

Analysis of Two Game Theory Models: Cournot Model and Stackelberg Model

Jiefu Gao*

School of Science, Xi'an University of Architecture and Technology, Xi'an, China

*Corresponding author: gaojkvera@chd.edu.cn

Abstract. Game theory has long been an important branch of economics, and many of its models have been able to find specific examples in real life. It solves practical problems by considering the predicted and actual behavior of individuals in the game and studying their optimization strategies. This article focuses on two classical models in game theory, the Cournot model and the Stackelberg model, and compares the two models from the perspective of the fundamental model and two special cases, respectively. Firstly, the paper introduces two basic models, enumerates real cases, and deduces the function relation of the optimal production quantity of each enterprise. Then, considering that production quantity and profit may be related to cost, number of enterprises and other factors, the paper changes the relationship between the number of enterprises and cost to explore the impact on the two models. To make the results more intuitive, the authors plug in two sets of data to calculate the profits in each case. The results show that when the number of firms increases, the profits obtained by both Firm1 and Firm2 in Cournot's model decrease. In Stackelberg, Firm 2 made half as much profit as Cournot, and Firm 1 remained the same. With different costs, firms with smaller production costs produce larger quantities and profits. In conclusion, this study compares the two models from the basic case and two additional cases, analyzes which model should be chosen respectively in which cases, and provides effective strategies for real life firms.

Keywords: Cournot model; Stackelberg model; multiple competitors.

1. Introduction

Game theory is a branch of mathematics and economics that deals with the analysis of strategic interactions between rational decision-makers. It provides a framework for understanding how individuals or entities make choices when their actions affect each other. It is used to analyze and understand phenomena like pricing strategies in markets, arms races between nations, evolution of animal behavior, decision-making in business negotiations, and more. Game theory, originating in the 18th century, saw significant advancements in the 1920s through contributions from Emile Borel and John von Neumann. Its pivotal moment came by von Neumann and Oskar Morgenstern, which laid the groundwork for applying game theory to economics, but in the 1950s and 1960s, mathematicians played a key role in its development. Nash's papers on equilibrium and bargaining, along with Shapley's contributions on the Shapley value and core for games with transferable utility, were significant during this period. Bondareva's work on the core also stood out [2]. In the 1970s, game theory made its debut in evolutionary biology, later becoming a cornerstone of microeconomic theory and expanding into various fields of economics, as well as other social and behavioral sciences. Game theory underwent a transformation upon its integration into economics. Initially dominated by a classical perspective, it emphasized viewing games as literal representations of the situations at hand, rather than mere approximations [3]. Evolutionary game theory emphasizes dynamic models over static equilibria, focusing on how outcomes are reached. This shift highlights the importance of historical and institutional factors, challenging the notion of general theories that disregard these influences [4]. The latest game-theoretic models see competition as strategic decision-making amid uncertainty, involving individuals and firms. Unlike perfect competition models, they consider the public disclosure of private information [5]. Another study examines a dynamic Cournot competition game among heterogeneous firms, choosing between two output strategies. Some firms use a simple rule, while others employ a more complex strategy. Initially, fixed fractions of firms adopt each approach, later switching based on profit-driven dynamics. Factors such as firm number, information

costs, and memory impact this evolution, which also delves into global properties of the oligopoly under these dynamics, especially when Nash equilibrium proves unstable [6]. One male(m) and one female(f) take part in this game. Each individual seeks to maximize their own utility. However, their decisions are interdependent. One family member's consumption decisions can impact the well-being of the other family member. The household allocation process occurs in two stages: first, spouses negotiate or agree on transfers, and then each member makes consumption decisions based on these allocations. Decisions can be made using backward induction [7]. Detailed solution processes will not be shown.

The relationship between a beginner and data creators can be seen as a game, where one player controls the predictive model and the other influences data generation. Mathematically, the impact on data generation is represented as a transformation applied to the distribution governing the training data, which affects the data distribution during application. The optimization criteria for both players depend on the chosen predictive model and the transformation. Usually, this scenario is modeled under the assumption of the worst-case scenario, where the adversary seeks to impose how much should the learner spend. This results in a zero-sum game where one player's loss corresponds to the other player's gain. In such a setup, both players can increase their outcomes by utilizing a minimax strategy [8]. Cognitive radio technology facilitates dynamic spectrum access, improving efficiency and addressing scarcity. Unlicensed users can access licensed bands through a collaborative framework where primary users engage secondary users as relays, leasing channel time for data transmission. Secondary users pay for proportional access time, aiming to maximize utility in transmission and revenue. This model represents a Stackelberg game with a distinct Nash Equilibrium point [9].

