A Study of Submersible Positioning and Search and Rescue Based on Inertial Navigation and Search Modeling

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Abstract. In the field of marine rescue, accurately locating and efficiently searching for lost submersibles is a crucial task. This paper presents a model for the positioning and search of lost submersibles based on inertial navigation, employing a grid search-creeping line search method for practical application. This paper explored a submersible positioning model that integrates Newtonian mechanics and the Strapdown Inertial Navigation System (SINS). The SITAN parallel Kalman filter algorithm was utilized to correct the submersible's position in real time, reducing the cumulative timing errors inherent in inertial navigation systems. To recommend initial deployment points and search patterns, this paper developed a set of search models and conducted a categorical discussion on submersible damage of varying degrees. Simulation data indicates that this study provides an efficient and accurate solution for the field of marine rescue.

Keywords: Trajectory Simulation, Strapdown inertial navigation System, SITAN Kalman Filter, Grid Search, Creeping Line Search.

1. Introduction

As the interest in underwater exploration grows, there is an increasing demand for submersible vehicles to carry tourists on expeditions to the depths of the ocean, such as the Ionian Sea, in search of shipwrecks. However, the safety of submersibles remains a contentious issue. Ensuring the safety of passengers traveling underwater is of paramount importance. Due to the complexity of the deep-sea environment, search and rescue operations for submersibles differ significantly from those on land or at the surface. The safety of submersibles is further challenged by the fact that water molecules can absorb and scatter electromagnetic waves across a wide range of frequencies, severely affecting communication and localization. Therefore, it is important to select accurate predictive models and comprehensive search and rescue models to cope with emergencies that may arise during rescue operations at sea.

2. Localization model

2.1. Mechanical modeling

As shown in the Fig 1, based on the classical mechanics model, this paper analyze the force on the submarine during dive. The formula for the combined force on the submarine is as follows:

$$F = G + B + f + F_p + L + F_t + F_c \tag{1}$$

Based on Newton's second law, this paper model the dynamics of a submarine:

$$F = (M_0 + M_c)\, a \tag{2}$$

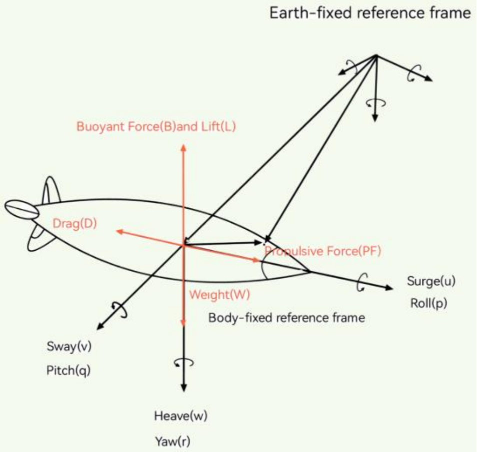

Figure 1. Stress analysis

2.2. Trajectory simulation

The purpose of the trajectory simulation is to generate the specific force and angular velocity, which are the sources of information for the inertial device, and to give the navigational parameters attitude, velocity, and position of the corresponding trajectory points. The specific force and angular velocity are used as data inputs for the Strapdown inertial navigation. This paper categorize and discuss the navigational motion states of the submersible and set the parameters of the navigational motion states. Based on the set navigational motion states, a set of trajectory differential equations is solved using the fourth-order Longe I Kuta method [1].

The motion states of the submersible can be categorized into stationary or uniform linear motion, accelerated or decelerated motion, and coordinated turns compared to the Earth's surface. The angular velocity and acceleration states of the three motion states are shown in the Table.1.

