Strategy for Water Level Control Based on Prediction Control Feedback Model

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Abstract. First, this article uses the fuzzy comprehensive evaluation algorithm to consider the interests and costs of all parties to construct a comprehensive satisfaction score as the objective function, with the safety needs of residents around the Great Lakes for water levels as constraints, and the optimal water level is obtained through nonlinear programming. Then, this article uses the Predictive-Control-Feedback (PCF) concept to construct a water level control management model. The predictive model is mainly built on the differential equation model of the flow rate change per second for each lake. The LSTM model predicts the per second precipitation, evaporation, and runoff. The input and output flow rates of the river are calculated based on the Manning formula. The PCF model dynamically minimizes the sum of squared errors between the predictive model and the target by using grid search to find the optimal control flow rate. In addition to the differential equations, the predictive model also includes a recursive predictive correction control term, which provides feedback to the current predictive model based on the deviation between the actual value and the predicted value from the previous period. Meanwhile, we have tested the sensitivity of this algorithm to dam outflow and environmental changes, and the result shows that the algorithm is stable for both types of sensitivity tests.

Keywords: Level Control, Prediction-Control-Feedback model, Fuzzy Comprehensive Evaluation, Differential Equations, LSTM, Dynamic Programming.

1. Introduction

The water level control issue of the Great Lakes has always been a focus of attention. On one hand, determining the target water level of the Great Lakes has a huge impact on stakeholders. For example, at low water levels, ship navigation in and out of ports may be restricted, and irrigation water sources for farmland may be insufficient; at high water levels, it may lead to flooding disasters, land erosion, and other natural disasters.

On the other hand, maintaining the water level of the Great Lakes in a suitable and stable state is not easy. The water level of the Great Lakes is influenced by multiple factors, including natural factors such as precipitation, evaporation, water depth measurement, river flow rates, as well as human factors such as reservoir policies and dam controls. In addition, managing the water level of the Great Lakes as a dynamic network flow problem involves complex interdependencies and inherent uncertainties.

Over the years, people have been trying to control the water levels of the Great Lakes through hydraulic engineering. The Soo Locks at Sault Ste. Marie and the Moses-Saunders Dam at Cornwall are the two main control facilities, but the current methods used still have significant shortcomings. Therefore, it is necessary to establish a more optimal water level management control model.

2. FCE-NLP model (Nonlinear Programming Algorithm based on Fuzzy Comprehensive Evaluation)

To meet the diverse stakeholder requirements for water levels and determine the optimal water levels of the Great Lakes at any time of the year, we have decided to utilize a nonlinear programming algorithm. This approach considers safety requirements as constraints and aims to maximize the aggregate satisfaction level of all stakeholders as the objective function. To quantify the demand for low, medium, and high water levels as well as the overall satisfaction level, we employ the fuzzy comprehensive evaluation method. This involves transforming water level demands into satisfaction scores using Gaussian membership functions, thereby establishing a mapping relationship between lake water levels and satisfaction scores. These scores are then aggregated through weighted summation to derive a composite satisfaction score[5].

2.1. Fuzzy Comprehensive Evaluation

The fuzzy comprehensive evaluation algorithm is commonly used to address issues involving fuzzy concepts. It attempts to map input values to a number between 0 and 1 using appropriate membership functions, aligning elements of a fuzzy set with fuzzy or uncertain concepts in the real world, laying the groundwork for subsequent data processing and calculation. The value between 0 and 1 is known as the membership value, indicating the degree to which an element belongs to a certain fuzzy set, i.e., how strongly or to what extent the element corresponds with the fuzzy set.

In this text, the fuzzy set comprises the diverse stakeholder demands for the Great Lakes' water levels, categorized into three levels: high, medium, and low. Through the mapping with membership functions, these demands are transformed into degrees of satisfaction. By using membership functions, we successfully establish the relationship between water level demands and satisfaction levels.

Water level demands are categorized into interest demands and environmental protection demands. The National Environmental Policy Act (NEPA) [2] states that environmental considerations, including social and economic aspects, must be taken into account before making decisions that could significantly impact the environment. It emphasizes the concurrent progression of economic development and environmental protection. Therefore, this paper decides to set the weights for interest demands and environmental protection demands at 0.5:0.5.

2.1.1 Benefit needs of people in various industries

luctuations in the water levels of the Great Lakes have significant impacts on various industries. Based on the Problem D Addendum document and relevant literature, water usage demands are categorized into seven types: navigation, recreation, irrigation, construction, power generation, fishing, and residential supply. Further analysis reveals the appropriate water levels for different water demands: navigation, power generation, residential supply, and irrigation require high water levels; recreation and fishing require medium water levels; construction requires low water levels.

After taking into account various factors such as the cost of water usage, benefits from water usage, and scale of use, as well as the economic overview of maritime and Great Lakes nations and the employment coefficient in the Great Lakes region, we have determined the weight ratios for each type of water demand as shown in Table 1:

Table 1. Weighting ratios for various industry groups

Shipping	Recreation	Irrigation	Cona=structing	Hydroelectric Generation	Fishing	Supply of Residents
0.1765	0.0588	0.1176	0.0882	0.2353	0.1741	0.1765

2.1.2 Environmental protection needs

The water level fluctuations of the Great Lakes affect the ecosystem of the region, further influencing the health of flora and fauna in and around the lakes. The habitat maintenance for species in the Great Lakes area of the United States and the cleanup of stagnant bays and tributaries require water level management strategies to be formulated based on the characteristics of the ecosystem, bird breeding seasons, fish migration seasons, and other ecological factors.

