# A Study on the Impact of Lamprey's Adaptive Gender Ratio Changes on Ecosystem Stability Based on Logical Growth and Predator-Prey Models

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**Abstract.** This paper aims to assess the ecological ramifications of lampreys' capacity to regulate population sex ratios in response to resource availability, exploring both the advantages and disadvantages associated with this unique ability. Initially, an enhanced logistic population growth model, derived from the Lotka-Volterra model, was developed to examine the broader ecological implications of lamprey influence. This involved integrating a logistic growth model into the traditional Lotka-Volterra framework to simulate the inhibitory impact of intrapopulation competition on population growth. The phase diagram with asymptotically stable fixed points further illustrates the advantage of lampreys using sex ratios to maintain population stability. Notably, distinctions in the survivability and resource-consuming capacity between male and female lampreys were considered by incorporating varying parameter values. The study posits that extreme sex ratios could lead to lamprey population extinction, while a balanced sex ratio correlates with a significant reduction in lamprey prey organisms within a larger ecological context.

**Keywords:** Logistic Growth, Lotka-Volterra model, lampreys.

## 1. Introduction

Many species in nature produce adaptive sex ratio variation in response to environmental changes, and their population sex ratios are influenced by many factors. Some research explores the significance of considering sex as a biological variable in nutrition research. It discusses the importance of understanding how sex differences influence nutritional needs and responses, spanning from human studies to animal models.[1] Invasive species also offer excellent opportunities to study basic processes in population biology. Life history studies and demographic models may be valuable for examining the introduction of invasive species and identifying life history stages where management will be most effective. Evolutionary processes may be key features in determining whether invasive species establish and spread.[2] However, environmental contaminants exert sublethal effects on lamprey. Sea lamprey (Petromyzon marinus in figure 1) and Pacific lamprey (Entosphenus tridentatus) are relatively high in total mercury (Hg) concentration compared with other fishes. Compared with other top predators, lamprey is relatively low in polychlorinated biphenyl (PCB) concentration and concentrations of organochlorine pesticides. We also conclude that regulations on the use of PCBs, beginning in the 1970s, were effective in dramatically reducing PCB concentration in lamprey since the 1970s. Emerging contaminants, such as pharmaceuticals, microplastics, and per- and polyfluorinated alkyl substances (PFAS), have yet to be studied in lamprey.[3] The role of lampreys in ecosystems is complex, and they are often recognized as an organism that has a significant impact on ecosystems. In some remote areas of the world, some indigenous people use the lamprey as one of their main daily food sources. The sex of the juvenile lampreys, which is sexless, is influenced by environmental resources, with juvenile lampreys having a higher probability of becoming males when they grow into adults when environmental resources are low, and vice versa for females . Therefore, the ratio of males to females in lampreys' populations observed under different environmental conditions is not the same.

**Figure 1.**Picture of lamprey

## 2. Improved Logistic Population Growth Model Based on Lotka-Volterra Modeling

Aiming to study the effects of species change on the surrounding ecosystem, this paper proposes model the dynamics of the population density of the lampreys. From the point of view of the population density of lampreys, there are many interactions between organisms in the same ecosystem, such as predation, prey and reciprocal and symbiotic relationships. Before discussing the effects of these complex interactions on the population density of lampreys, this paper proposes the growth-suppressive effects of population density on individual species.

### 2.1. Logistic Growth Model

In an ecosystem where resources are unlimited, the growth of an organism's population conforms to the following pattern:[4]

$$x'_{n+1} = Rx_n \tag{1}$$

where $x'_{n+1}$ is the number of individuals in the next generation of the population, $x_n$is the current number of individuals in the population, and $R$ is the relative growth rate of the population. Through equation 1, it can be inferred that the growth of a species will eventually take the form of exponential explosion. However, this does not align with reality. In actual ecosystems, resources are limited, and the maximum number of individuals that a single species can sustain is also finite. As the population of a species increases, its growth rate will slow down. This implies that larger population sizes will correspond to fewer natural resources and a smaller rate of population density growth. this paper refers to this phenomenon as density dependence in population growth. The phenomenon where the growth of individual species conforms to these patterns is known as Logistic growth.

Considering the simplest monotonic increasing linear model, which takes the form of $r\left(1-\frac{x_n}{k}\right)$, this paper has refined equation 1 to conform to the growth pattern of Logistic growth. Thus, this paper obtains the fundamental Logistic growth equation:

$$\dot{x}_{n+1} = Rx_n\left(1-\frac{x_n}{K}\right) \tag{2}$$

In equation 2, $K$ represents the maximum population size that the environment can support for the species. When the population size $X$ equals $K$ or $X = 0, \dot{x} = 0$, indicating the species ceases to grow. If $X$ exceeds $K$, the species will experience negative growth.

