

Research on Urban Rail Transit Operation Optimization Based on Target Planning

Jianglin Lu^{1,*}, Yaxin Zhang², Yiming Su¹

¹ School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China, 100044

² School of Mathematics and Statistics, Beijing Jiaotong University, Beijing, China, 100044

* Corresponding Author Email: 21252013@bjtu.edu.cn

Abstract. The optimization of train schedule is one of the classic problems in the traffic organization mode in the field of rail transit. In order to reduce the operation cost of operators and improve the service level of passengers, it is feasible and necessary to optimize the train operation road. This paper considers the factors of minimum number of trains and total mileage of trains, and minimizes the operating cost of enterprises and maximizes the passenger service level in the context of urban rail transit operation with large and small crossing schemes. We used the weighted coefficient method to combine multiple targets into a single optimization problem and perform the model solution using Lingo software. The results provide insights into the best train operation scheme, including the number of large and small crossings, operating distances, and the number of trains required for effective urban rail transit operations.

Keywords: Urban rail transit operation optimization, Target planning, Optimal operation scheme.

1. Introduction

The optimization of train schedule is one of the classic problems in the field of rail transit. Nowadays, China continues to promote the urbanization process, the scope of the urban circle is expanding, and the urban population is greatly increased, which makes the urban traffic problem increasingly prominent. As a transportation mode with large volume and fast and on time, urban rail transit has gradually become an important way to solve the problem of urban congestion. At present, some urban rail transit still implement the simple operation mode of single traffic, and do not adjust the actual situation of unbalanced passenger flow accordingly, leading to the waste of some transport capacity, which is not conducive to saving operating costs. Therefore, on the premise of ensuring passengers travel convenient, reduce the operating cost of rail transit, urban rail transit operation generally adopts the size of traffic operation mode, according to the passenger flow demand for different train operation order and operation interval, so as to clarify the train corresponding intersection type and to the moment, to achieve the goal of optimizing train schedule at the same time as far as possible to meet the lowest operating cost and the highest service level for passengers.

The train crossing scheme specifies the train running section, the return station and the operation log. Through the preparation of a reasonable train crossing scheme, the line transportation capacity and passenger service level can be effectively improved. Zhang [1] with the minimum passenger travel cost, minimum enterprise operating cost, passenger flow demand and line capacity matching prospects for the target function, to reconnecting marshalling train departure frequency and train marshalling number for the decision variable, considering the constraints of connecting marshalling train running scheme, considering the size of city rail reconnecting marshalling train running scheme optimization model, design with elite strategy of the dominant sorting genetic algorithm (NSGA-) to solve the model. An Zhilong [2] to passengers travel in transit waiting time cost and running train using minimum cost, with train tracking time interval and train passenger load rate model calculation constraints, establish dual target nonlinear planning model, using the shortest ideal point method to optimize the design target function, and using natural number coding genetic algorithm to solve the model; Li Zhengyang [3] By studying the multi-crossing and multi-formation scheme of urban rail lines, a mixed integer nonlinear planning model is established to determine the number of train formation and the number of trains in each crossing, and design a method to simplify the scale of

service network to improve the speed of model solving. Blanco class [4] Considering the problem of size crossing and train schedule compilation under the condition of urban rail transit network, and considering the constraints such as dynamic passenger flow, train accommodating capacity and transfer passenger flow, the mixed integer linear planning model is established. The objective function is to minimize the passenger cost and the line operation cost. According to the characteristics of the model structure, a solution algorithm based on the decomposition strategy is designed.

The train schedule specifies the arrival time, departure time, stop time and interval running time of the train number at the station, which is the basis for the preparation of the train bottom application plan. Yao Yu [5] By analyzing the coupling relationship between passenger flow demand, train schedule and train bottom application, the collaborative optimization model of train schedule and train bottom application plan is constructed, and the constraints include the minimum departure interval, dynamic passenger flow, train bottom number, train capacity, etc. Lu Lihong [6] This paper studies the schedule adjustment problem of urban rail transit line interruption, puts forward the adjustment measures of small crossing operation during the line interruption, and establishes two different schedule adjustment models. The model objective function is to minimize the number of stranded passengers on the platform, thus reducing the platform congestion and ensuring the smooth and safe operation of the line.