In the model, leaders confront endogenous entry, investing more than other firms. Among various applications, patent races are crucial, highlighting improved resource allocation when one firm acts as a leader and entry is endogenous. This analysis is closely linked to three ancient frameworks. Firstly, the literature on entry deterrence by Bain, Sylos Labini, and Modigliani considered the effects of entry on market leaders' behavior, albeit without a coherent game-theoretic framework. Secondly, dominant firm theory explains pricing decisions of market leaders facing a competitive fringe, assuming the supply of this fringe increases with price. This theory sheds light on market leaders' behavior under competitive pressure, complementing this work's game-theoretic foundation. Thirdly, the contestable markets theory, pioneered by Baumol, Panzar, and Willig, studies markets for homogeneous goods without sunk entry costs. It shows that potential entrants' "hit and run" strategies lead to an equilibrium price matching the average cost [10]. This equilibrium corresponds to price competition where incumbents set prices before entrants, fostering competitive behavior only when entry is endogenously induced by a potentially large set of competitors.

The purpose of this article is to compare two models in game theory: the Cournot model and the Stackelberg model. These two models share similarities, such as both involving only two competing firms initially, with identical production costs and the joint influence of both firms' outputs on market prices. However, there are also many differences, which will be elaborated on later in the text. The article aims to expand upon these two models by altering the initial assumptions for further comparison. Specifically, the research will focus on the following two aspects: more than 2 competitors and differing production costs between the two firms. The comparison will be conducted by defining variables and performing numerical calculations, supplemented by graphs when necessary. First of all, a brief introduction to the Cournot model is needed, explaining its principles and defining variables and relevant functions based on the initial assumptions of the model. This will be followed by the derivation of corresponding parameters and drawing conclusions. The same procedure will be applied to the Stackelberg model. Then, a comparison will be made between the principles and final parameter results of the two initial models. Next, the models will be expanded and generalized. Under two different expansion scenarios, the derivation of relevant parameters will be conducted, followed by the insertion of data sets for 1-2 groups for intuitive comparison. Finally,

the limitations of this study and areas for improvement will be summarized, with a prospective future outlook.

2. Cournot Model

2.1. Description

The Cournot model is an economic model used to analyze oligopoly market competition, first proposed by the French economist Cournot in 1838. In the Cournot model, it is assumed that two firms compete in the market selling the same product. They determine their optimal production levels based on their competitors' output. The key assumption is that each firm believes its competitor's output is fixed, so when making production decisions, they consider the total supply in the market. The interaction between firms leads to a market equilibrium known as Cournot equilibrium, where each firm's output choice results in total market supply equaling total demand, thereby determining the market price. The Cournot model is an important tool for understanding oligopoly market behavior and equilibrium outcomes, providing a framework for studying strategic choices among firms and the dynamic changes in market prices and quantities.

One classic example of the Cournot Model in action is the market for crude oil. Several oil-producing countries, such as Saudi Arabia, and the United States, are significant players in this market. Each country must decide how much oil to produce, taking into account the production decisions of other countries. Supposing Saudi Arabia, and the United States are the main producers of oil, and each country decides independently how much oil to extract. If one country decides to increase its oil production, it can potentially lower the market price of oil due to the increased supply. However, this decision also depends on how other countries respond. If both two countries increase their production simultaneously, the overall effect on market price may be limited. Followig assumptions are proposed:

- There are a small number of firms in the market, typically two or more.
- Firms compete by choosing the quantity of output they will produce.
- Firms make the decision at the same time, so they don't know the other's choice.
- The firms' production cost is the same.
- There are no barriers to entry or exit from the market.
- Firms aim to maximize their profits.
- The relationships are all linear.

2.2. Solution to the Model

Supposing there are two firms, i.e., Firm 1 and Firm 2, they both sell coke, and the relationship of the price and the amount of products is like:

$$p = a - q_1 - q_2 \quad (1)$$

Here, p is the price of per product, a can be any figure(must bigger or equal to p), q_1 is the amount of products produced by Firm 1, q_2 is the amount of products produced by Firm 2. If the amount of products increases, the price will decrease. Supposing the cost function is:

$$c(q_i) = c \cdot q_i \quad (2)$$

Here, $c(q_i)$ is the total production cost of Firm i ($i=1,2$), c is the per production cost (Firm 1 and Firm 2 are the same) and q_i is the amount of products produced by Firm i ($i=1,2$). Here comes the question that how many products should each firm produce to make the most profits individually. The profit is the total income minus the total production cost ,so the function can be :

$$\pi_i = p \cdot q_i - c \cdot q_i \quad (3)$$

where π_i is the total profits of Firm 1 ($i=1, 2$). Then it is not difficult to get:

$$\pi_1 = (p-c) \cdot q_1 = (a - q_1 - q_2 - c) \cdot q_1 = -q_1^2 + (a - q_2 - c) \cdot q_1 \quad (4)$$

$$\pi_2 = (p-c) \cdot q_2 = (a - q_1 - q_2 - c) \cdot q_2 = -q_2^2 + (a - q_1 - c) \cdot q_2 \quad (5)$$

