

A study of lamprey populations on ecosystem stability based on Lotka-Volterra modeling

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Abstract. The lamprey undergoes a major transition from larval to adult stages, and its sex ratio changes adaptively in response to food availability during the larval stage. This adaptive strategy allows lamprey to optimize their reproductive success under varying environmental conditions. However, the effects of this sex-ratio adjustment are not limited to the direct reproductive outcomes of lamprey, but also affect broader ecosystem dynamics and interrelationships between different species. To explore these effects, this study used mathematical and computational models to simulate the impacts of sex-ratio adjustments in lamprey populations in response to food availability. These models integrate ecological and evolutionary principles to predict how these adjustments will affect lamprey population dynamics and, in turn, the larger ecosystems in which they occur.

Keywords: Lotka-Volterra model, lamprey, sex ratio.

1. Introduction

The phenomenon of adaptive sex ratio variation observed in the animal kingdom is a fascinating aspect of evolutionary biology and ecology, with important implications for understanding population dynamics and ecosystem management. In particular, lamprey, a species known for its unique life cycle and ecological impacts, provides a compelling case for studying the effects of sex ratio adjustments on environmental cues such as resource availability [1].

Adaptive sex ratio variation is a remarkable phenomenon in biology. In lamprey, for example, the sex ratio of the species changes during the larval stage due to the effect of food availability on growth rate. This adaptive sex ratio change can greatly affect the ecological role of lamprey and its interactions with other species. The aim of this study was to explore the ecological effects of adjusting the sex ratio of lamprey under different resource availability conditions through modeling [2].

2. Changes in the sex ratio of lamprey populations

To adapt the classic Lotka-Volterra predator-prey model for considering sex ratio variations in lamprey populations, we introduce modifications to account for the influence of external environmental factors on sex ratios. The model is simplified into two subpopulations: females (represented by x) and males (represented by y), with a dynamic variable for the sex ratio (z). This approach allows for the simulation of population dynamics where the sex ratio can change in response to environmental conditions, reflecting the complexity of natural ecosystems more accurately.

Female population dynamics

$$\frac{dx}{dt} = r_x x \left(1 - \frac{x+y}{K}\right) - \alpha xy - \gamma xz \quad (1)$$

Male population dynamics

$$\frac{dy}{dt} = r_y y \left(1 - \frac{x+y}{K}\right) + \beta xy - \delta yz \quad (2)$$

Sex ratio

$$\frac{dz}{dt} = \epsilon z (1 - z) \left(\rho - \frac{x+y}{K}\right) \quad (3)$$

The model incorporates intrinsic growth rates r_x and r_y for females and males, carrying capacity K , predation intensity α and γ by females on males and vice versa, symbiotic strength β of females towards males, the effect of females on the sex ratio δ , the rate of change in the sex ratio ϵ , and the equilibrium value of the sex ratio ρ . It accounts for predation and symbiosis between sexes and how the sex ratio influences the entire population. The dynamic variation in the sex ratio, influenced by external environmental factors, simulates changes in the numbers and sex ratios of females and males under different conditions. By adjusting model parameters like predation and symbiosis strength, and the rate of sex ratio change, the model explores the impact of lamprey sex ratio variations on the broader ecosystem, especially observing how these changes affect the numbers and distribution of other species and overall ecosystem stability [3].

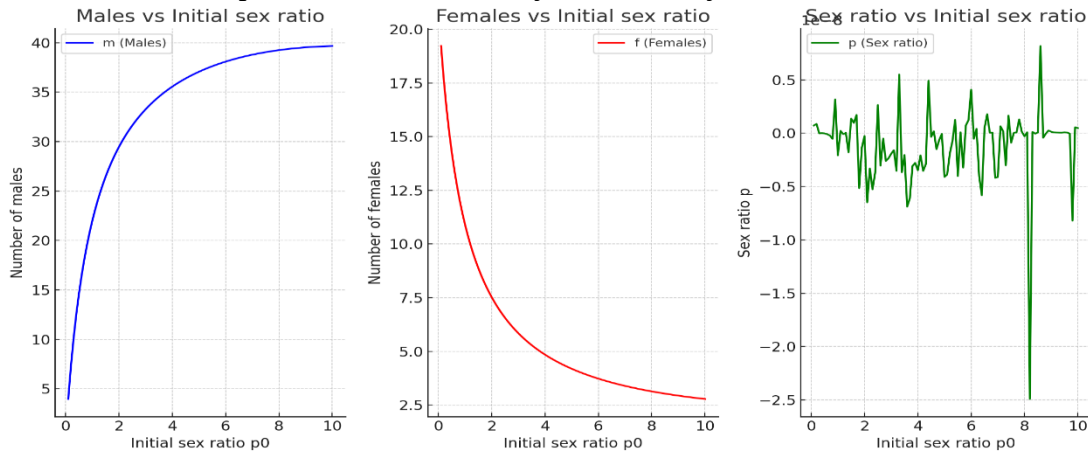


Fig. 1 The impact of the number of males and females, as well as the gender ratio, on population models