Typically, during the non-breeding season, especially in winter, to aid in the cleanup and maintenance of the bottoms of lakes, bays, and tributaries, reducing the accumulation of habitats and harmful substances, lower water levels should be maintained; before and after the bird breeding season, as well as before the fish migration season, to help provide suitable habitat conditions, supporting breeding and migration processes, medium water levels should be maintained; during the bird breeding season and the fish migration season, to help protect waterfowl nesting areas and provide suitable environments for fish, while minimizing disruptions to the breeding process, higher water levels should be maintained. Fish migration season usually occurs in spring and early summer, between March and July; bird breeding season usually occurs in spring and summer, between April and August. From this, the article derives the different water level requirements for the Great Lakes in December, with specific results as shown in Table 2:

Table 2. High and low water levels in different months

High water months	Medium water months	Low water months
4,5,6,7	3,8,9,10,11	12,1,2

2.1.3 Membership function

Considering the smooth shape, unimodal nature, and natural weight distribution of the Gaussian function, along with its two parameters: mean and standard deviation, which can be adjusted to shape the membership function to suit the nature of different fuzzy sets, this paper adopts a membership function that follows a Gaussian distribution. The membership functions for low, medium, and high water level demands of different stakeholders are as follows:

$$Q_{2j} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_{2j}-\mu)^2}{2\sigma^2}} \tag{1}$$

The formulas above map the water level demands of different users to satisfaction scores, where x_{2j} represents the j th type of water demand, Q_{2j} represents the satisfaction score for the j th type of water demand, and σ represents the standard deviation of historical lake water levels. For reasons of rationality, we referred to the historical water level data of the Great Lakes, setting the historical highest water level of each lake per month as the optimal value for high water level demands, the historical lowest water level as the optimal value for low water level demands, and the historical average water level as the optimal value for medium water level demands. Table 3 below presents the parameters μ and σ values for Lake Superior for each month.

Table 3. Lake Superior monthly parameter values

parameter	Jan	Feb	Mar	April	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
μ	74.33	74.29	74.76	75.30	75.80	75.91	75.80	74.97	74.78	74.63	74.56	74.28
sd	16.234716.239216.258116.306816.346416.580016.352816.308516.227416.240416.2167											

2.1.4 Composite Satisfaction

We construct a two-tier evaluation index system according to the schematic diagram. The first-tier evaluation indices are environmental protection satisfaction and interest demand satisfaction. The composite satisfaction is calculated using the following formula:

$$Q = w_1 Q_1 + w_2 Q_2 \tag{2}$$

Here, Q represents the composite satisfaction, Q_1 represents the satisfaction for environmental protection demands, and Q_2 presents the satisfaction for interest demands. $w_1 = w_2 = 0.5$ where w_1 represents the weight for the satisfaction of environmental protection demands, and w_2 is the weight for the satisfaction of interest demands.

The second-tier evaluation indices are the satisfactions for seven different types of water usage demands. The satisfaction for interest demands is calculated using the following formula:

$$Q_2 = \sum_{j=1}^{7} W_{2j} Q_{2j} \tag{3}$$

Here, Q_2 represents the satisfaction for interest demands, Q_{2j} epresents the satisfaction for the j th type of water usage demand, and w_{2j} represents the weight for the satisfaction of the j th type of water usage demand.

2.2. Nonlinear programming algorithm

Considering that if too little water is discharged or evaporated from the lakes, flooding may occur, causing potential damage to homes and businesses along the coastline. Therefore, this paper uses the historical maximum and minimum water levels as the safety constraints for water levels, and the satisfaction levels of various stakeholders regarding water levels as the objective function. By employing the fuzzy comprehensive evaluation method to construct the relationship between water level demands and satisfaction levels, we solve for the water level value that achieves the highest composite satisfaction score, i.e., the optimal water level. The nonlinear programming model is established as follows:

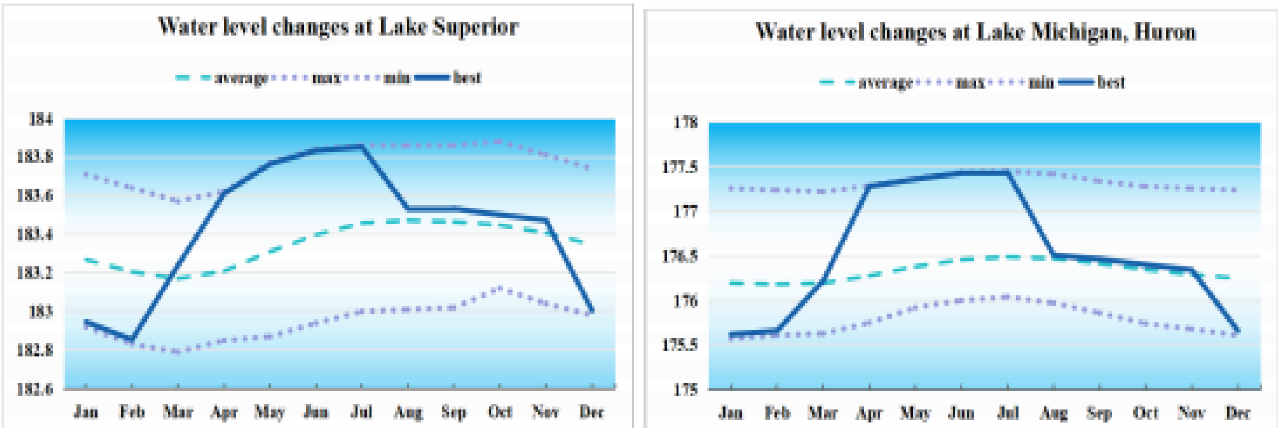
$$Q = w_1 Q_1 + w_2 Q_2 \tag{4}$$

$$Q_2 = \sum_{j=1}^{7} W_{2j} Q_{2j} \tag{5}$$

$$Q_{2j} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x_{2j}-\mu)^2}{2\sigma^2}} \tag{6}$$

$$\min L_{2j} \leqslant x_{2j} \leqslant \max L_{2j} \tag{7}$$

Here L_{2j} represents the demand for water levels by the j th category. After calculations in MATLAB, the optimal monthly water levels for the Great Lakes are obtained, and the comparison chart of the optimal monthly water levels, highest water levels, lowest water levels, and average water levels of the Great Lakes is presented as shown in Figure 1:



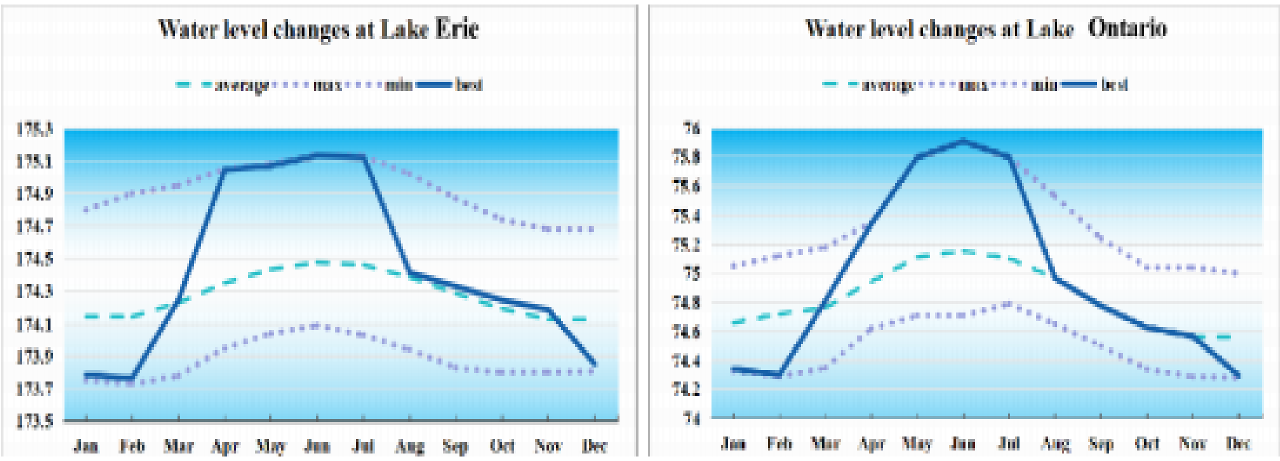


Fig. 1 Distribution of usage

From the above chart, it can be observed that from early spring to early summer, due to the melting of ice and snow flowing into the lakes, the optimal water level gradually increases, reaching its maximum in June and July. In the midsummer, the warmth of surface water leads to increased evaporation, causing the optimal water level to begin declining in August. In winter, some parts of the lakes and rivers are covered with ice and snow, leading to a continuous decline in the optimal water level, reaching its lowest point in January and February. This entire process is consistent with the descriptions in the Problem_D_Addendum document, and the changes in the optimal water level align with the natural seasonal variations, indicating that the derived optimal water levels are reasonable.

3. Water level control management model based on the PCF (Process Control Framework) concept

The water level control management model is divided into three parts: Prediction, Control, and Feedback.

The prediction part is primarily based on a water level forecast model that combines an LSTM model and differential equations, simulating and predicting water levels through the construction of differential equations for the rate of change in lake volume. In this model, the three natural variables of evaporation, precipitation, and runoff are predicted using the LSTM model, while the flow rates of upstream and downstream rivers are calculated using the Manning formula.

The control part adopts a dynamic programming control model that minimizes the sum of the squared differences between the controlled water levels of the five lakes and their optimal levels through grid search. It determines the flow rates controlled by two dams in the Great Lakes that minimize the objective function, thereby obtaining the optimal water level adjustment plan for the five lakes.

The feedback part uses the recursive prediction correction control item from the prediction model, which provides timely feedback to the current period's prediction model by calculating the sum of the deviations between the previous period's actual values and predicted values.

Finally, we compare the control water levels, actual water levels, and optimal water levels for the 12 months of 2017 to test the control effect. It was found that the lake water levels obtained using this model were closer to the optimal water levels compared to the actual data for 2017, indicating that our model achieves better control effects.

3.1. Water level prediction model based on LSTM, recursive prediction correction control item, and differential equations

The prediction model consists of two parts: the first part is obtained through simulation prediction constructed by differential equations, and the second part is the recursive prediction correction item.

The first part obtains the predicted value of L by simulating the physical process of water flow movement, while the second part provides feedback on the deviation from previous simulation predictions. Together, they adjust and enhance the model's adaptability to external environmental changes, ensuring that the predicted water levels remain close to reality even in the presence of significant disturbances. The basic formula of the prediction model is as follows:

$$\hat{L}_{pj} = L_{pj} + \lambda\left(L_{rj} - \hat{L}_{p,j-1}\right) \tag{8}$$

Where L_{pj} represents the uncorrected water level prediction value for day j, $\hat{L}_{pj}$ is the corrected water level prediction value for day j, L_{rj} is the actual water level for day j,and λ is the feedback coefficient.

3.1.1 Construction of differential equations

According to the text, the water level of each lake is determined by the amount of water entering and exiting the lake, a result of various factors acting over a long period. We categorize natural factors into precipitation, evaporation, and runoff, and human-controllable factors into river flow that can be controlled by two mechanisms (the compensating works at the Soo Locks and the Moses-Saunders Dam between Massena, New York, and Cornwall, Ontario), forming a first-order differential equation that can predict the daily water level of each lake. We use historical data of precipitation, evaporation, and runoff from the Great Lakes to train LSTM models, predicting future precipitation, evaporation, and net flow. Additionally, we utilize the Manning formula, widely used in water resources engineering and hydrology, to calculate the flow velocities of upstream and downstream rivers.

Based on the assumption that the instantaneous flow rate of each lake is equal to the sum of the lake area's precipitation, runoff, and upstream river flow rate per unit of time, minus the sum of the lake area's evaporation and downstream river flow rate per unit of time, and considering the Great Lakes and Lake St. Clair are ordered from Lake Superior to Lake Ontario as the $(1 \leqslant j \leqslant 5)$, and taking into account that Lake Superior has no river inflow, the differential equation is constructed as follows:

$$\frac{\sigma N_j(t)}{\sigma t} = -O_j - C_{evap}j + C\ j + C_{runo}j \qquad (j=1) \tag{9}$$

$$\frac{\sigma N_j(t)}{\sigma t} = I_j - O_j - C_{evap}j + C_{prec}j + C_{runo}j \qquad (2 \leqslant j \leqslant 5) \tag{10}$$

Where $N_j(t)$ represents the volume of the jth lake, I_j represents the inflow to the jth lake, O_j represents the outflow from the jth lake, and the precipitation, evaporation, and runoff per unit time for the jth lake are represented by $C_{evap}j$, $C_{pre}j$ and $C_{runo}j$, respectively. The units for all variables are in the International System of Units (SI).