In order to reduce the operating costs of operators and improve the service level of passengers, the train operation scheme route plan design, according to the target plan to establish multi-target linear planning model, from the perspective of system optimization, the linear planning model solution tries to seek the best train operation scheme, to determine the transport capacity matches the actual passenger flow demand. At the same time, according to the train departure interval and tracking interval conditions, the model is improved and optimized, and the target function of the train stop time condition is added to solve the train stay time in each station. On this basis, the departure interval and tracking interval conditions are considered, and the train operation schedule is optimized by equal interval requirements.

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2. Establishment and solution of the train operation scheme model

2.1. Model building

In the size of urban rail transit operation scheme, the goal for the lowest operating cost and the highest passenger service level, from the perspective of system optimization to determine the optimal scheme, trying to seek the size of the optimal road design scheme, at the same time to ensure that the actual train travel scheme can meet the actual demand of passenger flow, this paper establishes a multi-objective linear planning model, through the linear planning model to determine the optimal scheme.

Goal 1: Enterprise operating cost level

From the perspective of enterprise operation, the lowest cost is the minimum sum of fixed cost and variable cost, which is transformed into the minimum number of trains and the minimum total mileage of the train combined with actual problems.

Minimum number of trains:

$$\text{Min}Z_1 = \sum_{i=1}^k n_i \quad (1)$$

Minimum total train mileage:

$$\text{Min} Z_2 = \sum_{i=1}^k n_i s_i \quad (2)$$

Goal 2: Passenger service level

From the perspective of passenger service level, the shorter the waiting time on the platform, the faster the passenger can take the train, the higher the service level, and the lower the versa. Therefore, minimizing the waiting time of passengers at the platform is one of the indicators. Since there is no

traffic design scheme and the number of trains opened in this stage, the stay time between stations and the tracking time between trains cannot be evaluated, so the passenger service level in this section only considers the waiting time of passengers and train running time, but not the midway stop time of trains.

Set the train departure interval is t , in $[0, t]$, the number of passengers from s station to d station is q , including the first i passenger arrived at the station for t_i , in $[0, t]$ under the premise of q events, the moment of occurrence (t_1, t_2, \dots, t_q) can be regarded as mutually independent random variables, and obey uniform distribution on $[0, t]$, so there are:

$$E(t_i) = \frac{t}{2} \tag{3}$$

Therefore, the expected value of the total passenger waiting time is:

$$E[\sum_{i=1}^q (t - t_i)] = E(\sum_{i=1}^q t) - E(\sum_{i=1}^q t_i) = qt - \frac{qt}{2} = \frac{qt}{2} \tag{4}$$

Assuming that n train runs per hour, then

$$t = \frac{60}{n} \tag{5}$$

So there is

$$E[\sum_{i=1}^q (t - t_i)] = \frac{30q}{n} \tag{6}$$

Here n refers to the number of trains per h , so for a one-way line, the average waiting time for all passengers per h is:

$$Z_3 = \sum_{s=1}^{n-1} \sum_{d=s+1}^n E[\sum_{i=1}^{q_{sd}} (t - t_i)] = \sum_{s=1}^{n-1} \sum_{d=s+1}^n q_{sd} \times \frac{30}{\sum_{i=1}^k n_i \delta_{sd}^i} \tag{7}$$

$$Z_4 = \sum_{i=1}^k n_i T_i \tag{8}$$

For the solution of multi-objective function, this paper is usually a single objective function. There are two kinds of common transformation ideas: (1) weighting coefficient method, which assigns a weight for each target, merges multiple targets into one target, so as to turn into a single target optimization problem. (2) Hierarchical optimization method, that is, the optimization of the first target, the second target under the condition of the first target, namely the hierarchical optimization method.

First of all, this paper should first unify the target unit, with the cost price as the measurement standard.