As shown in Figure 1, in this model, the population peaks and subsequent decline rates are closely related to competition coefficients and natural mortality rates. Increased competition coefficients mean more intense resource competition among individuals, and higher mortality rates reflect greater survival challenges. High values of these parameters lead to rapid population peaks followed by swift declines, possibly due to competition leading to resource depletion and high mortality rates preventing population maintenance. These dynamics suggest population vulnerability, an important consideration for ecological management and conservation.

3. Impact of lamprey sex ratio on ecosystem stability

To simulate the impact of lamprey sex ratio variations on ecosystem stability, we modify an existing model to include dynamic changes in sex ratio. This model incorporates sex ratio variability, adjusting parameters to simulate various conditions. A new variable representing lamprey sex ratio is introduced, with parameters simulating its dynamic changes throughout the simulation process [4]. The dynamics of A population A's growth is influenced by predation B, and no other predators are available.

$$\frac{dA}{dt} = \alpha A - \beta AB \quad (4)$$

α is the natural growth rate of A.

β is the efficiency with which A preys on B

Dynamics of B population: growth of B is positively affected by the number of C and negatively by predation by A, taking into account the effect of sex ratio.

$$\frac{dB}{dt} = \sigma C - \gamma AB - \epsilon B(S) \quad (5)$$

σ is the rate at which B increases due to predation on C.

γ is the rate at which A preys on B.

ϵ is the effect of the sex ratio of population B on its mortality. ◦

S is the sex ratio of population B, which is affected by the number of population C

Dynamics of C populations: The growth of C is influenced by natural growth and predation by B.

$$\frac{dC}{dt} = \sigma C - \mu BC \quad (6)$$

σ is the natural growth rate of C.

μ is the efficiency with which B preys on C.

Dynamics of sex ratio: Assume that the sex ratio of B is inversely proportional to the number of C.

$$S = \frac{k}{C} \quad (7)$$

k is a constant, indicating that the sex ratio of population B decreases when the number of C increases.

To analyze the stability of this system, we need to find the equilibrium points of the system and then analyze the behavior around those equilibrium points. The local stability of the system was analyzed using the Jacobian matrix. The real part of the eigenvalues tells us how the system behaves near the equilibrium point: a positive real part indicates instability, and a negative real part indicates local stability.

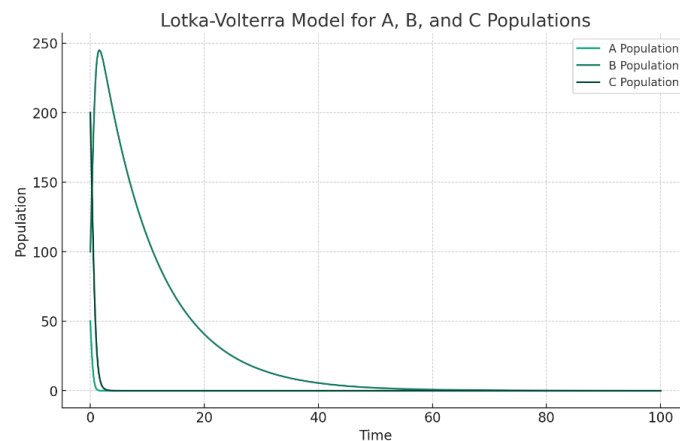


Fig. 2 Dominance of different populations under the Lotka-Volterra model

The Figure 2 shows the dynamic changes over time of populations A (top predators), B (mid-level predators), and C (bottom-level food resources) according to the Lotka-Volterra model. From the image, it can be observed that the quantities of the three populations fluctuate periodically over time, reflecting the complex interactions between predators and food in the natural world.

To analyze the impact of changes in the sex ratio of population B on the stability of the entire ecosystem, it is necessary to consider the relationship between the sex ratio of population B and the quantity of population C. In this model, we simplify by assuming that the sex ratio directly affects B's reproduction rate or mortality rate, but for more accurate analysis, the model needs to be further refined by introducing the sex ratio as a parameter affecting the reproduction rate or mortality rate of population B and re-analyzing the stability of the model.