3.2. Calculation of river flow rate based on Manning Formula

The Manning formula is an empirical equation used to estimate the average velocity of liquid in open channels (i.e., open channel flow) or in partially full pipes. It can be applied to various types of channels and fluids. The Manning formula has certain conditions for applicability:

a). It is applicable to fully water-filled open channel flows and not suitable for partially filled pipe flows.

b). It assumes the flow is turbulent and is not suitable for highly turbulent conditions or cases of extremely high flow velocities.

c). It is applicable to various types of channel materials and shapes, but the correct Manning roughness coefficient needs to be selected.

d). It is suitable for channels with slopes that are not particularly steep, to ensure the flow remains as open channel flow.

In this case, the rivers are considered fully water-filled open channels; under human control, the rivers will not experience excessively fast flow velocities; according to the data, the Manning roughness coefficient in natural river channels ranges from 0.03 to 0.06[1]; according to Problem_D_Addendum and related materials, the slopes of the rivers are not steep. All these conditions are met, thus, this paper uses the Manning formula for the calculation of river flow velocities.

The Manning formula is as follows[3]:

$$V=\frac{k}{n}R^{2/3}S^{1/2} \tag{11}$$

Where V is the average velocity per cross section; k is a conversion factor, and since the variables all use the International System of Units (SI), k=1; n is the Manning roughness coefficient. For the sake of realism, we use actual historical lake data to fit and solve for the Manning roughness coefficient of each river, with values all within the range of 0.03 to 0.06; R is the hydraulic radius, calculated as the cross-sectional area of flow/wetted perimeter, where the cross-sectional area of flow is the area enclosed by the water surface line and the river bottom line at a given research moment, and the wetted perimeter refers to the perimeter line where the fluid and solid wall surfaces contact in the flow cross-section. Since the width of the river is much greater than its depth, we approximate the hydraulic radius with the river depth; \(S\) is the hydraulic slope, equal to the slope of the conduit, which we calculate using the difference in height between lakes connected upstream and downstream of the river divided by the length of the river[6]. The specific calculation formula is as follows:

$$I_j=A_jV_j=\frac{k}{n_{j-1}}A_{j-1}{R_{j-1}}^{2/3}{S_{i-1}}^{1/2} \tag{12}$$

$$O_j=A_jV_j=\frac{k}{n}A_j{R_j}^{2/3}{S_j}^{1/2} \tag{13}$$

$$A=wd \tag{14}$$

$$R\approx d \tag{15}$$

$$S=\frac{L_u-L_d}{l} \tag{16}$$

$$N=a(L-B) \tag{17}$$

Where O_j is the outflow of the jth lake, I_j is the inflow of the jth lake, A is the cross-sectional area of the jth lake, R_j is the hydraulic radius of the jth lake, S is the surface width, w is the channel width, d is the river depth, I is the channel length, L_u is the upstream water level, L_d is the downstream water level, N is the lake volume, and a is the lake surface area.

3.3. Precipitation, evapotranspiration, and runoff prediction based on LSTM modeling

LSTM, short for Long Short-Term Memory, is a special type of Recurrent Neural Network (RNN). Unlike traditional feedforward neural networks, this network can effectively transfer and express information in long time sequences without causing the loss of useful information from the distant past. It avoids the gradient vanishing and exploding problems common in RNNs[4].

3.3.1 Predictions of precipitaion,evaporation and runoff volume

Through the LSTM model, with the parameters set as Number of LSTM Layers to 1, Number of Hidden Units to 100, and Probability in Dropout Layer to 0.2, using data from 1950 to 2016 as the training set and data from 2017 as the test set, we trained the LSTM model. This allowed us to derive the fitted values for the evaporation, precipitation, and runoff of the Great Lakes from 1950 to 2016, as well as the predicted values for 2017, and compared them with the actual values. Taking the evaporation of Lake E and the runoff of Lake H as examples, the following chart was produced. It can be seen that the model can reliably fit the evolution of the hydrological characteristics of the lakes, showing high consistency between the actual values and the fitted values. Based on the actual and predicted values for 2017, we can see that the trends of the predicted and actual values are similar, and as shown in Figure 2, the two curves are close together, with an RMSE of 17.52, indicating good prediction performance.

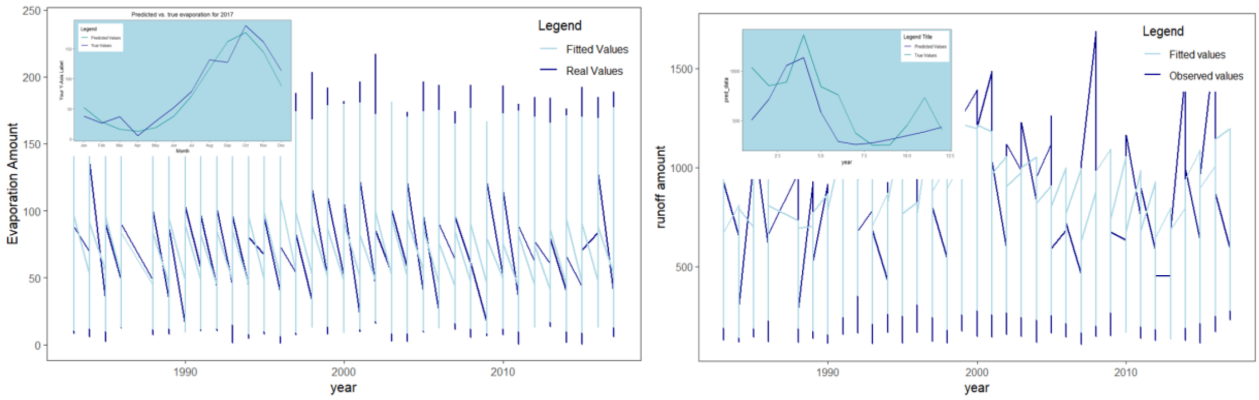


Fig. 2 Actual and predicted values of evaporation and runoff

3.3.2 Recursive Prediction Error Correction Term

The correction term represents the F stage of the PCF, its value is determined by the error between the true value and the predicted value from the previous period, and is used to correct the error between the predicted value and the true value. When the error is white noise, the result of this term is close to 0, having almost no effect on the predictive model, thus resulting in weak correction for the current period. When the true value consistently exceeds the predicted value, this term's result is positive and of significant absolute value, adjusting the initially smaller predicted value upwards to approach the true value. Conversely, when the true value consistently falls below the predicted value, this term's result is negative and of significant absolute value, adjusting the initially larger predicted value downwards to approach the true value.

