

Study On the Impact of Sea Lamprey Sex Ratio on Ecosystem Stability Based on The Lotka-Volterra Model

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Abstract. Sex ratio adjustment has a profound impact on the stability of the sea lamprey and its ecosystem, altering its reproduction and host relationship dynamics. To understand the complex relationship between sex ratio and population dynamics, and to reveal how changes in sex ratio affect reproduction, the present study was carried out to improve the model by establishing the basic Lotka-Volterra predator-prey model and improving the model by combining the characteristics of changes in sex ratio to obtain the results that the sex ratio of the sea lamprey is an important factor influencing the changes in the population size of the sea lamprey, and that the relationship between them shows significant differences in the different sex ratios. The relationship between the sex ratio and the population size of sea lampreys showed obvious differences, reflecting that the sex ratio not only determines the rate of increase and decrease of the population size but also indirectly affects the trend of the host population size.

Keywords: Sex Ratio, Sea Lampreys, Lotka-Volterra, Population Size, Ecosystem Stability.

1. Introduction

Sex ratio, as an important component of biological population structure, has far-reaching effects on population reproduction, survival, and the stability of the whole ecosystem^[1]. In the marine ecosystem, the sea lamprey, as a unique parasitic organism, changes in its sex ratio have complex effects on population dynamics and host relationships, which in turn affect the stability of the entire ecosystem^[2]. Therefore, studying the effects of sex ratio adjustment on the stability of the sea lamprey and its ecosystem not only contributes to an in-depth understanding of the ecological characteristics of this biological population but also is of great significance for maintaining the balance and sustainable development of the marine ecosystem.

This paper uses the Lotka-Volterra model, which is an important ecological model in ecosystems and mathematical ecology. In the 1920s, American mathematician Lotka and Italian mathematician Volterra proposed the Lotka-Volterra model in the study of chemical reactions and the explanation of the pattern of change in the fish population^[3], respectively, which promoted the development of mathematical ecology to reach a climax. The Lotka-Volterra model can describe the dynamic relationship between predators and prey and predict the trend of their population size over time. Han Baorui applied the improved Lotka-Volterra model to study the competitive relationship between rail transit and conventional public transportation and its development and evolution^[4], which can be also used to study the predator-prey interaction in the ecosystem in the natural world and provide a theoretical basis for the protection of biodiversity and ecological balance.

All data in this paper is based on the official website of the U.S. Mathematical Modeling Competition 2024 and the official website of the Great Lakes Department of Environmental Protection.

2. Constructing the Lotka-Volterra Predator-Prey Model

Clarification: Adult sea lampreys live primarily as parasites, with common hosts such as trout, smallmouth bass, and whitefish^[5]. Our prey-related metrics here are a combination of all the parasitic hosts of sea lampreys.

2.1. Establishment of Lotka-Volterra Equation

Predator(lampreys)

$$\frac{d(M + F)}{dt} = r(M + F) - d(M + F) + baP(M + F) \quad (1)$$

Prey(host)

$$\frac{d(P)}{dt} = r_p(P) - aP(M + F) \quad (2)$$

2.2. Parameter Setting

We used simulation experiments, assuming a closed environment, and initially set the population size of the seven-gill eel as 50, and the host population size as 300. After several tests of the stability of the simulation model and comparisons with the actual situation, the following parameters were obtained in Table 1.

Table 1. Model parameters in the ideal state

Symbol	Description	value
r	Reproduction rate of sea lamprey	0.4
d	Natural mortality rate of sea lamprey	0.5
b	Predation efficiency of sea lamprey	0.1
r_p	The natural growth rate of prey	0.5
a	Probability of prey predation	0.01

2.3. Model Result

The Lotka-Volterra equation is solved by Python, applying Euler's method. The model yields the following results.