Table 1. The angular velocity and acceleration states of the three motion states

Motion State	Attitude angle change	Acceleration changes
Stationary or uniform linear motion	0	0
Acceleration (deceleration)	0	$[0\ \ a\ \ 0]^T$
Co-ordinated turn (into turn)	$[0\ \ \omega_2\ \ 0]^T$	0
Co-ordinated turns (turning)	$[0\ \ \omega_1\ \ 0]^T$	$[-g\tan\gamma\ \ 0\ \ 0]^T$
Co-ordinated turns (re-levelling)	$[0\ \ \omega_2\ \ 0]^T$	0

After the navigational motion state is determined, the set of trajectory differential equations needs to be solved. The set of trajectory differential equations contains the attitude angle differential equation, the velocity differential equation, and the position differential equation. The attitude angle differential equation is as follows:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\gamma}\\ \dot{\psi}\end{bmatrix} = \omega(t) \tag{3}$$

Where $\theta\ \ \gamma\ \ \psi$ represent the pitch, roll and heading angles, respectively.

The velocity differential equation is as follows:

$$\dot{v}^n = a^n = C_t^n a^t(t) \tag{4}$$

The position differential equations are as follows:

$$\dot{L} = v_N^n \,/\, (R_M + h) \tag{5}$$

$$\dot{\lambda} = v_E^n \sec L \,/\, (R_N + h) \tag{6}$$

$$\dot{h} = v_U^n \tag{7}$$

Where v^n is the velocity of earthward motion in the navigational coordinate system, R_M is the radius of the earth's principal curvature along the meridian circle, R_N is the radius of the earth's principal curvature along the Uyguru circle, and v_E^n v_N^n v_U^n are the components of v^n in the east, north, and zenith directions, respectively.

The results of our trajectory simulation are shown in Fig 2. This paper simulated a section of the submersible's trajectory from the surface down to the seabed at 36.4°N latitude and 18.6°E longitude. The dive depth is 4068 meters. During the dive, the submersible carries ballast iron and dives with thruster power. Due to the complexity of the seabed and the purpose of the dive, not only is the speed kept low, but the lateral acceleration is constantly adjusted in order to avoid possible dangers or to reach the desired destination. The entire dive took nearly 1 hour and 20 minutes.

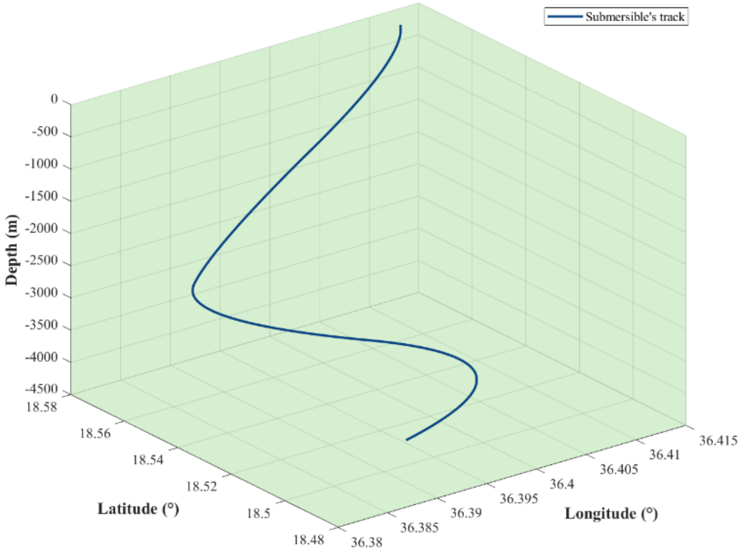

Figure 2. Diving Simulation Trajectory

2.3. Strapdown inertial navigation Analysis

Similar to the trajectory differential equation system, the Strapdown inertial navigation system also solves a system of differential equations to update the attitude, velocity and position. Before performing the Strapdown inertial navigation analysis, it is necessary to establish the transformation matrix between the coordinate systems, including the transformation between the carrier coordinate system and the navigation coordinate system (attitude matrix), and the transformation between the geographic coordinate system and the geocentric coordinate system (the paper use the WGS84 coordinate system) (position matrix) [2].