In practical applications, we need to find the optimal feedback coefficient λ that yields the best adjustment effect for the prediction. Since a large λ value leads to excessive adjustment of the prediction results, deviating them from the true values, we provide alternative values for λ as 0.01, 0.005, and 0.0025.

We utilize data from the Great Lakes for January to March 2017, creating scenarios where the true value consistently exceeds or falls below the predicted value by adjusting natural factors. We search for the optimal feedback coefficient by substituting different candidate values for λ, continuously increasing or decreasing precipitation and runoff in the differential equations to simulate scenarios where similar trends of anomalies persist in the natural environment. We obtain prediction results with and without recursive prediction error correction terms for different values of λ and calculate the errors between these results and the true values. Ultimately, we find that λ=0.0025 yields the best predictive performance.

Figure 3 illustrates a scenario where precipitation continuously and anomalously increases for Lake Michigan and Huron from January to March 2017. It can be observed that when λ=0.0025, the predicted values are closest to the true values and ultimately stabilize near the true values.

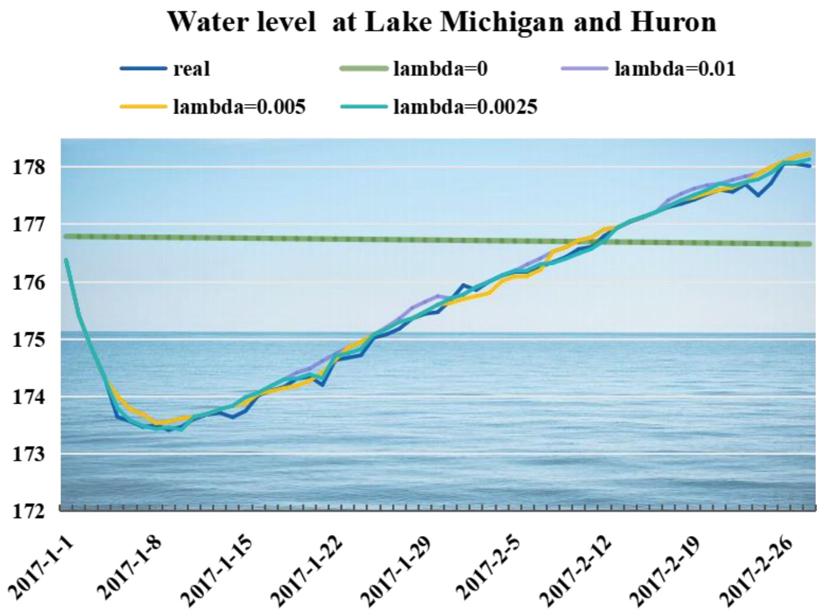


Fig. 3 Water level at Lake Michigan and Huron when Continued anomalous increase in Precipitation

3.4. Dynamic Programming Control Model

The dynamic programming control model is a model used in control systems to optimize decision-making using dynamic programming methods. Considering that river flow is a slow and continuous process, we set the control period to 1/3 months, adjusting the control system on the 1st, 11th, and 21st of each month, altering the inflow and outflow of the lake. A 10-day control period also enhances the system's resilience to noise. To transition the predicted monthly control water levels from the differential equations to align with the water levels of the previous and next months, we set three different optimal water levels for the early, middle, and late parts of each month.

The formula for the target water level in the three-stage dynamic programming is as follows, where $L_{goal,j}$ represents the target water level in stage j, $L_{best,j}$ represents the optimal water level in stage j , and L_j represents the actual water level in stage j .

$$\begin{aligned} &Phase\ I \quad L_{goal,j}=\frac{1}{3}L_{j-1}+\frac{2}{3}L_{best,j} \\ &Phase\ II \quad L_{goal,j}=L_{best,j} \\ &Phase\ III \quad L_{goal,j}=\frac{2}{3}L_{best,j}+\frac{1}{3}L_{best,j+1} \end{aligned} \tag{18}$$

According to the preceding assumption, the dam controls the variation of the lake's water level by adjusting the flow of the river passing through it. With human intervention, the outflow of Lake Superior, the inflow of Lake Michigan and Huron, and the outflow of Lake Ontario, which were originally calculated based on the Manning formula, are now determined by human control. At this point, the differential equations transform into the following formula:

$$\begin{aligned} &j=1 \quad \frac{\partial L_j}{\partial t}=\frac{1}{a}(-C_{evap}+C_{prec}+C_{runo}-D_1)\dots \\ &j=2 \quad \frac{\partial L_j}{\partial t}=\frac{1}{a}\left(-C_{evap}+C_{prec}+C_{runo}-D_1-\frac{k}{n_j}w_j d_j^{5/3}\left(\frac{L_j-L_{j+1}}{l_j}\right)^{1/2}\right)\dots \\ &j=3,4 \quad \frac{\partial L_j}{\partial t}=\frac{1}{a}\left(-C_{evap}+C_{prec}+C_{runo}-\frac{k}{n_j}w_j d_j^{5/3}\left(\frac{L_j-L_{j+1}}{l_j}\right)^{1/2}-\frac{k}{n_j}w_j d_j^{5/3}\left(\frac{L_j-L_{j+1}}{l_j}\right)^{1/2}\right)\dots \end{aligned} \tag{19}$$

$$j=5 \qquad \frac{\partial L_j}{\partial t}=\frac{1}{a}\left(-C_{evap}+C_{prec}+C_{runo}-\frac{k}{n_j}w_j d_j^{5/3}\left(\frac{L_j-L_{j+1}}{l_j}\right)^{1/2}-D_2\right)...$$

Up to this point, in order to obtain the optimal scheme for adjusting the lake water level, we must continuously strive to achieve the target water level on the lake surface, determining the best values for regulating river flow.