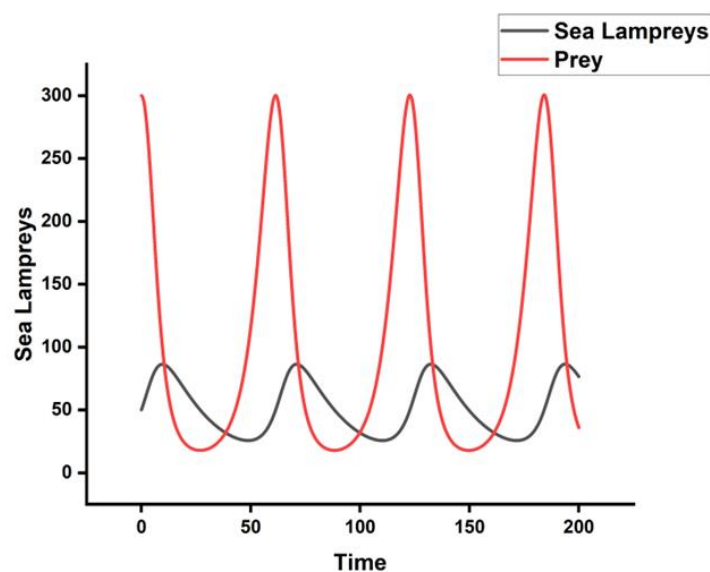


Figure 1. Trends in sea lampreys and prey population size over time

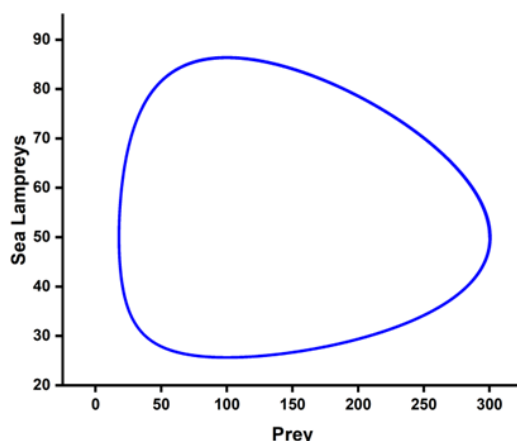


Figure 2. Phase diagram of the Lotka-Volterra equation for the sea lampreys and their prey

From Figures 1 and 2, it can be seen that the sea lamprey's population rises while the prey population declines over time, and vice versa the sea lamprey's population declines while the prey population rises cyclically. Under the idealized model, the long-term behavior of the system is that the sea lampreys, as the highest-level consumer, can suppress its growth rate while remaining indefinitely maintained at a non-zero level with its prey. Analysis of their phase diagrams shows that the efficiency of entry stabilization is sufficiently high, indicating that our parameter choices are more reasonable and consistent with the model.

3. Enhanced Model Incorporating Sex Ratio Dynamics

3.1. Causes of Changes in the Sex Ratio of Sea Lampreys

Studies have shown that the larger the body of female lampreys, the more eggs she produces [6]. Therefore, the number of offspring a female can leave behind will increase as her body grows. Under conditions of insufficient food resources, the growth rate of lampreys is slow, and females may not fully develop, which is not conducive to offspring reproduction. As a result, the proportion of males in the lamprey population becomes predominant.

Lampreys hatch in freshwater rivers or streams [7] and primarily feed on plankton during their larval stage [8]. The growth rate of the larvae varies depending on the availability of food resources in the freshwater environment [9], which, in turn, determines their sex. Consequently, different growth rates lead to varying sex ratios among adult lampreys in the ocean. The whole process is shown in Figure 3.

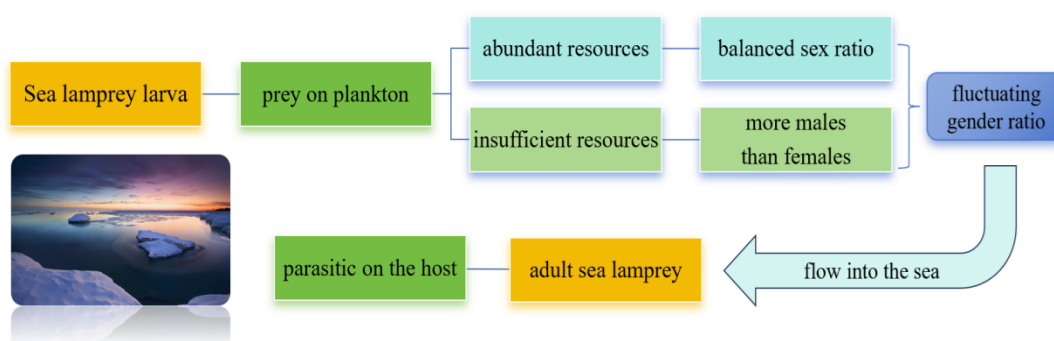


Figure 3. The transformation of sea lamprey from larval to adult stage

3.2. Establishment of Improved Lotka-Volterra Equation

The female and male sea lamprey populations are discussed separately here, and the environmental maximum carrying capacity K and Q of this lamprey and its host in a closed environment are considered. The Lotka-Volterra equations for female and male sea lampreys and prey are developed separately.