Attitude consists of pitch angle, heading angle and roll angle, which correspond to angular deflections in the x, y, z axes directions, respectively. The paper assume that the navigation coordinate system OXYZ is negatively rotated by angle ψ around the z-axis to obtain OX′Y′Z′, and OX′Y′Z′ is rotated by angle θ around the x′ axis to obtain OX″Y″Z″. Angle θ to obtain OX″Y″Z″, OX″Y″Z″ is then rotated around the Y″ axis by the angle γ to obtain the carrier coordinate system OX1Y1Z1. Therefore the transformation matrix between the carrier coordinate system and the navigation coordinate system is:

$$C_b^n = \begin{bmatrix} \cos\gamma\cos\psi + \sin\gamma\sin\psi\sin\theta & \sin\psi\cos\theta & \sin\gamma\cos\psi - \cos\gamma\sin\psi\sin\theta \\ -\cos\gamma\cos\psi + \sin\gamma\sin\psi\sin\theta & \cos\psi\cos\theta & -\sin\gamma\sin\psi - \cos\gamma\cos\psi\sin\theta \\ -\sin\gamma\cos\theta & \sin\theta & \cos\gamma\cos\theta \end{bmatrix} \tag{8}$$

After the coordinate transformation matrix is set up, the Strapdown inertial navigation differential equation system needs to be solved to update the attitude, velocity and position information. The Strapdown inertial navigation differential equation system contains attitude update quadratic differential equation, velocity update differential equation and position update differential equation [3]. The attitude update quaternion differential equation is as follows:

$$\dot{q}_b^n = \frac{1}{2} q_b^n \omega_{nbq}^b \tag{9}$$

$$\omega_{nb}^b = \omega_{ib}^b - \omega_{in}^b = \omega_{ib}^b - C_n^b \omega_{in}^n = \omega_{ib}^b - (C_n^b)^T \omega_{in}^n \tag{10}$$

$$\omega_{in}^n = \omega_{ie}^n + \omega_{en}^n \tag{11}$$

$$\omega_{ie}^n = [0 \;\; \omega_{ie} \cos L \;\;\; \omega_{ie} \sin L]^T \tag{12}$$

$$\omega_{en}^n = \left[-\frac{v_N^n}{R_M+h} \;\; \frac{v_E^n}{R_N+h} \;\; \frac{v_E^n}{R_N+h} \tan L\right]^T \tag{13}$$

Where q_b^n is is the transformation quaternion from the kinetic coordinate system to the reference coordinate system, C_n^b is the attitude matrix, ω_{ib}^b is the angular velocity of the gyro output, and ω_{ie} is the angular rate of rotation of the earth.

The velocity update differential equation is as follows:

$$\dot{v}^n = f_{sf}^n + g^n - (\omega_{en}^n + 2\omega_{ie}^n) \times v^n \tag{14}$$

Which

$$f_{sf}^n = C_b^n f_{sf}^q \tag{15}$$

$$g^n = [0 \;\; 0 \;\; -g]^T \tag{16}$$

$$v^n = [v_E^n \;\; v_N^n \;\; v_U^n]^T \tag{17}$$

f_{sf}^n is the projection of the acceleration into the dynamic coordinate system.

The position update differential equation is:

$$\dot{L} = \frac{V_N^n}{R_M+h} \tag{18}$$

$$\dot{\lambda} = \frac{V_E^n}{R_N+h} \tag{19}$$

$$\dot{h} = v_U^n \sec L \tag{20}$$

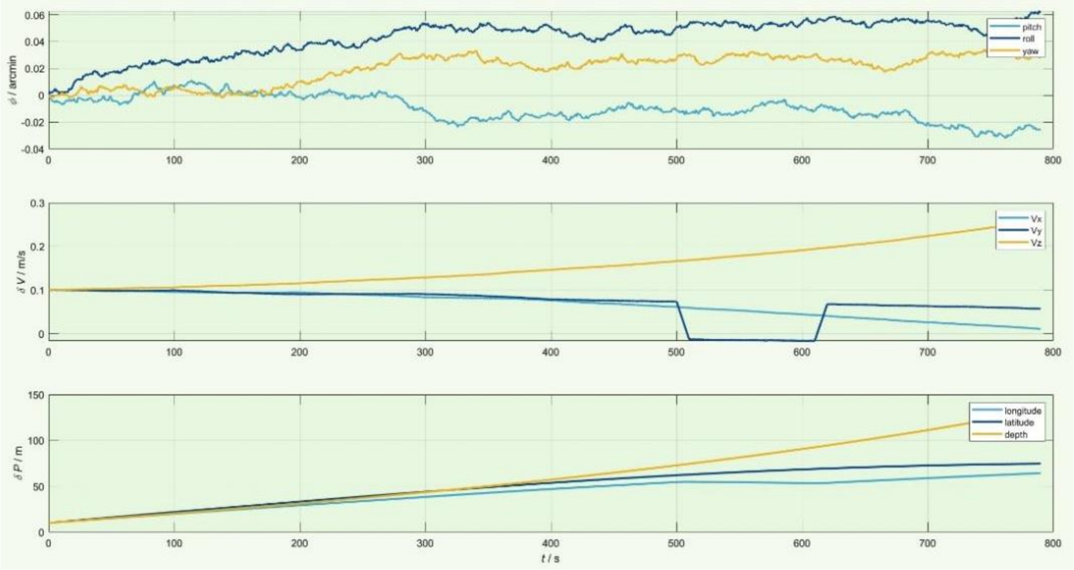

Figure 3. Inertial Navigation Results

Based on the simulated trajectory, the paper Strapdown inertial navigation system calculations and the results are shown in Fig 3. It can be found that the attitude, velocity and position deviate from the real trajectory as time goes by. This is because inertial navigation is autonomous navigation, and with the passage of time, the measurement error will gradually accumulate and eventually affect the prediction of the trajectory [4].

2.4. SITAN KALMAN filter

During the search, the approximate position provided by the inertial navigation system was used as a nucleus to create an area large enough to ensure that it would encompass the actual location of the submersible. Centered on this position, a circle with a radius of 5,160 m was set to cover the 99% probability area (corresponding to a circular probability error of 2,000 m, 2.58σ). Based on this, a series of parallel filters are arranged in the region for analysis, de-pending on the resolution of the gravity data. After each filter completes its computation, the filter that minimizes the smoothed weighted residual sum of squares is selected as the preferred object [5]. Then, the positions of this filter and its eight neighboring filters are combined to determine the final matching position by weighted averaging to complete the search phase.

In the process of Strapdown inertial navigation, the results after adding SITAN KALMAN filtering method for bias correction are shown in Fig 4. It can be found that after the filtering process of SITAN method, the deviations of attitude, velocity and position are obviously decreased, which proves that SITAN method has obvious optimization effect on the errors brought by Strapdown inertial navigation [6].