This planning process is dynamic and manifests in three aspects: Firstly, as we decide to control the dam every 10 days, at the onset of each new control cycle, we need to calculate the recursive prediction error correction control term from the previous period for use in the current prediction. Secondly, since we assume daily predictions of the lake water level, the prediction of the water level for the next day depends on the final lake water level obtained on the current day, serving as the initial value for the differential equation iteration when predicting the water level for the next day. Thirdly, at the start of each new month, the target water level for the first stage not only depends on the water level target for the new month but also on the current lake water level.

From this, it is evident that dynamic programming is required for optimizing the regulation of dam flow to achieve the best control scheme. Based on the differential equation model, we have established the relationship between river input, river output, and controlled water level. Let \(t \) represent the \(t \)-th control cycle starting from January. The objective function established by the dynamic programming control model on this basis is as follows:

$$f=\sum_{i=1}^{5}\sum_{j=1}^{10}\left(\hat{L}_{pj}^{(i)}-L_{goal,j}^{(i)}\right)^2 \tag{20}$$

Where $\hat{L}_{pj}^{(i)}$ represents the water level prediction for the ith lake on the jth day, and $\hat{L}_{goal,j}^{(i)}$ represents the current target water level for the ith lake. This equation represents the sum of squared differences between the actual lake water levels and the target water levels for the five Great Lakes in a certain stage. When the objective function is minimized, the corresponding river flow rates are the optimal control values sought for that stage.

In order to solve this dynamic programming problem and determine the control flow rates for the new stage, we need to traverse the control values of the dam's river flow within the constraints of historical maximum and minimum river flow data. We perform a grid search using GridSearchCV to attempt to find a set of flow control values that minimize the objective function, serving as the control scheme for this stage.

Figure 4 depicts a comparison of the optimal, actual, and controlled water levels for the five lakes each month in 2017 under this control method. It is evident from the figure that the controlled water levels are generally closer to the optimal water levels compared to the actual water levels. This clearly demonstrates the effectiveness of the dynamic control model, showing that it can effectively achieve the objectives of managing the water levels of the Great Lakes, meeting both economic and environmental preservation requirements.

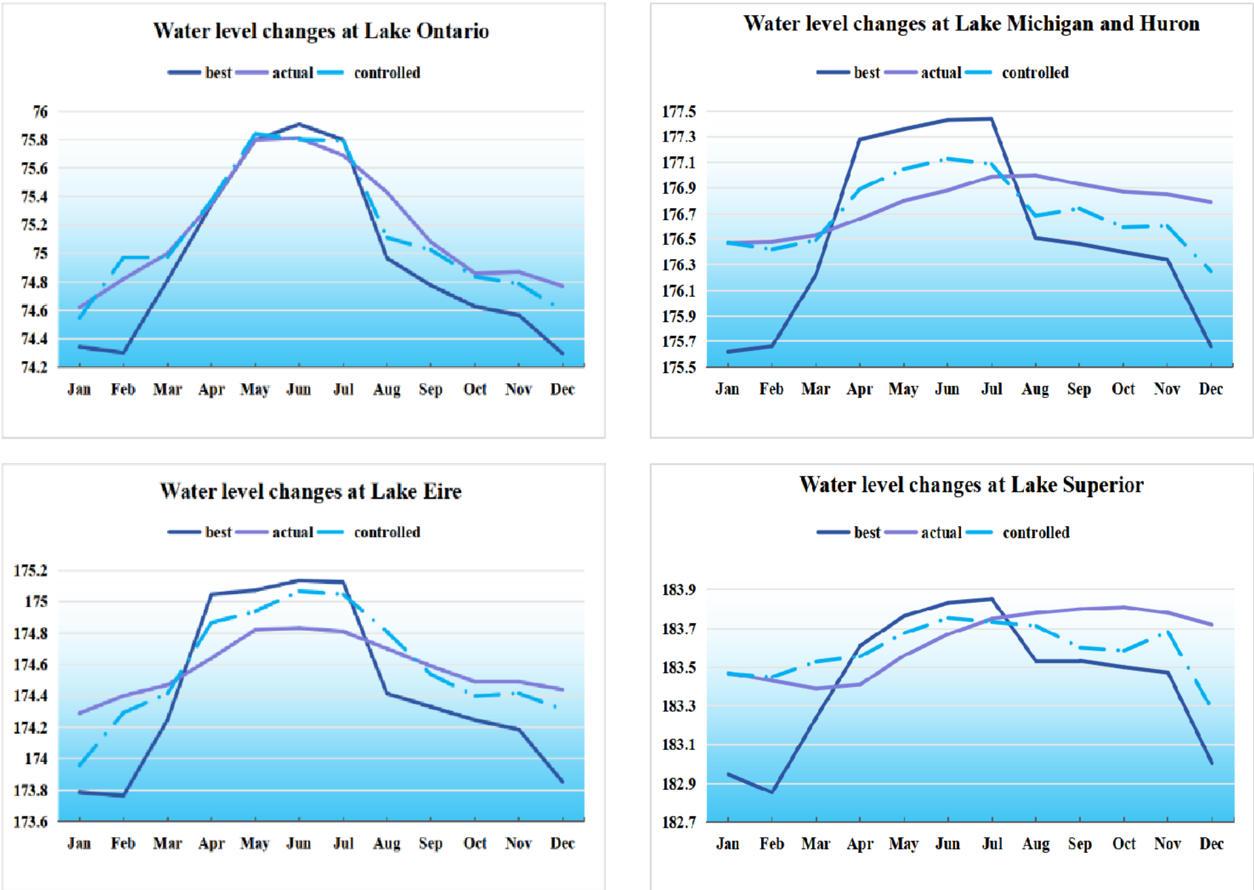


Fig. 4 Comparison of optimal,true and control water levels for four lakes on a monthly basis

4. Sensitivity Analysis

4.1. Environmental Sensitivity Analysis

As mentioned earlier, the PCF model we propose incorporates a recursive prediction error correction control term in the prediction phase, enabling model feedback. This feedback mechanism is particularly significant when the water level on the lake surface is greatly influenced by surrounding environmental factors. When environmental factors deviate significantly from their normal values, leading to a sustained high or low water level on the lake surface, the model retrieves this information from the anomalous changes in water level. Consequently, it adjusts the prediction of the next period's water level, yielding a more optimal water level control scheme.