- c_1 the constant is the fixed use cost of the train during the study period, and the unit is yuan/column;
- c_2 the constant is the running cost per kilometer of the train, and the unit is yuan/km;
- c_3 taking the constant is the cost of the average passenger waiting time value in yuan/min;
- c_4 the constant is the passenger time value cost in the unit of yuan/min.

Different indicators can be converted into the same measurement standard, and can be added.

$w_1 w_2$ in this paper, the weighting coefficient method is adopted, and the proportion of the passenger service level is set up. The optimal operation scheme, operation cost, passenger waiting time cost and total cost under different weights will also be different. With the increase of the weight of operating costs, that is, the decision makers pay more attention to the interests of the operators, the operating cost will gradually decrease, but at the same time, the cost of waiting time for passengers will rise, and the waiting time for passengers will increase. According to the evaluation, this paper compares the enterprise operating cost and passenger service level, which means $w_1 = w_2$.

The target function can be written as follows:

$$\begin{aligned}
 \text{Min}Z = & \sum_{i=1}^k n_i c_1 + \sum_{i=1}^k c_2 n_i s_i + \sum_{s=1}^{n-1} \sum_{d=s+1}^n q_{sd} \times \frac{30}{\sum_{i=1}^k n_i \delta_{sd}^i \times c_3} + \sum_{i=1}^k c_4 n_i T_i \\
 & \left\{ \begin{array}{l} \text{And for integers } n_i \geq 0 \\ 1860 \sum_{i=1}^k n_i \delta_{sd}^i \geq Q_{sd} \end{array} \right. \quad (9)
 \end{aligned}$$

2.2. Model solving

(1) Available from the data sheet, a total of 14 stations can be used as the beginning or end of the intersection, including stations 1, 2, 5, 8, 10, 14, 17, 18, 21, 22, 25, 26, 27, 30. On the premise that the minimum number of stations through the large intersection section is [3, 24] station, all possible intersections are constructed with a total of 76 qualified intersections, as shown in Figure 1.

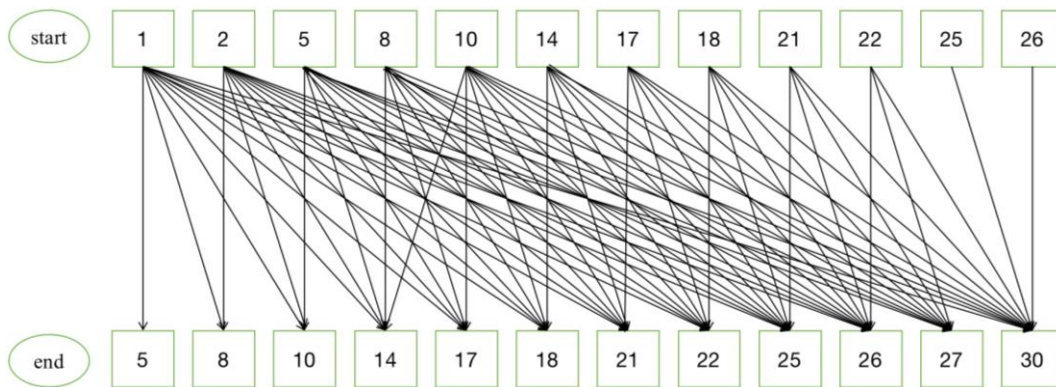


Figure 1. All possible train crossings

(2) Using the data table, we can solve the sum of the running time and the running distance of each intersection.

(3) Using the set of qualified intersection found in Step 1, we can judge the relationship between the starting and ending point of each passenger and the known intersection, that is, whether the passenger can take a train of a certain intersection from the departure station to the destination station. If the ride is available, the value is 1; if not, the value is 0.

(4) solve it using lingo software. Bring the passenger intersection table, operation schedule and operation distance table obtained in steps 2 and 3 into the lingo code for operation. Through the operation results, the train operation scheme can be obtained, that is, the operation number of big road, the operation interval and operation number of small road. The opening scheme is shown in Table 1.