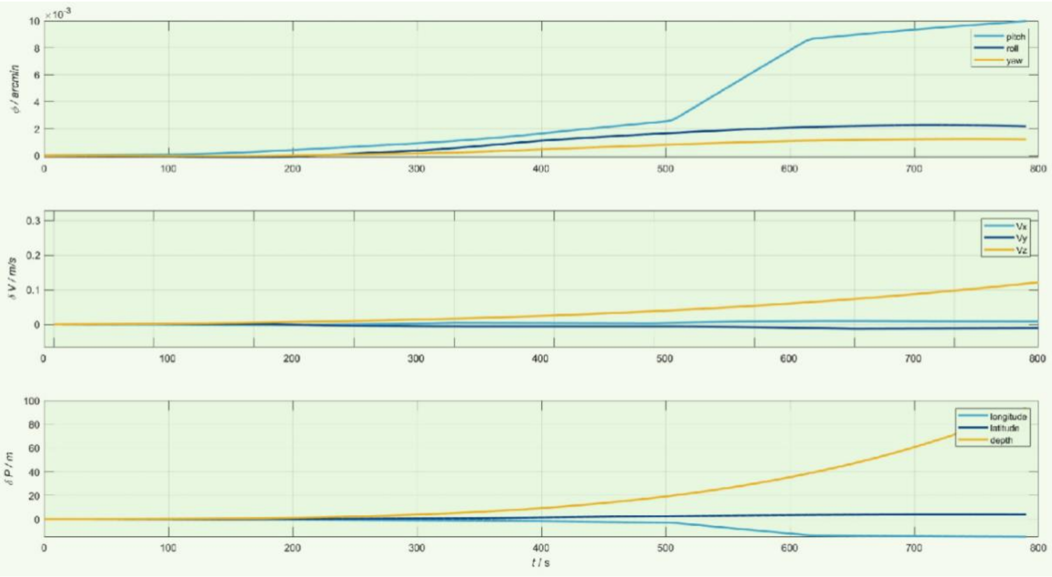

Figure 4. Kalman Filter Correction

3. Search Pattern Planning

In reality, after the submersible has left the mothership, it maintains a communication link with the mother ship, and the submersible reports to the mother ship every t minutes during the working period. If the submersible does not send information to the mothership at the reporting moment T, the mothership will immediately make a continuous call to the submersible through the emergency radio communication channel until the next reporting moment T+1, after a continuous call fort minutes and the submersible has still not responded, the mother ship will immediately carryout the rescue operation. The actual time the submersible is lost maybe any point in time from the T-1 moment to the T moment. In the best case, the submersible is lost moments before the T reporting moment; in the worst case, the submersible is lost immediately after sending the message at the T-1 reporting moment.

For the emergency program after the loss of the submersible, according to the actual situation the paper divide it into two categories. One is when the submarine's communications and power systems fail at the same time, and the submarine loses propulsion and sinks downward under the force of nature. In the other case, the submarine only has a communication system failure, the rest of the functions are intact, the submarine operator initiates the emergency escape procedure, and the submarine performs an in-situ float.

3.1. Failures in Both Communication and Power Systems

Based on the predicted drop point image, the paper draw a square with maximum Euclidean distance that completely covers the drop point range and use it as the actual search area. Considering the error in predicting the drop point, the paper can multiply the side length by a threshold ε (1 to 1.5) to make the search coverage wider. The paper choose the geometric center of this square as the device deployment point [7].

The paper develop the Grid Searching method -Creeping Line Searching method to search for the lost submersibles. We use the grid search method to encode the diver's position data, and store the results of the grid encoding, and use the encoded two-dimensional string data to represent the three-dimensional latitude, longitude and depth spatial information of the diver, which greatly simplifies the complex spatial information [8]. The actual search is then carried out using AUVs through the creeping line search method, which maximizes the search coverage in each subdivided area and ensures that nothing is missed [9]. Through this combination, the paper were able to quickly narrow down the scope of the search initially, identifying key areas to focus on, and then using Creeping Line Search to drill down to every possible area.

As time passes, our probability of finding the submersible decreases. In this section, the paper use the ratio of time to quantify the search success probability x. The numerator A0 is the primetime for search and rescue: 4320min (72h), a concept in the context of Search and Rescue (SAR) [10]. The denominator is the actual time to search the full range of drop points, which can be derived from metrics such as sailing speed and exploration width.

$$x = \frac{A_0}{A_t} \tag{21}$$

The time required to search the entire fallout range using AUVs according to the grid search-Creeping line search method:

$$A_{2t} = \frac{4\,dr^{+4}r^2}{dv} + T_{extra} \tag{22}$$

$$T_{extra} = T_u + T_d + T_p \tag{23}$$

d is the maximum width of the single search range of the side-scan sonar device carried by the AUV; vis the average search speed of the AUV during the search; r is the length of the side of the square of the fallout range; T_u is the time taken by the AUV to dive from sea level to the seafloor, and T_d is the time taken by the AUV to surface from the seafloor to thesea level (the AUV will need to return to the mothership to transmit the data after the completion of its work, and thus the surfacing time needs to be calculated); T_p is the processing time to get the position data of the lost submersible after the AUV returns to the mother ship.