### 2.2. Prey-Predator Biological Evolution Model

In an ecosystem, a species often plays the roles of both predator and prey, and the lampreys is no exception. For predators, their population size primarily depends on the abundance of their prey. Conversely, for prey, their population size is often influenced by both environmental resources and the number of predators. Based on this relationship, this paper derives a system of differential equations for the Prey-Predator Model [5]:

$$\begin{cases} \frac{dU}{dt} = \alpha U - \gamma UV \\ \frac{dV}{dt} = -\beta V + e\gamma UV \end{cases} \tag{3}$$

In equation 3, $U$ represents the population size of the prey, while $V$ represents the population size of the predator. $\alpha$ denotes the prey's natural growth rate, and $\gamma$ represents the proportion of prey captured by predators per unit time. The left side of the equation describes the inherent changes in each population within their natural environment, while the second term depicts changes caused by predation, where $\beta$ stands for the predator's natural death rate, and $e$ represents the proportion of newborn juvenile predators after predation, known as the predation conversion rate. Equation 3 constitutes the classical Lotka-Volterra equations.

Equation 3 elucidates the dynamic relationship between prey and predator populations. However, in fact, species often play dual roles as both predator and prey. Therefore, it is necessary to formulate separate Lotka-Volterra equations for a species acting as both predator and prey, integrating them into a cohesive whole.

Through research, this paper has discovered that the lamprey has few natural predators in the wild, with human predation being its primary cause of mortality. As humans are omnivores at the top of the food chain, their food sources are diverse. Although the lamprey is a significant food source for indigenous populations, its population fluctuations have minimal impact on human population dynamics due to alternative food sources. While the relationship between the lampreys and humans fits the predator-prey dynamic, their population ratios do not conform to the Lotka-Volterra equations. Hence, in our model, this paper only considers the population dynamics of the lampreys when it acts as a predator.

Conventional Lotka-Volterra equations solely consider the interaction between prey and predator. This paper incorporates Logistic growth into the Lotka-Volterra equations to introduce a population density-dependent effect into the prey-predator biological evolution model. It is expressed as

$$\begin{cases} \frac{dU}{\mathrm{d}t} = \alpha U\left(1 - \frac{U}{U_{\max}}\right) - \gamma UV \\ \frac{dV}{dt} = -\beta V + e\gamma UV \end{cases} \tag{4}$$

In equation 4, the model incorporates Logistic growth, effectively restraining the growth rate and population size of the prey, rendering the entire model more realistic.

## 2.3. Prey-Predator Biological Evolution Model Considering Sexual Relationships

For the lampreys, due to its ability to alter the ratio of male to female individuals in the population, it is essential to consider both sexes and the influence of juvenile growth into adult eels on the population dynamics. Therefore, this paper proposes a Prey-Predator model that accounts for sexual relationships.

$$\begin{cases} \frac{dU}{dt} = \alpha_u U\left(1 - \frac{U}{U_{\max}}\right) - \gamma U(M+F) \\ \frac{dM}{dt} = \alpha_m M - \beta_m M + e\gamma U(M+F)Z \\ \frac{dF}{dt} = \alpha_f F - \beta_f F + e\gamma U(M+F)(1-Z) \end{cases} \tag{5}$$

In the model, $e\gamma U(M+F)$ represents the efficiency of converting prey into the next generation of individuals. By multiplying this term by the gender conversion ratio $Z$, this paper obtains the number of new male individuals added to the population after juvenile eels grow into adults. Similarly, by multiplying this term by $1-Z$, this paper obtains the number of new female individuals added to the population after juvenile eels grow into adults. If the population of either male or female lampreys decreases, the number of newly born individuals will correspondingly decrease. Additionally, the model introduces two distinct parameters, $\beta_m$ and $\beta_f$, to characterize the different survival abilities of male and female lampreys in their natural environment [6-8]. Figure 2 is the schematic of the model.