Table 1. Train operation plan

| | Run interval | operating mileage | Number of open line |
|------------|----------------------|-------------------|---------------------|
| Big road | Station 1-30 | 40.168km | 25 column |
| Small road | Station 1-25 | 33.398km | 3 column |
| Small road | Station 2-Station 25 | 32.018km | 2 column |

3. Establish and solve the operation scheme model of equal interval trains

3.1. Model building

On the basis of the solution of the solution in the previous chapter, the operation design is the lowest operating cost and the highest passenger service level.

First of all, in the previous chapter, because the train plan has not been determined, the train stop time cannot be considered. After solving the train plan, this paper considers the stop time constraint according to the stop of each transit line. In the solution scheme of the previous chapter, the value i of n_i is certain, and the number of open rows is also certain, so the value of the original objective

function has been determined. If the objective function continues to be established with the goal of the lowest operating cost and the highest passenger service level, the fixed value can be ignored. Here, this paper only considers the stay time of the passengers in the train time index in the passenger service level. The objective function is as follows:

$$\begin{aligned} \text{Min-Z} &= n_9 \sum_{j=2}^{24} T_j + n_{11} \sum_{j=2}^{29} T_j + n_{19} \sum_{j=3}^{24} T_j \\ &\begin{cases} 420 \leq T_j \leq 120 \\ T_j \geq q_j \times 0.04 \end{cases} \end{aligned} \tag{10}$$

The length of departure interval will be limited: too short departure interval will affect the safety of train operation; and too long departure interval will increase the average waiting time of passengers, thus affecting the service level. Similarly, the stop time is limited. Generally speaking, the stop time at the station is proportional to the number of passengers on and off the station. In addition, it should be noted that the two trains should keep a certain safety interval (tracking interval time). Based on this model, this paper adjusts the train scheme according to the above specified conditions, and constantly updates and iterates until the optimal scheme [7] is produced.

The goal of the equal interval driving adjustment is to keep the tracking interval between each train consistent, [8], which can be abstractly expressed as shown in Figure 2. In the figure, the circumference of the large circle is the time required for the train to complete a crossing according to the default stop time and the default running time. A small dot is a train, the next dot in the clockwise direction is the previous train in its running direction, and the side length between the dots is the tracking interval between the trains in the default way. The large circle on the left is called the "initial state circle", which represents the relative position of the initial state train in the time dimension, and the right large circle is called the "equally spaced state circle", which represents the relative position of the equally spaced running state train in the time dimension. The goal of equal interval adjustment is to adjust the chaotic left circle to the evenly distributed right circle, as shown in Figure 2.

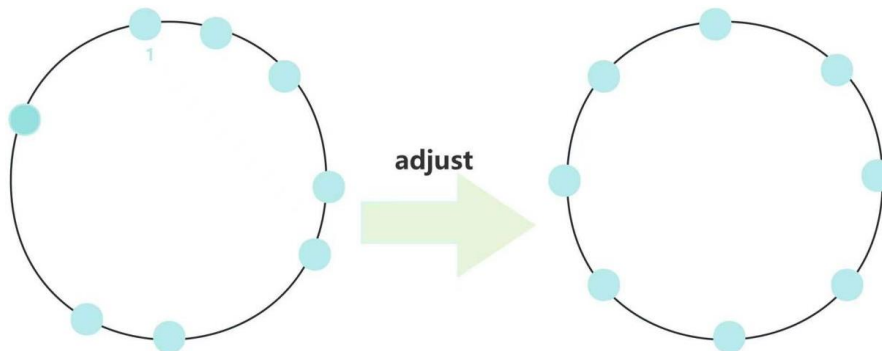


Figure 2. Train equal interval adjustment diagram

To move the position of the small dot on the large circle, you can only change the stop time and running time. On the basis of the default time, increase the stop time and running time, the interval between the train and the front car will be enlarged, and the interval with the rear car will be smaller, so the small dots move counterclockwise; similarly, shorten the stop time and running time, and the small dots move clockwise. It should be noted that the adjustable range of stop time should be limited (passengers may be too late to get on and off; because the stop time is too long, passengers may experience anxiety); and each platform has different passenger traffic, so the minimum and maximum stop time [9] for each platform should be set reasonably.

