meet the ecological needs, which is to achieve the ecosystem's self-regulation.

2.3.3 Inorganic Environment impacted by lampreys' sex ratios

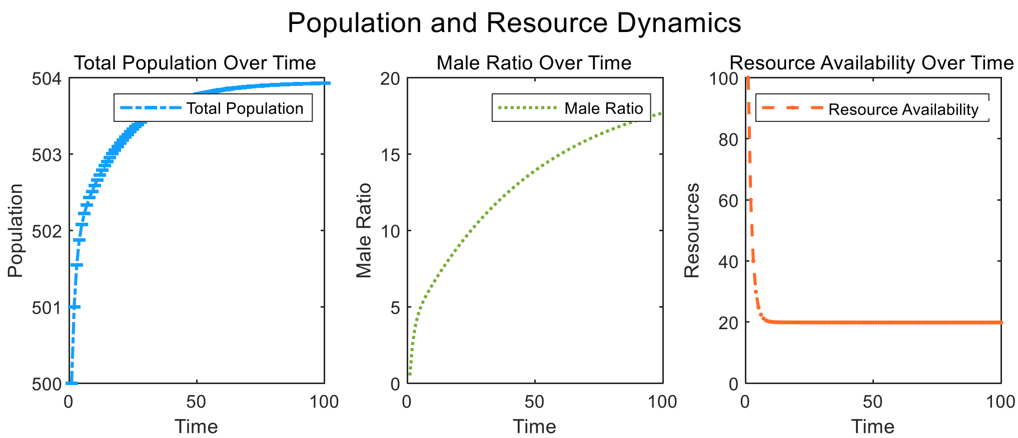

Figure 7. Population and resource dynamics

As can be seen from the Fig 7, the sex ratio of lampreys may also affect their population dynamics and ecological balance. Suppose an area has a disproportionately high proportion of male lampreys. This tendency may increase population size, as male lampreys grow faster and reproduce better than female lampreys. Overpopulation may impact the local ecosystem, as the lamprey is a link in the food chain, and an increase or decrease in its population may affect the survival of other species.

3. Conclusion

The changing trend of the sex ratio of lamprey provides a foundation for analyzing overall characteristics and establishing prediction models. In this study, the Von Bertalanffy model was used to analyze its growth. In the meantime, the Lotka-Volterra population dynamic model was established to analyze the influence of its sex ratio on the biological community, and an improved system-based dynamics model was established to analyze the impact on the inorganic environment. Finally, this paper simulates the change process of lamprey population by cellular automata. Experimental results demonstrated that variations in sex ratio significantly influenced ecosystem stability, ultimately reaching equilibrium through self-regulation mechanisms. However, extreme imbalances in sex ratio were found to cause irreversible damage to larger ecosystems.

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