Male sea lampreys

$$\frac{dM}{dt} = (r_M M - d_M M) \left(1 - \frac{M + F}{K}\right) + b_M a P M \quad (3)$$

Female sea lampreys

$$\frac{dF}{dt} = (r_F F - d_F F) \left(1 - \frac{M + F}{K}\right) + b_F a P F \quad (4)$$

Prey(hosts)

$$\frac{dP}{dt} = r_P P \left(1 - \frac{P}{Q}\right) - a P (M + F) \quad (5)$$

In which the sex ratio is varied by changing the intrinsic birth rate of male and female sea lampreys. Finally, the intrinsic birth rate function of male and female sea lampreys according to the male sex ratio is obtained.

Male sea lampreys

$$r_M = r_0 (1 + g(S_m - 0.5)) \quad (6)$$

Female sea lampreys

$$r_F = r_0 (1 - g(S_m - 0.5)) \quad (7)$$

3.3. Parameter Setting

Because female and male sea lampreys are introduced as predators, the simulated closed environment was expanded here for subsequent images to be more intuitively clear. In the initial case, the population size of male sea lampreys is 200, females are also 200, and the population size of hosts is 1000. Ideally, the relevant parameters are set as in Table 2.

Table 2. Model parameters in the ideal state

Symbol	Description	value
r_0	Initial intrinsic birth rate	0.4
S_m	The proportion of male sea lampreys in the population	0.5,0.6,0.7, 0.8,0.9
d_m	Natural mortality rate of male sea lampreys	0.45
b_m	Predation efficiency of male sea lampreys	0.025
d_f	Natural mortality rate of female sea lampreys	0.45
b_f	Predation efficiency of sea lampreys	0.025
r_p	The natural growth rate of prey	0.3
a	Probability of prey predation	0.01

3.4. Model Result

The sex ratio of males and females exported from juveniles to adults is controlled by varying the intrinsic birth rates of females and males. Results are obtained for male sex ratios at 0.5 (equal male-to-female ratio), 0.6, 0.7, 0.8, and 0.9, respectively.

a) Population size change curves of the sea lampreys and their hosts

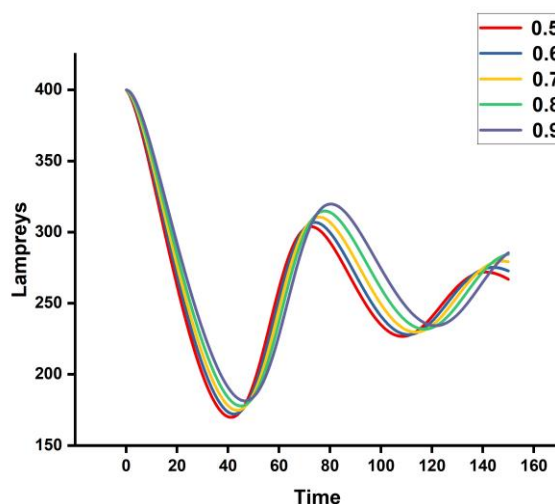


Figure 4. Trends in sea lampreys' population size over time

As can be seen from the sea lamprey's population size change curve in Figure 4, the population size of the sea lampreys rises and falls alternately under different male ratios, and the rise and fall show a clear cyclic pattern. Interestingly, this cyclic pattern is closely related to the male ratio. Specifically, when the proportion of males is higher, the population size of the lampreys declines more slowly, but at the same time, the population size increases more rapidly. In addition, as the proportion of males rises, the peak of the population size of the sea lamprey becomes larger. This phenomenon may be related to the importance of males in the sea lamprey population. For sea lampreys, which are monogamous aquatic organisms^[10], the higher the proportion of males during the breeding period, the faster the reproduction rate and the faster the population size will rise.