So a quadratic function can be shown in Fig. 1. Then, one sets the first derivative of this quadratic function to zero (Firm 1 as an example):

$$\frac{\partial \pi_1}{\partial q_1} = -2q_1 + (a - q_2 - c) = 0 \quad (6)$$

At this time, when Firm 1 produces, it can get the most profits:

$$q_1^* = \frac{1}{2}(a - q_2 - c) \quad (7)$$

Similarly, one derives

$$q_2^* = \frac{1}{2}(a - q_1 - c) \quad (8)$$

Combining Eq. (6) and Eq. (7), one derives

$$q_1^* = q_2^* = \frac{1}{3}(a-c) \quad (9)$$

At this time, there is a Nash equilibrium, both of them get the most profits, if any of them deviate, both of them cannot get the most profits:

$$\forall q_1' \in A_1, \pi_1(q_1^*, q_2^*) \geq \pi_1(q_1', q_2^*) \quad (10)$$

$$\forall q_2' \in A_2, \pi_1(q_1^*, q_2^*) \geq \pi_1(q_1^*, q_2') \quad (11)$$

It also means they need to produce the same amount of product, and because of the price is the same and the cost is the same, the profits they make is also the same.

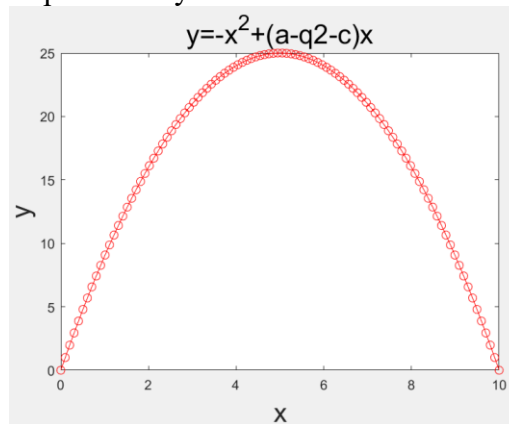


Fig. 1 Quadratic function of Firm 1.

3. Stackelberg Model

3.1. Description

The Stackelberg model is an economic model used to analyze oligopoly market competition, named after the German economist Heinrich von Stackelberg. In this model, there are two firms competing in the market. In the Stackelberg model, one firm, the leader, determines its amount first, while the other firm, the follower, observes the leader's choice and then decides its own amount. This sequential decision-making process results in a hierarchical structure, where the leader has a strategic advantage over the follower. The Stackelberg model provides insights into how firms in an oligopoly market strategically choose their quantities and how the leadership position influences market outcomes such as prices and quantities. An example of the Stackelberg model in reality can be observed in the automobile industry. Consider a scenario where there are two major automobile manufacturers, A and B. Manufacturer A, being the larger and more dominant player in the market, acts as the leader, while manufacturer B follows its lead. In this scenario, manufacturer A strategically

sets its production quantity first, taking into account market demand, production costs, and potential reactions from manufacturer B. Manufacturer B, as the follower, observes the production quantity set by manufacturer A and then decides its own production quantity accordingly.

This sequential decision-making process in the Stackelberg model reflects the real-world dynamics often seen in industries where one company holds a dominant position over its competitors. The leader-follower relationship in the Stackelberg model provides insights into how firms strategically interact in oligopoly markets, influencing market outcomes such as prices, quantities, and overall industry dynamics. Following assumptions are prosposed:

- Two Firms. There are only two firms in the market.
- Leadership and Followership: one firm acts as the leader and sets its quantity first, while the other firm follows by observing the leader's quantity and then determining its own quantity.
- Perfect Information: both firms have perfect information about market demand, costs, and the actions of their competitors.
- Homogeneous Products. The products offered by both firms are identical or highly similar.
- No Strategic Entry or Exit. The number of firms in the market remains constant; there are no new entrants or exits during the period of analysis.
- No Collusion or Cooperation. Firms act independently and do not collude or cooperate with each other.
- The leader will consider the follower's action carefully before it makes decision,so it is clear that the follower's function should be analysed first.