The parameters of Remus 6000 and Ping 360 mentioned before: d is 150m, v is 4 knots, and r is 9.87km. Roughly counting $T_u + T_d$ is 600min, T_p is 60min, and calculating A_t is 6648min, so the probability of search success x is 64.11%.

3.2. Failures in Only Communication System

In such cases where the submersible is lost and still has power, the emergency escape program is usually activated immediately and the submersible will float in place until it reaches the surface and wait for rescue. The submersible may lose power at any point in time from moment T-1 to moment

T (totaling t minutes). We extrapolate the range of trajectories of the submersible by modeling the forces for which power exists during that time period based on the original predicted trajectory of the submersible.

As in Fig 5, the paper use the extended square method for rescue. Extended square searching is used when the location of the search target is in a relatively close area [11]. In this method, the starting point of the search is always the datum position, and the search operation extends outward in concentric squares, thus covering the area centered on the datum almost uniformly. The length of the two search routes is equal to the spacing of the search lines, and the spacing of the search lines is increased by one for every two subsequent search lengths. If a continuous search is carried out in the same area, the search route is normally turned at 45° to continue the search. Extended square searching requires that the search facility be capable of precise navigation, and in order to minimize navigational errors, the first search route is usually upwind. The extended square search is usually applied to operations by ships or small craft searching for persons overboard or other search targets. In this case, the ship should navigate according to the search method based on the trajectory projection and utilize the search method similar to the fan search that automatically compensates for the effect of the total current pressure. We continue to select the previously mentioned search range of the lost power square and use the geometric center position of the drop range as the center point of the extended square search method, i.e, the deployment point.

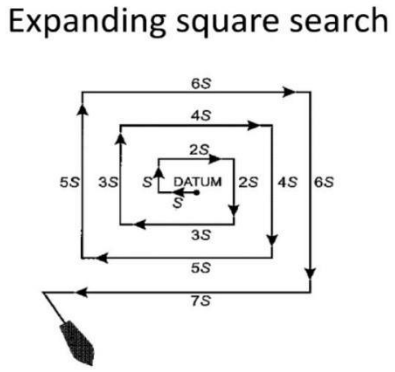

Figure 5. Expanding Square Search

This paper follow the formula of 3.1 and quantify the ratio of times as a probability x.

$$x = \frac{A_0}{A_t} \tag{24}$$

Typically, a rescue helicopter has a cruise SAR efficiency of 130 nautical miles per hour and a field of view of 5 nautical miles. We still choose the aforementioned square of lost power as the search and rescue range, and search on the sea level. A simple calculation shows that the value of A_t is less than A_0, so the search success rate is 100% in this case. This result maybe surprising, but in reality, assuming that the submersible can successfully surface, whether by helicopter or satellite positioning and other technologies, based on the current capabilities and experience of mankind's search at sea, the paper have reason to believe that the success rate of the search can reach 100%.

4. Conclusion

This paper presents a model for the positioning and search of lost submersibles based on inertial navigation, employing a grid search-creeping line search method for practical application. By integrating Newtonian mechanics and the Strapdown Inertial Navigation System (SINS), the paper developed a submersible positioning model and used the SITAN parallel Kalman filter algorithm to correct the submersible's position in real-time, reducing the cumulative timing errors inherent in inertial navigation systems. Additionally, the paper developed a set of search models and conducted a categorical discussion on submersible damage of varying degrees, providing a basis for recommending initial deployment points and search patterns. Simulation data indicates that this study provides an efficient and accurate solution for the field of marine rescue. Future research could explore more search strategies and further optimize the positioning and search models to improve the efficiency and success rate of marine rescue operations.

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