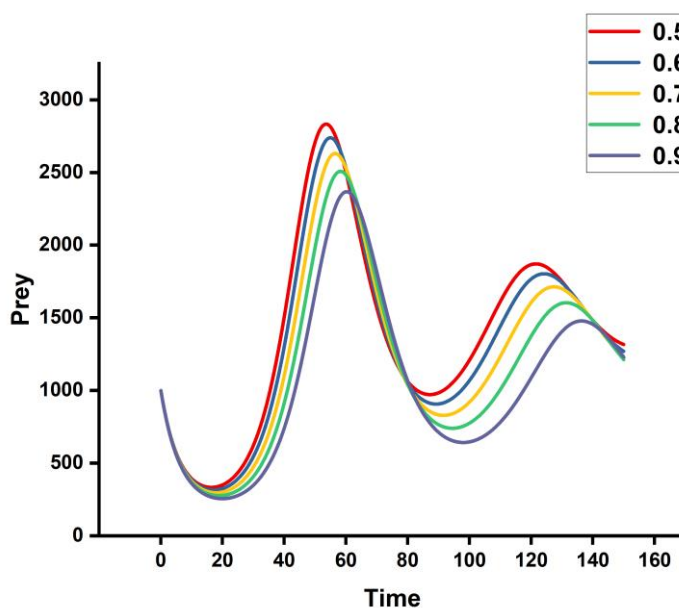


Figure 5. Trends in sea lampreys' population size over time

As shown in Figure 5, changes in the size of the host population of the sea lampreys, as time progresses, the higher the proportion of males in this model, the smaller the population size of their hosts, and the fluctuation in the population level of the hosts becomes smaller and smaller with the development of time, and ultimately shows a small range of upward and downward fluctuations between a certain value. This is also similar to the variation in population size of the sea lamprey itself. Under this model, the effect of changes in the sex ratio of the sea lampreys, on changes in host numbers is obvious, with the maximum value of the difference in population size between a curve with a male ratio of 0.5 and a curve with a male ratio of 0.9 approaching 1500 at the same point in time, which is equivalent to a doubling of the difference.

b) Change curves of male and female population size of the sea lampreys.

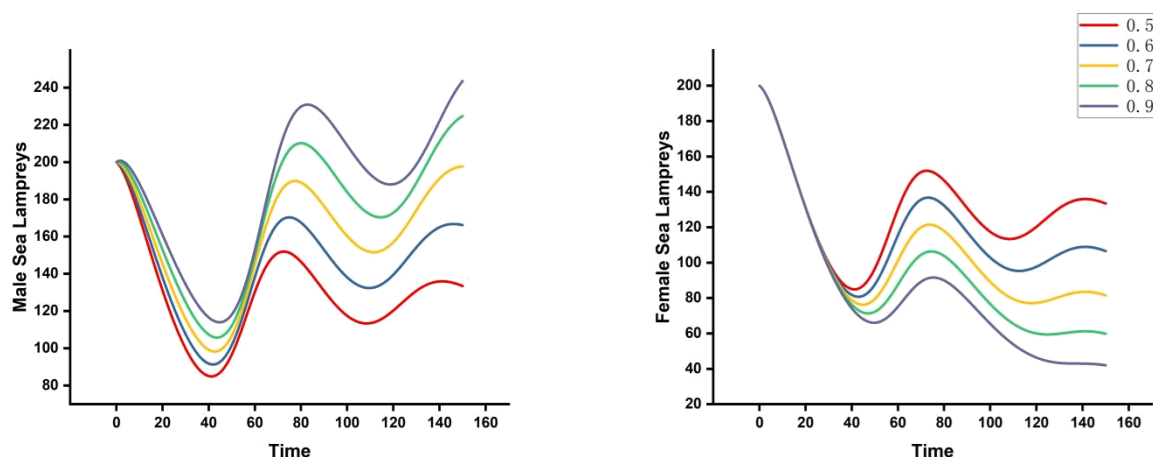


Figure 6. Trends in male and female sea lampreys' population size over time

Male and female populations of sea lampreys show a broadly consistent pattern of change over time.