3.2. Solution to the Model

Supposing there are two firms:Firm 1 and Firm 2,they both sell coke,Firm 1 is the leader and Firm 2 is the follower,and the relationship of the price and the amount of products is the same as the Cournot Model:

$$p = a - q_1 - q_2 \quad (12)$$

Here, p is the price of per product, a can be any figure(must bigger or equal to p), q_1 is the amount of products produced by Firm 1, q_2 is the amount of products produced by Firm 2. Supposing the cost function is also the same:

$$c(q_i) = c \cdot q_i \quad (13)$$

where $c(q_i)$ is the total production cost of Firm i ($i=1,2$), c corresponds to per production cost (Firm 1 and Firm 2 are the same) and q_i is the amount of products produced by Firm i ($i=1, 2$). Here comes the question: how many products should each firm produce to make the most profits individuall. The profit is the total income minus the total production cost ,so the function can be:

$$\pi_i = p \cdot q_i - c \cdot q_i \quad (14)$$

Here, π_i is the total profits of Firm 1 ($i=1, 2$). Then, it is not difficult to get:

$$\pi_1 = (p-c) \cdot q_1 = (a - q_1 - q_2 - c) \cdot q_1 = -q_1^2 + (a - q_2 - c) \cdot q_1 \quad (15)$$

$$\pi_2 = (p-c) \cdot q_2 = (a - q_1 - q_2 - c) \cdot q_2 = -q_2^2 + (a - q_1 - c) \cdot q_2 \quad (16)$$

and another quadratic function can get as given in Fig. 2. Then set the first derivative of this quadratic function to 0,but what should be paid attention to is that Firm 1 will consider Firm 2's action carefully before it makes decision, so it is clear that the Firm 2 should be analyzed first:

$$\frac{\partial \pi_2}{\partial q_2} = -2q_2 + (a - q_1 - c) = 0 \quad (17)$$

At this time, the relationship of q_2 and q_1 can be concluded:

$$q_2 = \frac{1}{2}(a - q_1 - c) \quad (18)$$

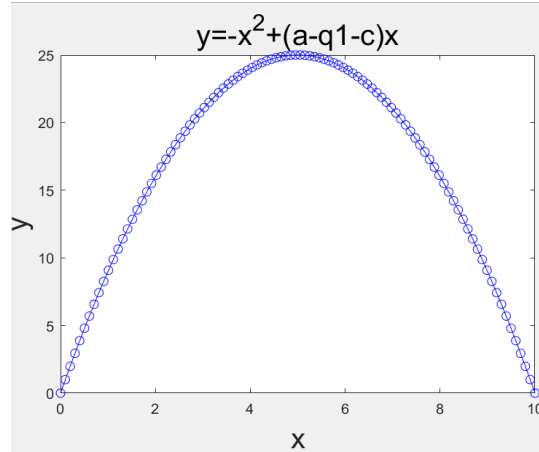


Fig. 2 Quadratic function of Firm 2

Next, one comes to Firm 1, it is significant to figure out the best choice of Firm 1, because it makes the decision first. Only when q_1^* is figured out, can the q_2^* be known. Combining Eq. (4) and Eq. (18), one derives:

$$\pi_1 = -q_1^2 + (a - q_2 - c) \cdot q_1 = -q_1^2 + \left[a - \frac{1}{2}(a - q_1 - c) - c \right] \cdot q_1 \quad (19)$$

Finally it can be simplified:

$$\pi_1 = -\frac{1}{2}q_1^2 + \frac{1}{2}(a - c) \cdot q_1 \quad (20)$$

Then, let the first derivative of this quadratic function to 0:

$$\frac{\partial \pi_1}{\partial q_1} = -q_1 + \frac{1}{2}(a - c) = 0 \quad (21)$$

At this time, q_1 can be the maximum:

$$q_1^* = \frac{1}{2}(a - c) \quad (22)$$

After knowing q_1^* , q_2^* can be calculated:

$$q_2^* = \frac{1}{2}(a - q_1^* - c) = \frac{1}{4}(a - c) \quad (23)$$

Through the results, it can be known that in Stackelberg Model, the leader will usually take the advantage because he can anticipate what will happen next, thus he can make the decision more carefully and correctly