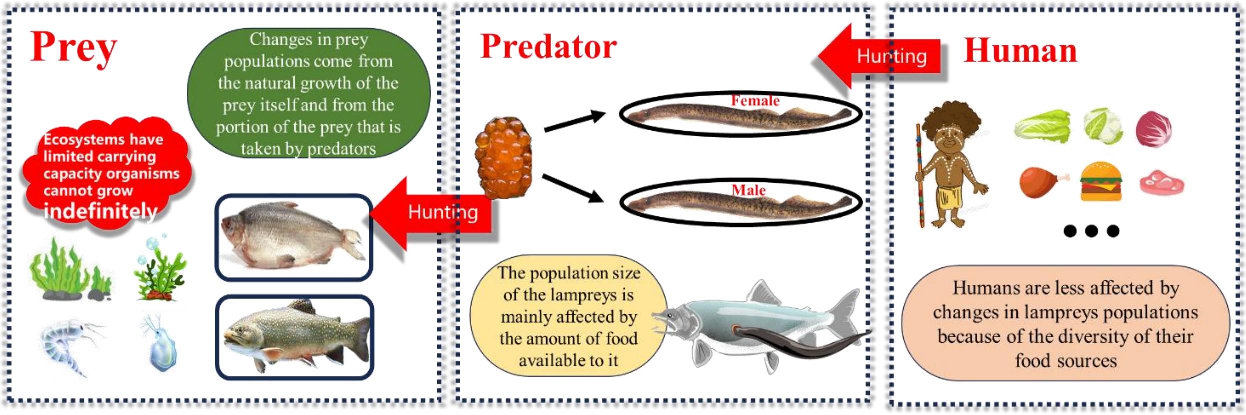


**Figure 2.**Schematic of the model

## 3. Results

It is important to note that in the Prey-Predator biological evolution model considering sexual relationships, there are many uncertain parameters. This paper requires a significant amount of prior information to help determine the parameter values or narrow down their ranges. By obtaining frequency and temporal information about the occurrence of lampreys from the Global Biodiversity Information Facility(https://www.gbif.org/), and some related data from The Mathematical Contest in Modeling, this paper fit the model using the least squares method to obtain the parameter set with the minimum loss for this model. Subsequently, this paper analyzes the results using this parameter set and draw conclusions. The optimal parameter results obtained from the fitting process are illustrated in the figure 3 below:

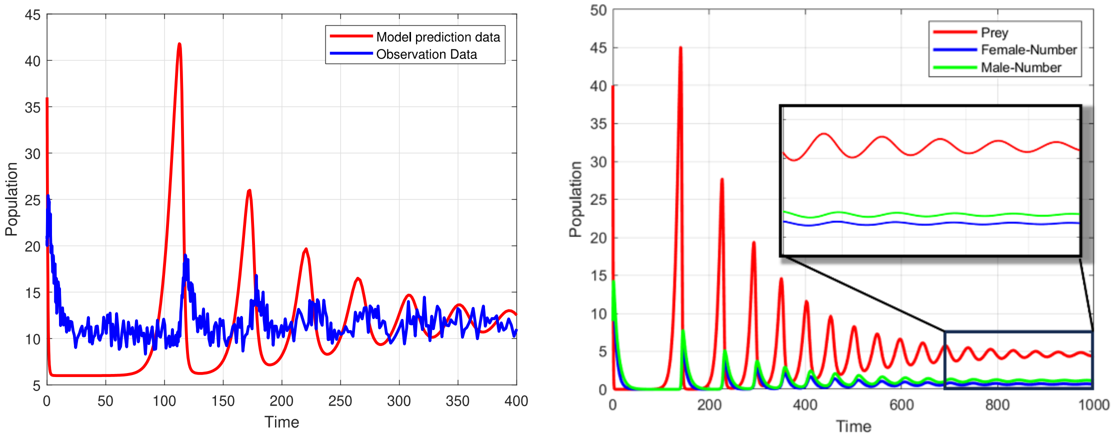


**Figure 3.** Changes in the population size of the lamprey as a function of time (the left panel shows the results of the model fit, and the right panel shows the results of the model prediction)

Using the fitted optimal parameters, this paper can make preliminary predictions about the frequency and temporal information regarding the occurrence of lampreys in the middle Volga and upper Don regions of Russia. From figure 3, this paper can observe the relationship between the population size of lampreys and time. In this model, this paper set $Z = 0.5$, and from the final result, it can be observed that the ratio of male to female lampreys approaches 1. The model results indicate that, in the absence of significant and sudden changes in the external environment, the population sizes of lampreys and the organisms they prey on will eventually stabilize and undergo periodic fluctuations over time. This paper proposes such results acceptable and reasonable [9].