Specifically, if precipitation consistently deviates from the expected increase, the variable C_{prec} in the actual differential equation will increase; conversely, it will decrease. If winter snowfall experiences a significant increase compared to usual conditions, it will result in increased runoff from the melting snow during the thawing period, i.e., C_{runo} will increase. Conversely, if snowfall is less than usual, the reduced amount of melted water released will lead to runoff smaller than the predicted value. Moreover, if ice accumulates in the river channels forming ice jams, it will increase the Manning roughness coefficient n in the differential equation. This obstruction to water flow will consequently increase the water level in the lake.

To examine the sensitivity of our PCF model to various sudden environmental anomalies, we introduced different variations in the three parameters mentioned earlier: C_{prec}, C_{runo}, and n. These variations were designed to simulate different environmental changes, including abnormal precipitation, unusual snow accumulation, and river channel ice jams. We observed the adjustment schemes of the PCF model for the water levels of the five Great Lakes under these three abnormal conditions and compared them with the water level adjustment scheme in the absence of abnormal environmental factors.

Taking Lake Superior's response to the artificially induced heavy precipitation in January 2017 as an example, from Figure 5, we observe that when the precipitation increases by 5% or 10%, during the initial stages of the anomaly, the dam continues to control the river flow according to the original forecasted plan despite the high precipitation environment, resulting in unstable increases in the lake's water level. By January 10th, the model starts to feedback on the water levels of the preceding 10 days, detecting the discrepancy between the actual and predicted water levels, thus adjusting the water levels for the next 10 days. By January 20th, further adjustments are made based on the water level situation from January 11th to January 20th, and this cycle continues. Generally, the greater the deviation of abnormal precipitation from the normal predicted values, the longer it takes for feedback and correction. As time progresses, the actual water level gradually approaches the normal predicted water level shown in the graph, eventually fluctuating around the original forecasted water level. This demonstrates the PCF model's stability in response to heavy precipitation. Similar trends are observed when the model encounters decreases in abnormal precipitation.

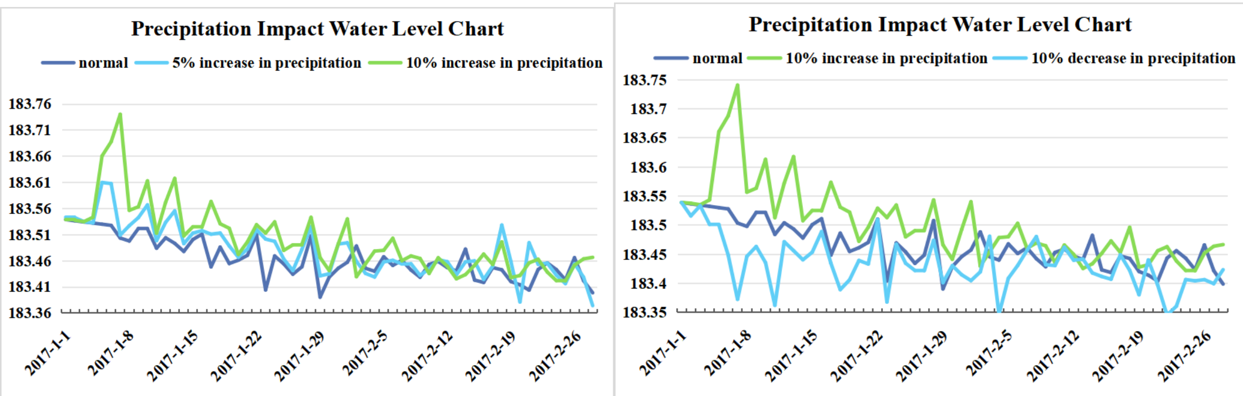
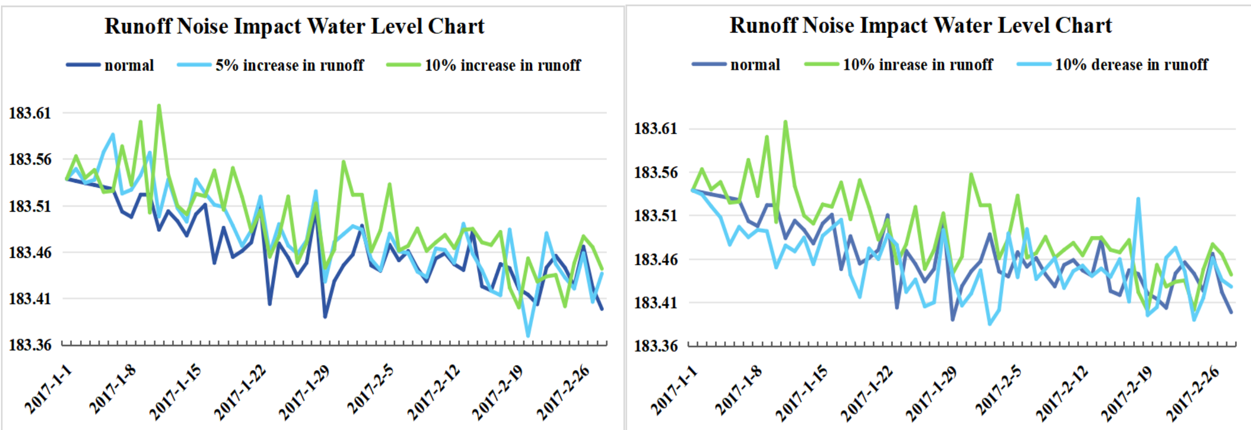


Fig. 5 Precipitation impact water level chart

Similarly, when there are anomalies in runoff or ice jams occur in the river channel, the PCF model gradually provides feedback and corrections to the water level, ultimately restoring it to a position close to the predicted water level. From Figure 6, it is evident that our PCF model demonstrates robustness against changes in environmental factors.