4. Comparison

4.1. Basic Models

In views of time of decision, for Cournot Model, firms make decisions simultaneously regarding the quantity of output they will produce. It means they cannot discuss with each other and cannot cooperate, they need to make decisions independently. Regarding to Stackelberg Model, one firm (the leader) makes its production decision first, followed by the other firm(s) (the follower(s)), so the leader will anticipate the follower's action carefully before making the decision, and the follower can know the leader's production decision. Regarding to leadership and Market Power, in Cournot Model firms have equal market power and act independently of each other. For Stackelberg Model the leader firm has a strategic advantage and exercises market power by setting its quantity before the other firm(s) react. For Quantity Produced, in Cournot Model, firms produce quantities that result in a Nash equilibrium, where each firm's output maximizes its profit given the output of its competitors. Both of them can benefit the most by producing the same quantity of products when the

production cost is the same. For Stackelberg Model, the leader firm produces a larger quantity than it would under Cournot competition, while the follower(s) produce less. According to the calculation before, when the relationship of the price and the amount of products is $p = a - q_1 - q_2$, the most suitable amount of Firm 1 is $q_1^* = (a - c)/2$, the Firm 2 is $q_2^* = (a - c)/4$. So if the production cost is the same, q_1^* doubles q_2^* . For Profit Maximization, in Cournot Model, firms maximize profits given the quantities produced by their competitors. For Stackelberg Model, the leader firm maximizes its profit considering how the follower(s) will respond to its output decision. For Stability, in Cournot Model, it can lead to unstable outcomes or cycles due to firms not fully anticipating each other's decisions. Stackelberg Model shows a more stable outcome often arises because the leader firm can carefully guess and respond to the actions of the follower(s). For Information Assumptions, in Cournot Model, Firms have perfect information about market demand and the actions of their competitors. For Stackelberg Model, one assumes the leader firm has perfect information about market demand and can perfectly anticipate the reactions of the follower(s).

To make the difference clearer to see, using statistical comparison is necessary. Next, there will be several groups of figures and calculations. Among these figures, where and how it makes a difference can be seen. Assuming that $a=100, c=40$, then the situation will be shown in Table. 1 Substituting another set of data: $a=1200, c=300$, the situations of the two models are still shown in Table 1. It can be found that Firm 1 tends to get more profit in Stackelberg than in Cournot, this is because the production increases, and the profit of per production is the same. Similarly, Firm 2 tends to get fewer than Cournot. In Cournot model, the total profits made by 2 firms are more than in Stackelberg. So, it is important and essential for Firm 1 to make the choice and anticipate Firm 2's action first.

Table 1. Two basic models comparing.

a	c	Model	q_1^*	q_2^*	p	π_1	π_2	$\pi_1 + \pi_2$
100	40	Cournot	20	20	60	400	400	800
100	40	Stackelberg	30	15	55	450	225	675
1200	300	Cournot	300	300	600	90000	90000	180000
1200	300	Stackelberg	450	225	525	101250	50625	151875

4.2. Situation 1: More than 2 Competitors

In the basic Cournot model, Firm 1 and Firm 2 makes the same amount of profit, and it is both $(a-c)/3$. When the competitors increases, it will be different. First, one supposes that there are 3 firms: Firm 1, 2 and 3. Then, the relationship of price and the production will be:

$$p = a - q_1 - q_2 - q_3 \quad (24)$$

Here, q_3 is the amount of products produced by Firm 3. The production cost and function is the same as the basic model. It is not difficult to get:

$$\pi_1 = (p-c) \cdot q_1 = (a - q_1 - q_2 - q_3 - c) \cdot q_1 = -q_1^2 + (a - q_2 - q_3 - c) \cdot q_1 \quad (25)$$

Then, let the first derivative of this quadratic function to 0:

$$\frac{\partial \pi_1}{\partial q_1} = -2q_1 + (a - q_2 - q_3 - c) = 0 \quad (26)$$

At this time, when Firm 1 produces q_1 , it can get the most profits:

$$q_1^* = \frac{1}{2}(a - q_2 - q_3 - c) \quad (27)$$

Similarly:

$$q_2^* = \frac{1}{2}(a - q_1 - q_3 - c) \quad (28)$$

$$q_3^* = \frac{1}{2}(a - q_1 - q_2 - c) \quad (29)$$

Then, one derives:

$$q_1^* = q_2^* = q_3^* = (a-c)/4 \quad (30)$$

So, it is the profits made by 3 firms. They are also the same, but every firm makes fewer. The main reason can explain the phenomenon is that an increase in competitors means fewer resources available to each company, resulting in reduced output. Next, one comes the situation of n competitors:

$$p = a - q_1 - q_2 - q_3 - \dots - q_n \quad (31)$$

It is easy to get:

$$\pi_1 = -q_1^2 + (a - q_2 - q_3 - \dots - q_n - c) \cdot q_1 \quad (32)$$

Similarly:

$$\pi_i = -q_i^2 + (a - q_1 - q_2 - \dots - q_{i-1} - q_{i+1} - \dots - q_n - c) \cdot q_i \quad (33)$$

Combining those formulae

$$q_1^* = q_2^* = \dots = q_n^* = \frac{1}{n+1}(a-c) \quad (34)$$

In this situation, all of the products are divided into n+1 parts, the profits made by each firm will decrease sharply.

