Sex Ratio of Lampreys Based on Logistic and Lotka-Volterra Model

Yiqiao Wang*, Boyu Xiao and Xiangrui Zhi

School of Haide, Ocean University of China, Qingdao, China

*Corresponding author: 3253462071@qq.com

Abstract. In this paper, we investigated the ecosystem impacts of changes in the sex ratio of the lampreys by means of a logistic population model and a Lotka-Volterra predator model. The results showed that the sex ratio of lampreys was closely related to predator scarcity, and that self-regulation of sex ratio promoted population growth but had a time lag with the number of predators. The Lotka-Volterra model was further improved to verify the ecosystem stability of the sea lamprey under different sex ratios. The study suggests that regulating sex ratios maintains ecosystem stability and provides insights into ecosystem interactions.

Keywords: Logistic; Lotka-Volterra; Jacobian determinant discrimination.

1. Introduction

Sex plays an important role in nature, however, some species such as the lampreys exhibit adaptive sex ratio changes that are critical to ecosystem impacts [1]. The aim of this paper is to explore the effects of lamprey's sex ratio changes on ecosystem interactions and their advantages and disadvantages. Through logistic population modeling as well as Lotka-Volterra predator modeling, we seek to understand how changes in sea lamprey sex ratio affect population dynamics and ecosystem stability. We revealed the effect of sex ratio on population growth through logistic population modeling. Further, we used the Lotka-Volterra model to simulate the effects of sex ratio changes on sea lamprey species and found that sex ratio is directly related to predator scarcity. By improving the model and examining Jacobi determinants under different sex ratios, we conclude that changes in sex ratio do not destabilize the ecosystem. These methods will contribute to an in-depth understanding of the impacts of sea lampreys and their changing sex ratios on ecosystems and provide important references for biodiversity conservation and ecosystem management.

2. Population Change Model

2.1. Logistic Model

The population change curve is a continuous curve that changes over time. When solving this curve, many related factors of population changes should be taken into account, and this is clearly a complex and ever-changing situation. Considering that population growth is mainly constrained by the total population, growth rate, maximum environmental carrying capacity, etc. in a certain environment, the differential equation of population growth relative to time, namely the logistic model, can be considered [2].

$$\frac{dM}{dt} = b_1 r M\left(1 - \frac{M}{M_{\max}}\right) - m_1 M \tag{1}$$

$$\frac{dF}{dt} = b_2 r F\left(1 - \frac{F}{F_{\max}}\right) - m_2 F \tag{2}$$

Where M stands for the population of male, F stands for the population of female, r is sex ratio; b_1, b_2 are the birth rate for male and female, respectively. $M_{\max}, F_{\max}$ are the maximum population that the environment could afford for male and female, respectively. m_1, m_2 are the morality rate for male and female, respectively.

Hansen (2016) described how adult lampreys in the two tributaries of Lake Ontario have a natural mortality rate of between 6 and 30 percent [3]. In this model, for natural mortality rate is set to 6% to improve the simulation. Chipman (2013) believed that the female bias in human beings is related to the higher birth rate in the poorest areas, and drew an analogy to lampreys, which assumes that the proportion of women directly affects birth rates [4]. The simulation by constant selection of parameters found that when 10%-20% is a reasonable interval, so we set the growth rate as $b_1 = b_2 = 0.5r$, where r is a variable that changes over time and is the proportion of females in the total population.

Thus, to simplify the problem, consider b_1, b_2 and m_1, m_2 in the equation as the same and take them as fixed values. $M_{\max}, F_{\max}$ are considered as constant as well.

Programming in MATLAB can provide a more intuitive understanding of the fitting effect of the model. Firstly, some parameters were defined, including the simulated time range and time interval. Determine the initial value of sex ratio r by initializing the initial values of M and F. Next, use a loop to simulate the passage of time, starting from the second time step $(i = 2)$, and calculate the rate of change in the number of males and females in each time step. Subsequently, the Euler method is used to update the population of males (M), population of females (F), and sex ratio (r), multiplying the current rate of change by the time step (dt) and adding the result to the number of previous time steps.

