

# Evaluation of Uncertainty in Linear Plane Angle Measurement based on Coordinate Measuring Machines

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**Abstract.** According to the measurement method of linear plane angle based on coordinate measuring machine (CMM), establish a black box evaluation model for the uncertainty of CMM linear plane angle measurement, and analyze the sources of each uncertainty. Combining measurement examples, this paper elaborates on the uncertainty evaluation process of linear plane angle measurement based on CMM, calculates the extended uncertainty of linear plane angle measurement based on CMM, and provides a reference for the uncertainty evaluation of linear plane angle measurement based on CMM.

**Keywords:** metrology; coordinate measuring machine; linear plane angle; uncertainty in measurement.

## 1. Introduction

Coordinate Measuring Machine (CMM) is an efficient and multifunctional precision geometric measurement instrument that can quickly and accurately measure the dimensions, shapes, positions, and other geometric quantities of components with complex shapes. Due to the diversity and complexity of CMM measurement objects, the evaluation of measurement uncertainty is relatively difficult. The main reason is that there are multiple sources of CMM measurement uncertainty, the construction of uncertainty models is complex, and the uncertainty components are difficult to accurately quantify. Generally speaking, when evaluating the uncertainty of CMM measurement, it is usually aimed at specific measurement tasks, which requires specific clarification of a certain measurement object, as well as relevant measurement environment elements, detection strategies, and other influencing conditions. To address the assessment of measurement uncertainty in CMM for specific tasks, relevant scholars have conducted research. Reference [2] studied the uncertainty evaluation method for CMM size measurement and established a black box model for measurement uncertainty evaluation. Using the method of measurement system analysis, quantitatively analyze the uncertainty components introduced by the six characteristic indicators of quantity values. Reference [3] adopts the measurement system analysis method to evaluate the uncertainty of coordinate measuring machines in product inspection based on quantity characteristic indicators. Reference [4] studied the uncertainty evaluation problem of measuring end face distance using coordinate measuring machines (CMMs). The national standard [5] GB/T 38762.3-2020 establishes a default specification operation set for angle dimensions and defines a series of specific specification operation sets for the following various angle dimension elements. To further standardize and guide the evaluation of measurement uncertainty in CMM, the national standard [6] GB/T 24635.1-2020 provides technical guidelines for determining measurement uncertainty in CMM. On the basis of the aforementioned research, this article focuses on the uncertainty evaluation of linear plane angle measurement tasks based on CMM.

## 2. Measurement model

The angle between linear planes belongs to the direct global angular dimension in linear dimensions, and it is required that the dimension has a clear and specific specification operation set annotation [5]. The planar evaluation methods include the minimum containment area method and the least squares method [7]. When using CMM for linear plane angle measurement, based on the

representation of the input and output of linear plane angle measurement, a black box modeling method based on quantity characteristic analysis is adopted to establish the following model

$$\alpha = A + \delta_1 + \delta_2 + \delta_3 + \delta_4 \quad (1)$$

where,  $\alpha$  is the linear plane angle measurement result,  $A$  is the estimated value of linear plane angle,  $\delta_1$  is the impact of CMM dimension measurement indication error on measurement results,  $\delta_2$  is the impact of repeatability in measuring the angle between linear planes on the measurement results,  $\delta_3$  is the impact of the reproducibility of linear plane angle measurement on the measurement results,  $\delta_4$  is the influence of the flatness of the measured plane on the angle between the planes.  $\delta_1$  is a uniform distribution with an expected value of 0, and the standard deviation is the uncertainty  $u_1$  introduced by the CMM detection error.  $\delta_2$  is a normal distribution with an expected value of 0, and the standard deviation is the uncertainty  $u_2$  introduced by measurement repeatability.  $\delta_3$  is a normal distribution with an expected value of 0, and the standard deviation is the uncertainty  $u_3$  introduced by measurement reproducibility.  $\delta_4$  is a uniform distribution with an expected value of 0, and the standard deviation is the uncertainty  $u_4$  introduced by the influence of the flatness of the measured plane on the angle between the planes.