For Stackelberg model. What need to pay attention to is that there are 2 potential situations. One is that Firm 1 is the leader, Firm 2 and 3 both make their decisions at the same time. The other situation is that Firm 1 is the leader, Firm 2 follows Firm 1, and Firm 3 follows Firm 2. Here, one uses the latter one, because it better aligns with the principles of the Stackelberg model. If it is the first situation, for Firm 2 and Firm 3, it is a second Cournot model, since they make decisions at the same time. So the assumption is that, Firm 1 is the leader, Firm 2 follows Firm 1, and Firm 3 follows Firm 2. Firstly, Firm 3's choice should be analyzed:

$$\pi_3 = -q_3^2 + (a - q_1 - q_2 - c) \cdot q_3 \quad (35)$$

Let the derivative be 0:

$$\frac{\partial \pi_3}{\partial q_3} = -2q_3 + (a - q_1 - q_2 - c) = 0 \quad (36)$$

$$q_3 = \frac{1}{2}(a - q_1 - q_2 - c) \quad (37)$$

Secondly, Firm 2's choice can be figured out:

$$\pi_2 = (a - q_1 - q_2 - q_3 - c) \cdot q_2 = -q_2^2 + (a - q_1 - q_3 - c) \cdot q_2 \quad (38)$$

Then substituting q3 for q3, combine above, one derives

$$\pi_2 = -\frac{1}{2}q_2^2 + \frac{1}{2}(a - q_1 - c) \cdot q_2 \quad (39)$$

Let the derivative be 0:

$$\frac{\partial \pi_2}{\partial q_2} = -q_2 + \frac{1}{2}(a - q_1 - c) = 0 \quad (40)$$

$$q_2 = \frac{1}{2}(a - q_1 - c) \quad (41)$$

Thirdly, one can calculate Firm 1's most profits, the function is the same as earlier and one derives:

$$\pi_1 = -q_1^2 + (a - q_2 - q_3 - c) \cdot q_1 = \frac{1}{2}q_1q_2 = \frac{1}{4}q_1(a - q_1 - c) = -\frac{1}{4}q_1^2 + \frac{1}{4}(a - c)q_1 \quad (42)$$

Let the derivative be 0, at this time, Firm 1 will make the most profits:

$$\frac{\partial \pi_1}{\partial q_1} = -\frac{1}{2}q_1 + \frac{1}{4}(a - c) = 0 \quad (43)$$

$$q_1^* = \frac{1}{2}(a - c) \quad (44)$$

Finally, q_2^* and q_3^* can be obtained:

$$q_2^* = \frac{1}{4}(a - c) \quad (45)$$

$$q_3^* = \frac{1}{8}(a - c) \quad (46)$$

By examining the formula introduced, it can be observed that the maximum output of Firm 1 and Firm 2 remains unchanged, while Firm 3 can only produce half of Firm 2, as it makes decisions later than Firm 2. Similarly, when it comes to n competitors:

$$q_n^* = \frac{1}{2^n}(a - c) \quad (47)$$

In this situation, Firm n is the biggest ‘victim’ because it can only make decisions in the last place and its production is bound to be the fewest. To allow comparison between the new and original models, the same values as the original model were selected here. Assuming that $a=100, c=40$, just like before, then the situation is shown in Table 2. Substituting another set of data, $a=1200, c=300$, the situations of the two models are also shown in the table.

Table 2. Two models in situation 1 comparing.

a	c	Model	q_1^*	q_2^*	q_3^*	p	π_1	π_2	π_3	sum
100	40	Cournot	15	15	15	55	225	225	225	675
100	40	Stackelberg	30	15	7.5	47.5	225	112.5	56.25	393.75
1200	300	Cournot	225	225	225	525	50625	50625	50625	151875
1200	300	Stackelberg	400	225	112.5	412.5	50625	25312.5	12656.25	88593.75

It can be found that Firm 1 tends to produce a half products of Stackelberg in Cournot, but the profits may not be fewer than in Stackelberg. This is because the production increases, but the price decreases, so the profit of per production decreases, and the total profits decreases. For Firm 2, the amount of products is the same in two models, but the price is lower in Stackelberg, so the Cournot can make more profits. For Firm 3, it is clear that Cournot will help to benefit more, because the price and the amount of products are both higher than Stackelberg. Usually, the whole profits made by 3 firms is higher in Cournot, because Firm 2 and Firm 3 are bound to make more profits in such a situation. As for n competitors, it is the same as 3 competitors.