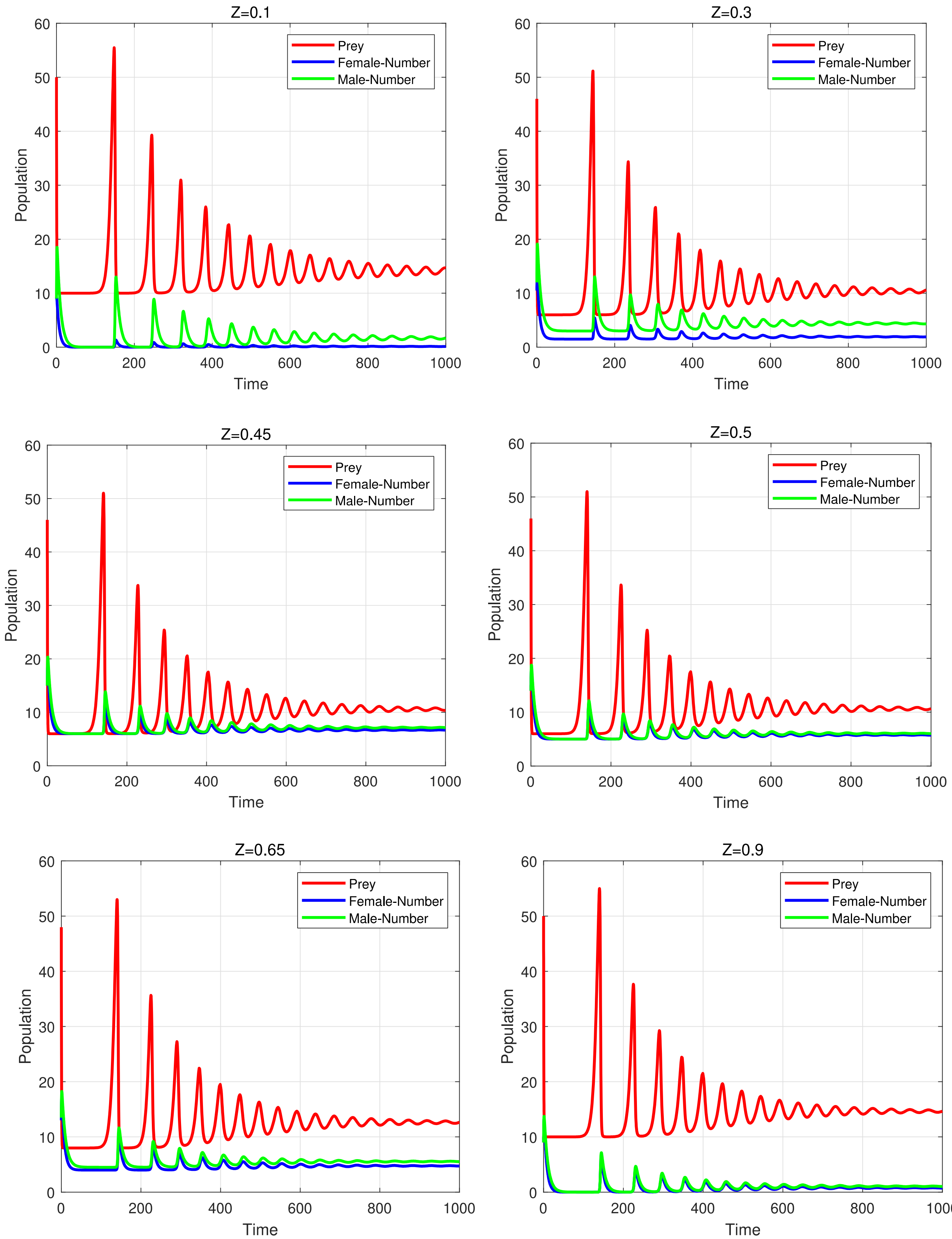


**Figure 4.** Schematic diagram of the variation of the number of lampreys with time for different Z-parameters

From Figure 4, it draws the following conclusions: When $Z = 0.1$ and $Z = 0.9$, the entire population of lampreys stabilizes and eventually tends toward 0 over time, indicating that extreme gender ratios will lead the entire population of lampreys towards extinction. When $Z = 0.3$ or $Z = 0.65$, the population of lampreys will maintain at a lower level because an unreasonable male-to-female ratio will significantly reduce the reproductive capacity of the entire population. When $Z = 0.45$ or $Z = 0.5$, both the population of lampreys and the organisms they prey on will maintain at a relatively normal level. However, regardless of the value of $Z$ in the model, the population of lampreys will eventually stabilize at a certain value after the ecosystem stabilizes, and the population will undergo periodic fluctuations over time. Clearly, the reason for such periodic fluctuations is the periodic changes in prey populations. The populations of prey and predators always follow a certain rule: when the prey population increases, the predator population also increases, with a lag effect. After the predator population increases, the predation of prey leads to a brief decrease in prey population. Subsequently, the predator population decreases due to the decrease in prey population.