According to the measurement model equation (1), ignoring the correlation between components, the formula for calculating the uncertainty of the composite standard is

$$u_c^2(\alpha) = c_1^2 u^2(\delta_1) + c_2^2 u^2(\delta_2) + c_3^2 u^2(\delta_3) + c_4^2 u^2(\delta_4) \quad (2)$$

where,  $c_1$  is 1,  $c_2$  is 1,  $c_3$  is 1,  $c_4$  is 1.

Then

$$u_c(\alpha) = \sqrt{u^2(\delta_1) + u^2(\delta_2) + u^2(\delta_3) + u^2(\delta_4)} \quad (3)$$

### 3. Evaluation of measurement uncertainty

#### 3.1 Measurement example

Under laboratory environmental conditions, use CMM to measure the angle between two planes of a certain part, as shown in Figure 1. According to the measurement requirements, suitable measuring needles and rods are selected. In this example, a measuring head with a diameter of 8mm, a measuring rod length of 114mm, and a contact measuring force of 150mN are used. Contact detection is carried out by uniformly distributing several measurement points on each measured plane (in this example, there are 2 measured planes, with 25 measurement points evenly distributed on each plane). The minimum area method is used to fit the measured plane, and the complementary angle between the two planes in the normal direction is calculated, which is the angle between the two planes. Repeat 3 measurements and take the average of the 3 measurements as the measurement result.

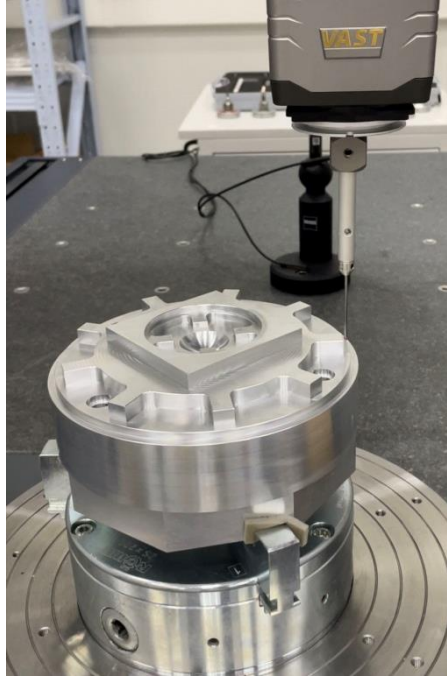


Figure 1. Schematic diagram of linear plane angle measurement process based on CMM

### 3.2 Quantization of uncertainty components

The above analysis shows that the uncertainty components of linear plane angle measurement based on CMM mainly include uncertainty  $u_1$  introduced by CMM size measurement indication error, uncertainty  $u_2$  introduced by measurement repeatability, uncertainty  $u_3$  introduced by measurement reproducibility, and uncertainty  $u_4$  introduced by the influence of the flatness of the measured plane on the plane angle. Below, we will analyze and calculate the quantification of each component.

The CMM dimension measurement indication error can be directly referenced from the CMM traceability certificate. According to the traceability certificate, the MPE is  $\pm (0.4) \mu\text{m} + 1.2 \times 10^{-6} L$ , where  $L$  is the dimension distance value. The distance between the center points of the two planes in this example is 210mm, and the maximum distance from the center points of the two planes to the intersection point is 150mm. Assuming that MPE follows a uniform distribution, the standard uncertainty  $u(\delta_1)$  introduced by the CMM size measurement indication error is

$$u(\delta_1) = \frac{(0.4 + 1.2 \times 10^{-6} \times 210 \times 10^3) \mu\text{m}}{\sqrt{3} \times (150 \times 10^3) \mu\text{m}} \times 206265'' \approx 0.5'' \quad (4)$$