4.3. Situation 2: Different Production Costs

In this situation, there are only 2 firms, but the per production cost is different, which means that profit is not determined solely by price. In order to simplify the model, the cost function is supposed to be linear. For Cournot model, the function of the price is the same as the basic model, so it will not be shown here. What makes a difference is the cost function:

$$c(q_i) = c_i \cdot q_i (c_1 \neq c_2) \quad (48)$$

At the same time, other functions also change:

$$\pi_1 = (p - c) \cdot q_1 = (a - q_1 - q_2 - c) \cdot q_1 = -q_1^2 + (a - q_2 - c_1) \cdot q_1 \quad (49)$$

$$q_1^* = \frac{1}{2}(a - q_2 - c_1) \quad (50)$$

$$q_2^* = \frac{1}{2}(a - q_1 - c_2) \quad (51)$$

Thus,

$$q_1^* = \frac{1}{3}(a + c_2 - 2c_1) \quad (52)$$

$$q_2^* = \frac{1}{3}(a + c_1 - 2c_2) \quad (53)$$

So, q_1^* and q_2^* depends on c_1 and c_2 . If $c_1 > c_2$, $q_1^* < q_2^*$; if $c_1 < c_2$, $q_1^* > q_2^*$. For Stackelberg model The price and the cost function is the same as above. Next, Firm 2's decision should also be analyzed first:

$$\pi_2 = -q_2^2 + (a - q_1 - c_2) \cdot q_2 \quad (54)$$

$$\frac{\partial \pi_2}{\partial q_2} = -2q_2 + (a - q_1 - c_2) = 0 \quad (55)$$

At this time, the relationship of q_2 and q_1 can be concluded:

$$q_2 = \frac{1}{2}(a - q_1 - c_2) \quad (56)$$

Next comes to Firm 1:

$$\pi_1 = -q_1^2 + (a - q_2 - c_1) \cdot q_1 \quad (57)$$

$$q_1^* = \frac{1}{2}(a - q_2 - c_1) \quad (58)$$

Combining them together,

$$q_1^* = \frac{1}{2}(a + c_2 - 2c_1) \quad (59)$$

Thus, q_2^* can be concluded:

$$q_2^* = \frac{1}{4}(a + 2c_1 - 3c_2) \quad (60)$$

The result is similar to the same situation in Cournot model. What makes differences are that Firm 1 has an inclination to produce more products, while Firm 2 tends to produce fewer. It is Assuming that $a=100, c_1=50 > c_2=30$, then the situation is presented in Table 3. Substituting another set of data, $c_2=500 > c_1=300$, $a=1200$, the situations of the two models are also shown in Table 3. It can be found that if $c_1 > c_2$, then Firm 1's production is fewer than Firm 2, and Firm 1's per production profit is also fewer than Firm 2, so of course Firm 2 gets much more profits. If $c_1 < c_2$, the fact will be opposite. Firm 1's production is more than Firm 2, and Firm 1's per production profit is also more than Firm 2, so undoubtedly Firm 1 gets much more profits than Firm 2. It means that the firm with a lower production cost tends to produce more to increase the income. As a result, it is always the firm that can get the more profits.

Table 3. Two models in situation 2 comparing.

a	c1	c2	Model	q1*	q2*	p	π_1	π_2	$\pi_1 + \pi_2$
100	50	30	Cournot	10	30	60	100	900	1000
100	50	30	Stackelberg	15	27.5	57.5	112.5	756.25	868.75
1200	300	500	Cournot	336.67	166.67	666.67	134444.44	27777.78	162222.22
1200	300	500	Stackelberg	550	75	575	151250	5625	156875

5. Limitations and Prospects

The article's reliance solely on linear functional relationships is a significant limitation. By neglecting consideration for nonlinear functions, it oversimplifies complex real-world scenarios. While this approach streamlines research and data computation, it overlooks the intricacies inherent in market dynamics. Incorporating nonlinear functions would better capture the dynamics of supply and demand, pricing strategies, and market equilibrium, leading to a more comprehensive analysis. Furthermore, the article's exploration of only two scenarios lacks depth. To address this limitation, it should delve into more intricate and realistic situations. For example, introducing multiple companies with varying production costs or altering market supply and demand functions to include quadratic

or exponential relationships would enhance the article's relevance and applicability. The presentation of data in the article is also simplistic, primarily relying on tables. To improve this aspect, incorporating visual aids such as charts, graphs, and diagrams would make the data more accessible and understandable to readers. Visual representations can highlight trends, patterns, and correlations, facilitating a deeper understanding of the concepts discussed. Expanding the article's model variety by including well-known models like the Bertrand and Sweezy models would enrich the analysis. Contrasting different models can empower businesses to tailor their strategies based on collaboration and competition dynamics, leading to more effective decision-making and competitive advantage. In addition, exploring these models could stimulate fresh perspectives on game theory, fostering further exploration and innovation in the field. By considering a wider range of models and scenarios, the article can provide valuable insights for practical applications in real-world businesses and contribute to advancements in economic theory and strategy.