The initial state circle and the equally spaced state circle are overlapped together, as shown in Figure 2. It is clear how much time each car needs to adjust to change from the initial state (solid point) to an ideal equal-interval state (hollow point). The time to be adjusted is defined as the sum of the time changes of the train from the initial state to the ideal interval state. If the duration to be adjusted is positive, it needs to increase the stop time, or increased by adjusting the running level; if

the adjustment duration is negative, the stop time needs to be reduced or reduced by adjusting the running level length; if the duration is zero, adjustment is not required [10].

In this regard, the following algorithm model is established (as shown in Figure 3) to adjust the train tracking time interval and the departure time interval:

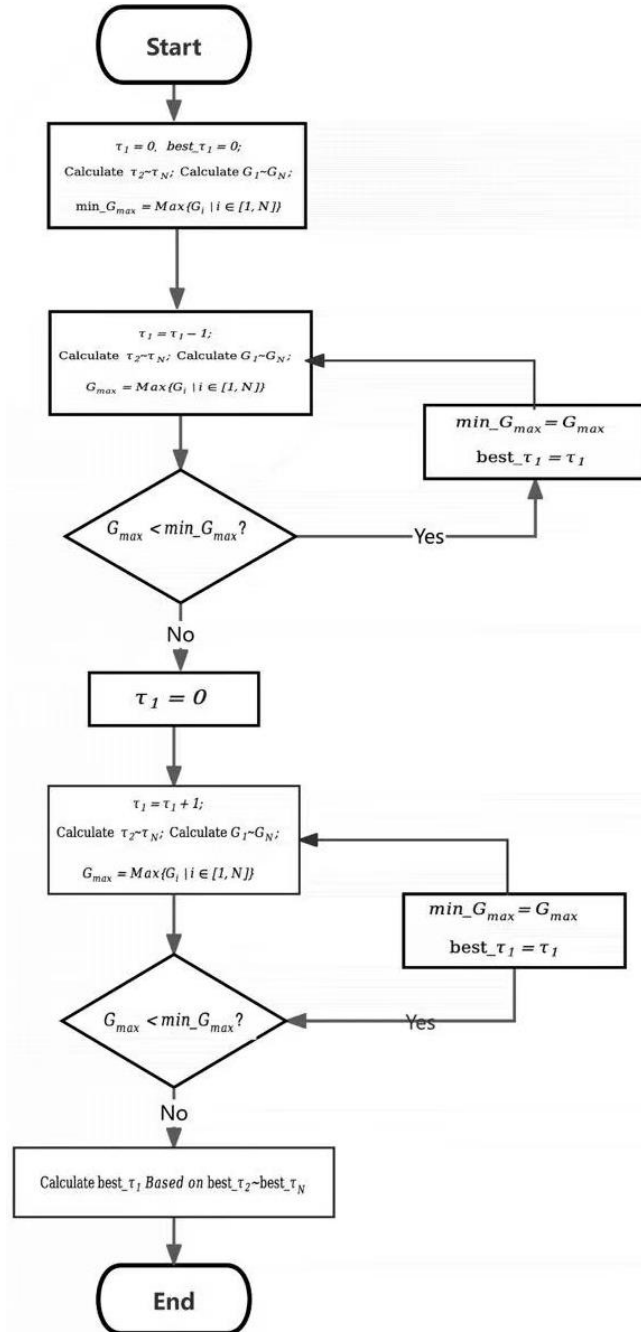


Figure 3. Flowchart of the train-equal interval adjustment algorithm

After time iteration, the absolute value of the train will gradually decrease; when the train is adjusted to zero (or close to zero).