This analysis involves modifying the model equations to include the impact of the sex ratio on the dynamics of population B, followed by another stability analysis and numerical simulation. This may include adjusting the nonlinear terms of the model equations and considering how the sex ratio changes with changes in the population size of C.

Ultimately, by comparing simulation results under different sex ratios, we can understand how changes in the sex ratio affect the stability of the entire ecosystem. For example, if a significant reduction in the sex ratio of population B leads to a decrease in the reproduction rate, it may reduce the quantity of population B, thereby affecting the dynamic balance between populations A and C, possibly leading to decreased stability of the ecosystem.

This analysis method provides a tool to understand and predict the complex interactions of biological population dynamics in the natural environment, especially when considering biological characteristics such as sex ratios.

4. The impact of changes in lamprey's sex ratio on its own

The improved model incorporates the dynamics of parasite populations, where the birth rate of parasites is influenced by the sex ratio of lampreys. This approach allows for the exploration of how changes in the lamprey sex ratio can affect the breeding behaviors, fluctuations in parasite host numbers, and consequently, the population dynamics of other species within the ecosystem. Through simulation, the model examines the impact of parasite populations on the dynamics of other species, highlighting the interconnectedness of ecosystem components [5].

Consider lampreys as prey and parasites as predators

The female predator

$$\frac{dF}{dt} = \beta F \left(1 - \frac{\alpha M}{F}\right) \quad (8)$$

Male of the predator

$$\frac{dF}{dt} = \beta M \left(1 - \frac{\alpha F}{M}\right) \quad (9)$$

Females of prey

$$\frac{df}{dt} = r_f F - d_f F + b_f \alpha F \quad (10)$$

Male of prey

$$\frac{dm}{dt} = r_m M - d_m M + b_m \alpha M \quad (11)$$

Total prey

$$\frac{dP}{dt} = r_p - aP(M + F) \quad (12)$$

β : thermal expansion coefficient

α : electrical resistivity

r_m, r_f : the thermal resistance of the material and the film's thermal resistance respectively

d_m, d_f : the thickness of the material and the film's thickness respectively

b : width of the film

a : film thickness

r_p : thermal resistance of the film

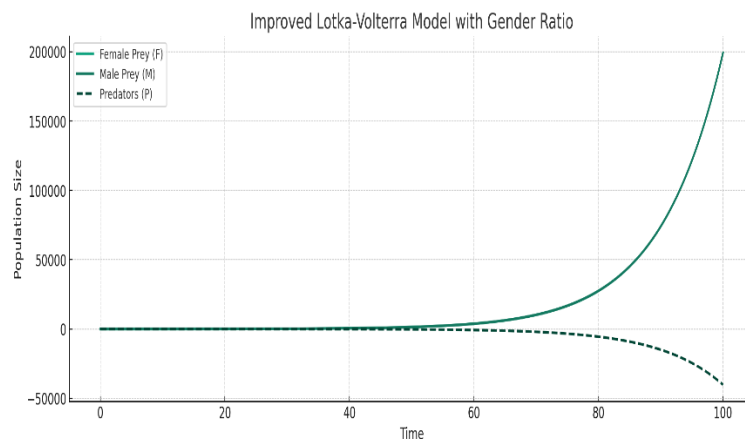


Fig. 3 The impact of changes in the number of males and females on populations under the Lotka-Volterra model

The Figure 3 depicts an improved Lotka-Volterra model that includes gender ratio as a factor. There are three lines on the graph representing female prey (F), male prey (M), and predators (P). Both the female and male prey populations show a steep increase over time, with the female population eventually outgrowing the male population significantly. The predator population initially remains constant but then drops off steeply, potentially going into negative values, which is not possible in a real-world scenario and suggests that the predators go extinct.