Under repeatability conditions, the linear plane angle of the measured plane is measured 10 times, and the standard deviation  $s$  of the repeated measurement experiment is calculated to be 1.3". The uncertainty  $u(\delta_2)$  introduced by the repeated measurement of the linear plane angle is

$$u(\delta_2) = \frac{1.3''}{\sqrt{3}} \approx 0.8'' \quad (5)$$

Regarding the impact of measurement reproducibility on measurement results, considering that the CMM measurement program operates automatically, the linear plane angle is a small error, and environmental temperature changes have little impact on it, this article sets the reproducibility difference measurement settings as changes in probe diameter, rod length, and measurement force, ignoring the influence of measurement personnel, measurement environment, and other factors. Measure the measured plane three times under each reproducibility condition, and take the average of the three measurements as the measured value under that condition. The measurement reproducibility results are detailed in Table 1.

Table 1. Linear plane angle measurement results under the conditions of reproducibility difference measurement setting

| No. | Repeatability measurement settings<br>(Measuring head diameter/measuring rod<br>length/measuring force) | Measured<br>value |
|-----|---------------------------------------------------------------------------------------------------------|-------------------|
| 1   | 8mm / 114mm / 150mN                                                                                     | 90°00'32"         |
| 2   | 8mm / 114mm / 100mN                                                                                     | 90°00'33"         |
| 3   | 8mm / 100mm / 150mN                                                                                     | 90°00'34"         |
| 4   | 8mm / 100mm / 100mN                                                                                     | 90°00'32"         |
| 5   | 5mm / 75mm / 150mN                                                                                      | 90°00'29"         |
| 6   | 5mm / 75mm / 100mN                                                                                      | 90°00'30"         |
| 7   | 5mm / 135mm / 150mN                                                                                     | 90°00'31"         |
| 8   | 5mm / 135mm / 100mN                                                                                     | 90°00'31"         |
| 9   | 3mm / 58mm / 150mN                                                                                      | 90°00'31"         |
| 10  | 3mm / 58mm / 100mN                                                                                      | 90°00'32"         |
| 11  | 3mm / 130mm / 150mN                                                                                     | 90°00'30"         |
| 12  | 3mm / 130mm / 100mN                                                                                     | 90°00'30"         |

According to the measurement results in Table 1, the uncertainty  $u_3$  introduced by the reproducibility of linear plane angle measurement can be calculated using the Bessel formula as follows

$$u(\delta_3) = 1.4'' \quad (6)$$

The flatness of two planes was measured to be 0.8 $\mu$ m and 1.0  $\mu$ m. The widths of the two planes are 280mm and 300mm respectively. Assuming that the influence of flatness follows a uniform distribution in this area, a B-class method is used for evaluation. There are

$$u(\delta_4) = \sqrt{\left(\frac{0.8\mu\text{m}}{280 \times 10^3 \mu\text{m}}\right)^2 + \left(\frac{1.0\mu\text{m}}{300 \times 10^3 \mu\text{m}}\right)^2} \times 206265'' \approx 0.9'' \quad (7)$$

### 3.3 Combined standard uncertainty

The composite standard uncertainty of linear plane angle measurement based on coordinate measuring machine is calculated according to equation (3) as follows

$$u_c(\alpha) = \sqrt{0.5''^2 + 0.8''^2 + 1.4''^2 + 0.9''^2} \approx 1.9'' \quad (8)$$

### 3.4 Expanded uncertainty

If the inclusion factor  $k$  is 2 is taken, the extended uncertainty of linear plane angle measurement based on coordinate measuring machines is

$$U = ku_c(\alpha) \approx 4'' \quad (9)$$

## 4. Conclusion

According to the measurement method of linear plane angle based on CMM, establish a black box evaluation model for the uncertainty of CMM linear plane angle measurement, and analyze the sources of each uncertainty. Combining measurement examples, this paper elaborates on the uncertainty evaluation process of linear plane angle measurement based on CMM, calculates the expanded uncertainty of linear plane angle measurement based on CMM, and provides reference for the evaluation of uncertainty of linear plane angle measurement based on CMM.

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