To further enhance the article, it's crucial to consider scenarios that better reflect the complexities of real-world market dynamics. One approach is to incorporate multiple companies, each with distinct production costs. This addition would introduce competition and variability into the analysis, offering a more realistic portrayal of market interactions. Additionally, modifying the market supply and demand functions to reflect quadratic or exponential relationships would better capture nonlinear effects, providing a more accurate representation of how changes in factors like price and quantity affect market equilibrium. Furthermore, incorporating extra factors such as discounts on certain goods could provide deeper insights into consumer behavior and market trends. By accounting for these nuances, the analysis would better align with real-world scenarios, enabling more informed decision-making for businesses. Expanding the range of models considered in the article would also enrich the analysis. Introducing well-known models like the Bertrand and Sweezy models alongside or instead of the Cournot and Stackelberg models would provide a broader perspective on market dynamics. This expansion would offer readers a more comprehensive understanding of various competitive strategies and their implications for firms operating in different market environments. Moreover, to improve the presentation of data, incorporating more datasets and considering a wider range of scenarios would be beneficial. Visual aids such as line charts, scatter plots, and histograms would enhance the readability of the article, making trends and relationships more accessible to readers. This visual representation would facilitate comparisons between different scenarios or models, enabling a more nuanced interpretation of the findings.

In conclusion, by embracing more realistic scenarios, incorporating additional models, and improving data presentation methods, the article can evolve into a more robust and insightful resource for both academics and practitioners in the field of economics and business strategy. These enhancements would enable a deeper understanding of market dynamics and better equip readers to navigate complex real-world situations.

6. Conclusion

To sum up, this article compares the two models from the basic situation and two additional situations. The quantity that each firm should produce and the profit obtained in each situation are calculated, which helps to understand the differences more clearly. First comes to two basic models: In Cournot, Firm 1 and Firm 2 preferably produce the same amount of product, which is $1/3$ (a-c). Since the selling price and production cost are the same, the profits of the two firms are the same; In Stackelberg, Firm 1 produced more production and profits than Cournot, while Firm 2 did the opposite. Next is the conclusion about the expansion model. When there are multiple firms, all firms in Cournot's model still make the same profit, but they all reduce compared with the base model. In Stackelberg, firm 1 made the same profit as Cournot, but firm 2 and 3 made less profit than Cournot. When the two firms have different costs, whether in Cournot or Stackelberg, the firm with lower production costs will produce more to increase profits, and that firm tends to be the one with more profits. The main limitations of this paper are the lack of consideration for more complex cases, such

as nonlinear functions, and the lack of diversity of data, which is only reflected in tables. The above can be improved by introducing more models and drawing scatter plots, line plots, etc. Generally, the article helps to understand firm behavior and market equilibrium in different market competition environments. These models provide powerful theoretical support and practical guidance for enterprises to formulate market strategies, predict market changes, analyze market competition structure, and make policy and supervision.

References

- [1] Osborne M J. An introduction to game theory. New York: Oxford university press, 2004.
- [2] Peters H. Game theory: A Multi-leveled approach. Springer, 2015.
- [3] Samuelson L. Game theory in economics and beyond. *Journal of Economic Perspectives*, 2016, 30(4): 107-130.
- [4] Weibull J W. Evolutionary game theory. MIT press, 1997.
- [5] McAfee R P, McMillan J. Competition and game theory. *Journal of marketing research*, 1996, 33(3): 263-267.
- [6] Bischi G I, Lamantia F, Radi D. An evolutionary Cournot model with limited market knowledge. *Journal of Economic Behavior & Organization*, 2015, 116: 219-238.
- [7] Chen Z, Woolley F. A Cournot–Nash model of family decision making. *The Economic Journal*, 2001, 111(474): 722-748.
- [8] Brückner M, Scheffer T. Stackelberg games for adversarial prediction problems. *Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining*. 2011: 547-555.
- [9] Zhang J, Zhang Q. Stackelberg game for utility-based cooperative cognitive radio networks. *Proceedings of the tenth ACM international symposium on Mobile ad hoc networking and computing*. 2009: 23-32.
- [10] Etro F. Stackelberg competition with endogenous entry. *The Economic Journal*, 2008, 118(532): 1670-1697.