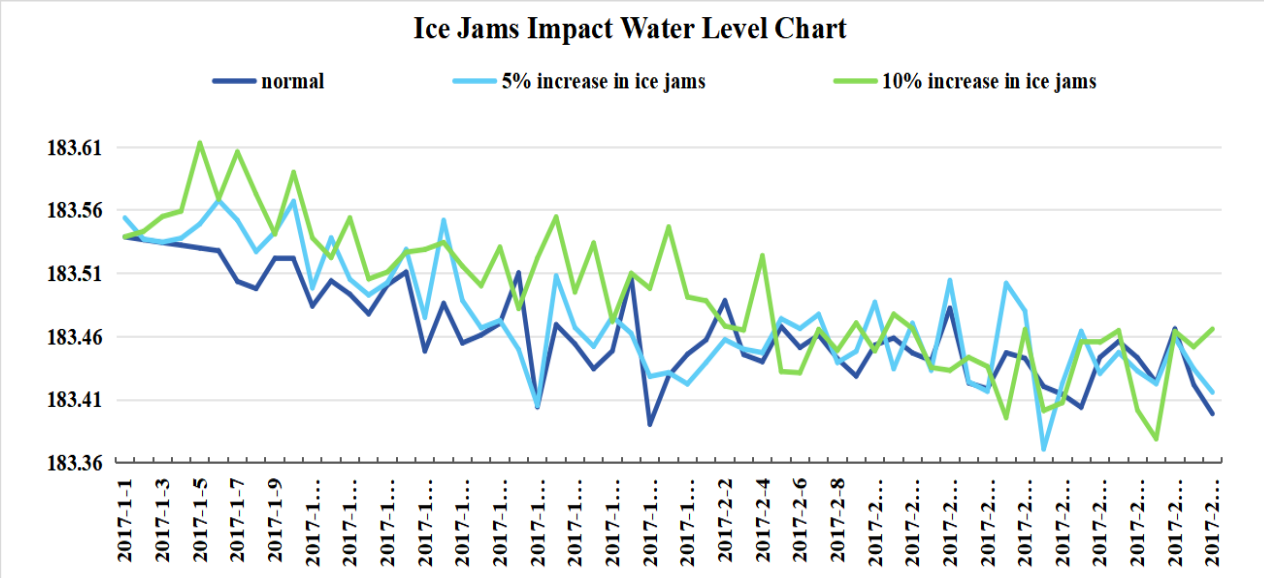


Fig. 6 Ice jams impact water level chart

4.2. Sensitivity of dam control algorithms

During the actual process of controlling the lake water level, deviations may occur between the actual river flow controlled by the dam and the expected controlled flow obtained from the model due to reasons such as equipment aging and operational errors. Consequently, this may lead to imprecise adjustments in the lake water level. To assess the sensitivity of the PCF model to dam control, we introduced Gaussian distributed white noise of 5% and 10% on parameters D1 and D2 in the differential equations, simulating system errors when controlling river flow through the dam. We then observed the resulting water level adjustment schemes and compared them with the originally predicted adjustment schemes.

In Figure 7, even with the addition of noise, the actual controlled water level by the dam still fluctuates around the predicted water level. This indicates that despite the dam itself not being able to precisely control the water level according to the plan, thus having some deviation, it does not significantly affect the water level control of the PCF model. Therefore, the PCF model exhibits good stability in dam control.

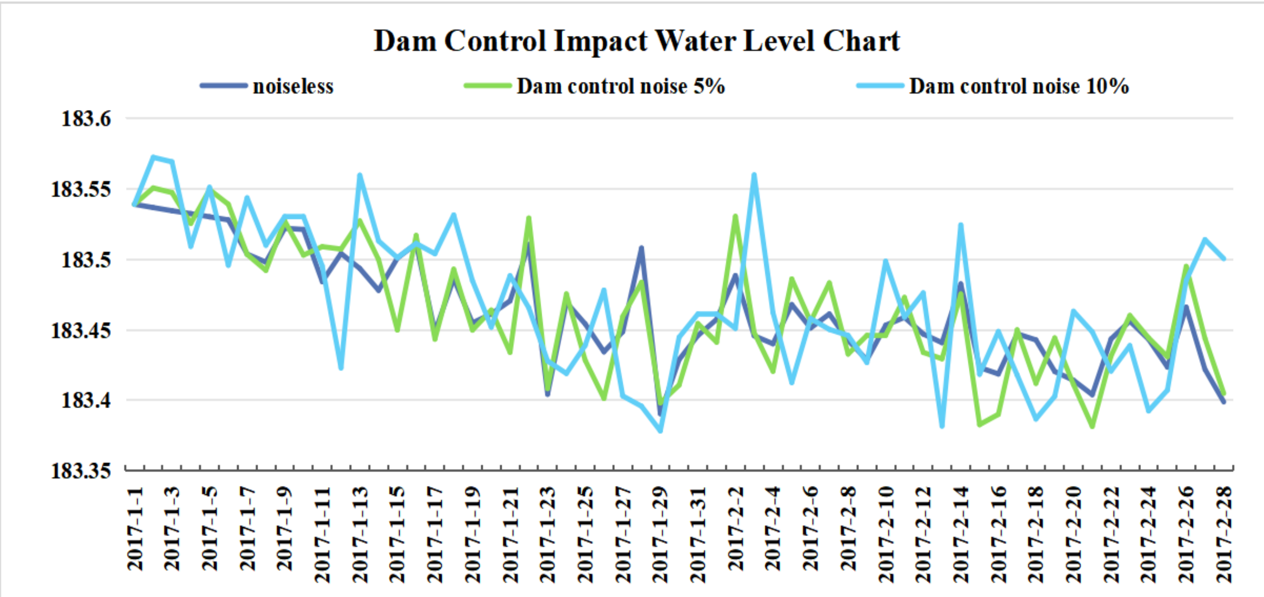


Fig. 7 Dam control impact water level chart

5. Summary

The water level control strategy proposed in this paper, based on the Predictive-Control-Feedback (PCF) model, has been validated for its effectiveness and stability in controlling the water levels of the Great Lakes through comparative analysis with the actual, optimal water levels for the year 2017. The model not only meets the comprehensive satisfaction of various stakeholders' interests and costs but also effectively responds to environmental changes and operational errors in dam control. Through its built-in feedback mechanism, the model self-adjusts to ensure water level control is close to the predicted levels. Sensitivity analyses for environmental factors and dam control algorithms further demonstrate the model's stability and robustness in addressing these changes. Overall, the study provides an optimized solution for the water level management of the Great Lakes, capable of effectively handling various challenges in practical applications.

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