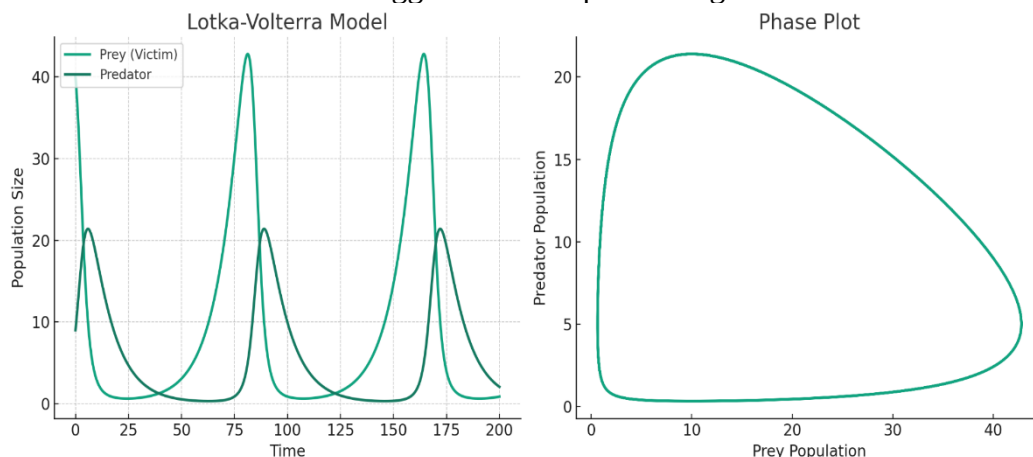


Fig. 4 Population changes between predators and prey resulting in a graph

The Figure 4 shows two plots. The left plot is the classic Lotka-Volterra model, displaying the oscillatory nature of predator and prey populations over time. As the prey (victim) population increases, the predator population also increases but with a delay. After the predators reach a peak, the prey population begins to decrease due to increased predation, followed by a subsequent decrease in the predator population because of a lack of food. This cycle repeats, creating the characteristic oscillatory pattern.

The right plot is a phase plot, showing the predator population as a function of the prey population. This type of plot is used to analyze the stability and behavior of the system without respect to time. The closed loop indicates that the system is cyclic and that the populations are interdependent.

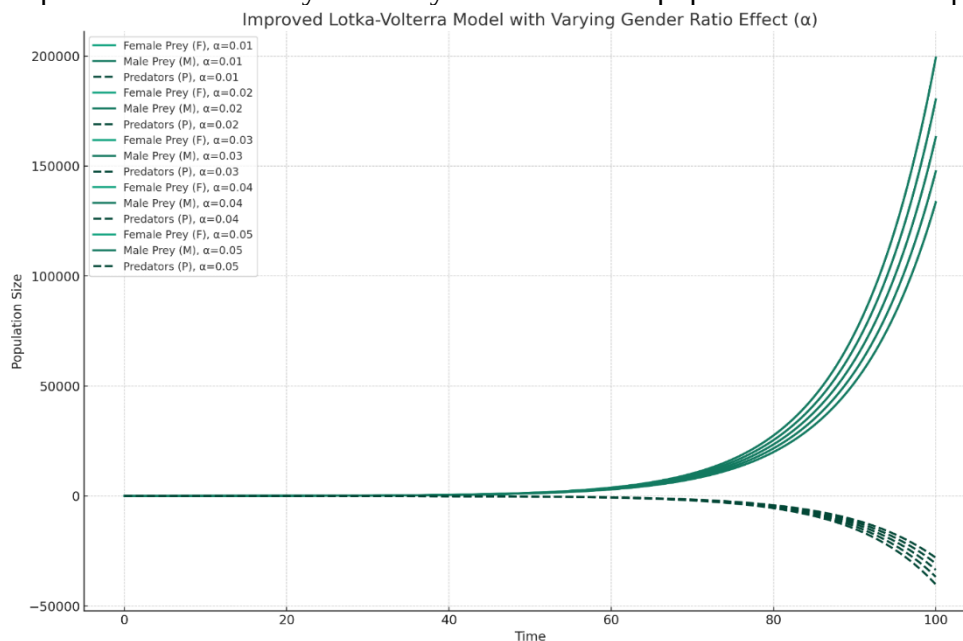


Fig. 5 Population changes under different parameters

The Figure 5 provides a more detailed look at the improved Lotka-Volterra model, showing the effects of varying the gender ratio effect (α) on the populations. Multiple lines represent different values of α , ranging from 0.01 to 0.05. As α increases, the population size of both female and male

prey at any given time increases, and the predator population decreases more steeply. This suggests that changes in the gender ratio have a significant impact on the dynamics of the system, potentially affecting the survival and growth rates of the species involved.

5. Summary

Findings suggest that adaptive changes in lamprey sex ratios can lead to large fluctuations in their populations with cascading effects on ecosystem structure and stability. For example, changes in lamprey sex ratios can alter predation patterns, resource competition, and the dynamics of species interactions within an ecosystem. These changes, in turn, affect the distribution and abundance of other organisms, including organisms that serve as lamprey prey or predators, and species that occupy different positions within the same ecological community.

In addition, this study discusses the dual nature of adaptive sex ratio variation. While sex-ratio variation provides lamprey with the flexibility to adapt to changing environmental conditions, potentially increasing their survival and reproductive success, it also introduces unpredictability and instability into their populations. Fluctuations in sex ratios can lead to oscillations in population sizes, affecting the intricate balance of interactions between species and thus potentially driving the ecosystem to a new equilibrium.

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