3.2. Model solving

In this section, this article first uses the lingo program to solve the target function and get the residence time of the train at each station. The output results process some data that does not meet the maximum stop time interval again, taking the maximum time interval of 120s input, and the results are shown in Table 2:

Table 2. Stop time of trains

| | | | | | | | | | | |
|--------------------|-------|-------|-----|----|-------|--------|-------|-------|-------|-------|
| station | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| station dwell time | 20 | 20 | 20 | 20 | 90.08 | 100.76 | 101.4 | 39.04 | 57.28 | 53.08 |
| station | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| station dwell time | 120 | 93.44 | 120 | 91 | 120 | 91.88 | 120 | 71.52 | 24 | 49.4 |
| station | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| station dwell time | 37.32 | 25.64 | 120 | 20 | 92.44 | 47.16 | 55.44 | 92.48 | 93.32 | 120 |

Equal interval train adjustment:

According to the results of the previous chapter, a total of 25 big road and 5 small intersection, ratio of 5:1, conform to the size of the scheme mentioned n: 1 mode, so need to consider the subject of the operation requirements: the size of the operation mode of train operation, big trains and small trains will alternate, such as when the big train and small train ratio of 2:1, to every 3 train for a combination (before 2 train for big train, the third train for small intersection train) rolling departure.

Under the initial driving scheme, consider the tracking interval condition substitution model to continue to iteration until the adjustable time approaches 0. The output results are partially shown in Table 3:

Table 3. Train departure time table

| | Train number 1 | Train number 2 | Train number 3 | Train number 4 | Train number 5 | Train number 6 | Train number 7 |
|-----------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Station 1 / departure | 7:00:00 | 7:02:00 | 7:04:00 | 7:06:00 | 7:08:00 | 7:10:00 | 7:12:00 |
| Station 2 / arrive | 7:02:00 | 7:04:00 | 7:06:00 | 7:08:00 | 7:10:00 | 7:12:00 | 7:14:00 |
| Station 2 / departure | 7:02:20 | 7:04:20 | 7:06:20 | 7:08:20 | 7:10:20 | 7:12:20 | 7:14:20 |
| Station 3 / arrive | 7:03:57 | 7:05:57 | 7:07:57 | 7:09:57 | 7:11:57 | 7:13:57 | 7:15:57 |
| Station 3 / departure | 7:04:17 | 7:06:17 | 7:08:17 | 7:10:17 | 7:12:17 | 7:14:17 | 7:16:17 |
| Station 4 / arrives | 7:05:58 | 7:07:58 | 7:09:58 | 7:11:58 | 7:13:58 | 7:15:58 | 7:17:58 |
| Station 4 / departure | 7:06:18 | 7:08:18 | 7:10:18 | 7:12:18 | 7:14:18 | 7:16:18 | 7:18:18 |
| Station 5 / arrives | 7:07:47 | 7:09:47 | 7:11:47 | 7:13:47 | 7:15:47 | 7:17:47 | 7:19:47 |
| Station 5 / departure | 7:09:17 | 7:11:17 | 7:13:17 | 7:15:17 | 7:17:17 | 7:19:17 | 7:21:17 |
| Station 6 / arrives | 7:10:59 | 7:12:59 | 7:14:59 | 7:16:59 | 7:18:59 | 7:20:59 | 7:22:59 |
| Station 6 / departure | 7:12:40 | 7:14:40 | 7:16:40 | 7:18:40 | 7:20:40 | 7:22:40 | 7:24:40 |
| Station 7 / arrives | 7:15:04 | 7:17:04 | 7:19:04 | 7:21:04 | 7:23:04 | 7:25:04 | 7:27:04 |
| Station 7 / departure | 7:16:45 | 7:18:45 | 7:20:45 | 7:22:45 | 7:24:45 | 7:26:45 | 7:28:45 |

4. Conclusion

This paper establishes multiple objective functions, and adopts the weighting coefficient method to transform the multi-objective integer linear planning problem into single objective linear planning

problem. Based on the premise of the lowest operating cost and the highest passenger service level, the train stop time establishes the algorithm model, and makes the optimal scheme by iteratively adjusting the train time. The train timetable optimization system established according to the model and algorithm established by us can quickly provide a relatively reasonable operation plan for urban rail transit with almost the lowest cost under the premise of satisfying the passenger flow demand as much as possible, and provide a feasible system model for the development of urban rail transit with universality.

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