It is also worth noting that when the gender ratio of lampreys is in an extreme state, the decrease in the population of lampreys will significantly increase the number of organisms in a larger ecological environment captured by them (figure 3) [10]. When the gender ratio of lampreys is in equilibrium, the increase in the population of lampreys will lead to a decrease in the number of organisms in a larger ecological environment captured by them. In summary, the overall change in the population of lampreys caused by changes in the male-to-female ratio affects the population of organisms interacting with them (primarily in the context of predation and prey relationships).
According to the fixed-point theory, under a period T, we can determine the fixed points of the Nicholson-Bailey model as follows:

$$\bar{U} \quad = \lim_{T\to\infty} \frac{1}{T}\int_0^T \ U(t)dt \tag{6}$$

$$\bar{M} + \bar{F} \quad = \lim_{T\to\infty} \frac{1}{T}\int_0^T \ \big(M(t) + F(t)\big)dt \tag{7}$$

In the phase diagram (figure 5), within each different period, the densities of prey and predator exhibit periodic oscillations. The amplitude and frequency depend on the parameters we fitted. They can evolve to stable points from any arbitrary initial values. However, the average density remains constant, and the values of the stationary point are $(\bar{U}, \text{bar } F) = (3.559, 1.800)$.According to the graphical representation of derivatives and the eigenvalues of the Jacobian matrix, the real parts of the eigenvalues of the Jacobian matrix are all less than 0. We can observe that this stationary point is asymptotically stable. Asymptotic stability is much stronger than stability. A small disturbance to the stationary point can be immediately counteracted by the dynamics, as the solution tends back to the stationary point. This kind of motion is somewhat akin to a spring returning to equilibrium. Conversely, in fact, a series of disturbances may transition the state from one periodic orbit to another, corresponding to the extinction of a population. Additionally, minor changes in the vector field only slightly alter the location of the stationary point, fundamentally without changing the nature of the orbit. The orbit still revolves towards the stationary point, further demonstrating the stability of the population.

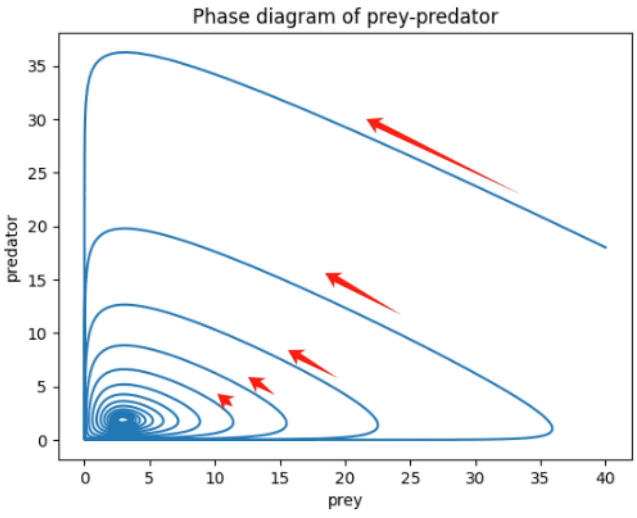


**Figure 5:**Phase diagram of prey-predator

## 4. Conclusions

This paper explores the ecological ramifications of lampreys' capacity to regulate population sex ratios in response to resource availability. It introduces an enhanced logistic population growth model, derived from the Lotka-Volterra model, to examine the broader ecological implications of lamprey influence. The phase diagram with asymptotically stable fixed points further illustrates the advantage of lampreys using sex ratios to maintain population stability. The study considers the survivability and resource-consuming capacity differences between male and female lampreys and posits that extreme sex ratios could lead to lamprey population extinction. Additionally, it discusses the prey-predator biological evolution model and its integration with logistic growth to introduce a population density-dependent effect into the model. The results section presents the model's predictions regarding the population size of lampreys and the organisms they prey on, based on different gender ratios. It concludes that extreme gender ratios will lead the entire population of lampreys towards extinction, while a balanced gender ratio will maintain the population at a relatively normal level. The study also notes that changes in the male-to-female ratio affect the population of organisms interacting with the lampreys, primarily in the context of predation and prey relationships. Overall, this paper proposes insights into the ecological ramifications of lampreys' population sex ratios and the implications for the broader ecosystem.

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