c) Three-dimensional plot of the number of males, females, and host population

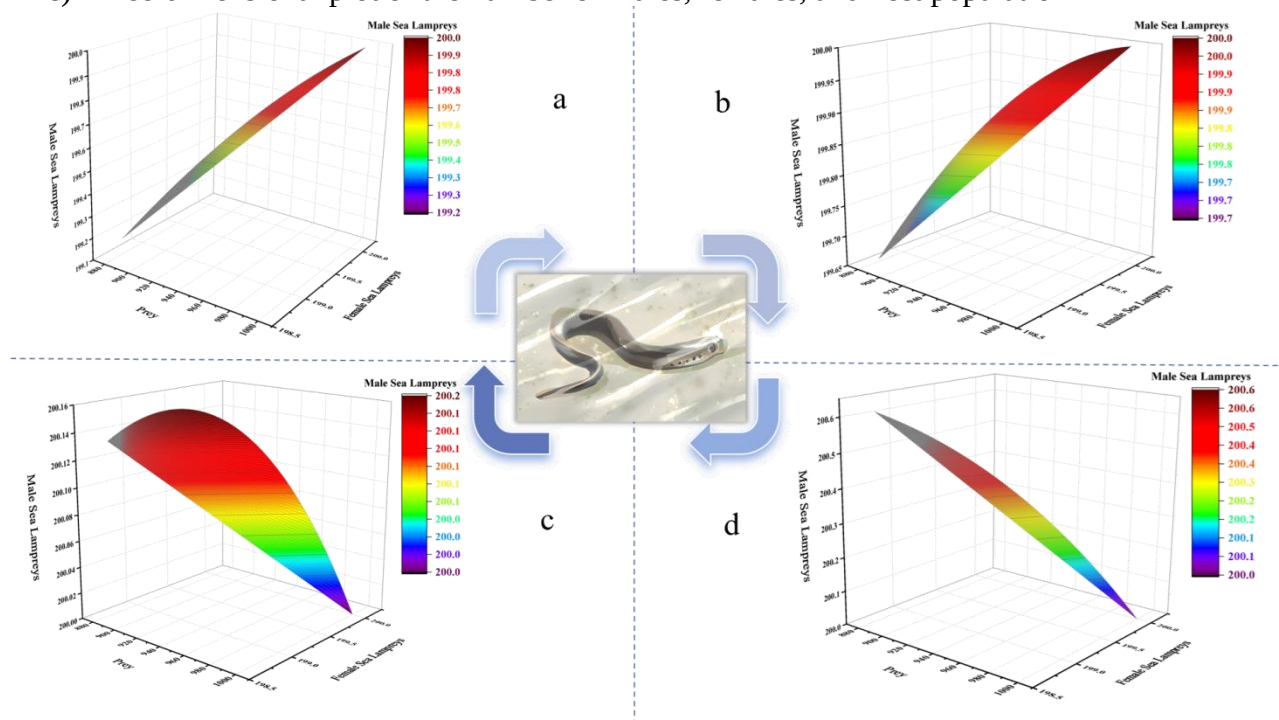


Figure 7. Three-dimensional plot of the number of populations:

a. Gender ratio of 0.6; b. Gender ratio of 0.7; c. Gender ratio of 0.8; d. Gender ratio of 0.9

As shown in Figure 6 and Figure 7, when sex ratios are different, the graph of male, female, and host population size relationships shows a prominent monosurface shape, resembling a leaf. It shows a continuous surface in which changes in the number of males, females, and host populations are correlated with each other. There is a clear difference in the shape of the relational surface for the four sex ratios. This difference indicates that the sex ratio has a significant effect on the relationship between the number of males, females, and host populations.

4. Conclusions

In this study, the Lotka-Volterra predator-prey model was established to investigate that the population size change of the sea lampreys under the ideal state showed a cyclic pattern of rising and falling, then, combined with the changing characteristics of the sex ratio, the improved model was established and the parameters were estimated by simulation experiments, and the population size pattern of the sea lampreys was closely related to the level of the proportion of males. From the point of view of male, female, and host population number relationship, the number relationship of the three under different sex ratios showed obvious changes. The sex ratio of the sea lampreys not only determines the rate of increase and decrease of the population size but also influences the trend of its host population size. These findings are important for understanding the ecological characteristics and ecosystem stability of the altered sex ratio of the sea lampreys.

The results of this study provide new ideas and frameworks for the field of ecological assessment. By considering the effect of sex ratio on population dynamics, we can better understand and predict the evolution and fluctuation of species populations. This has practical application value for ecosystem management and conservation and helps to develop effective conservation strategies and management measures.

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