Therefore, by taking the population change rates of male and female sea lampreys over time under different sex ratios and placing curves with different sex ratios in one figure can provide a more intuitive observation of some of their properties, as shown in Fig. 1.

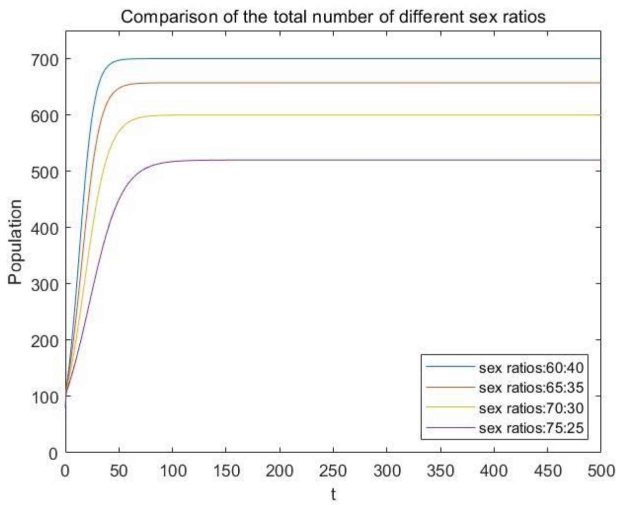


Fig. 1 Population change rate under different sex ratios

From the figure, it can be observed that as the sex ratio increases, i.e., the proportion of male sea lampreys increases, the total population actually decreases. This indicates that in populations with a higher proportion of males, there are not enough females to mate and reproduce offspring. The sex ratio will affect population reproduction, and groups with a higher proportion of males have a smaller total population.

2.2. Lotka-Volterra Predator Model

In the food chain where the sea lamprey is located, considering its identity as a predator and prey, the Lotka Volterra equation can be considered. It can provide information on how changes in the population size of predators and prey depend on the number and interaction of both parties. Considering different genders of sea lampreys, the equation for different populations over time can be listed as follows. By solving the equation, the population changes of different roles in the food chain can be obtained.

$$\frac{dM}{dt} = b_1 R_0 F - m_1 M - \alpha_1 M P \tag{3}$$

$$\frac{dF}{dt} = b_2 R_0 F - m_2 F - \alpha_2 F P \tag{4}$$

$$\frac{dP}{dt} = -m_3 P + \alpha_3 (F + M) P \tag{5}$$

Where M stands for male, F stands for magnetic, P stands for predator. R_0 is resource availability. b_1, b_2 are the birth rate of male and female, respectively. m_1, m_2, m_3 are the morality rate of male, female lamprey and predator, respectively. α_1, α_2 can be regarded as the proportion of male and female prey captured by predators per unit time. α_3 is a conversion relationship between the number of prey captured and the number of newborn predators.

Similarly, consider m_1, m_2 and α_1, α_2 in the equation as the same and take them as fixed values. m_3, α_3 are considered as constant. Thus consider $m_1 = m_2 = 0.06, \alpha_1 = \alpha_2 = 0.08$, and α_3 =0.02, resource availability R_0 =0.5.

Specifically, for b_1, b_2, they satisfy the following equations:

$$r_{\min} = \frac{(1+b_1 R_{\max} - (m_1+\alpha_1))F}{(1+b_1 R_{\max} - (m_1+\alpha_1))F + (1+b_2 R_{\max} - (m_1+\alpha_1))M} \tag{6}$$

$$r_{\max} = \frac{(1+b_1 R_{\min} - (m_2+\alpha_2))F}{(1+b_1 R_{\min} - (m_2+\alpha_2))F + (1+b_2 R_{\max} - (m_1+\alpha_1))M} \tag{7}$$

Where $R_{\max} = 0.8, R_{\min} = 0.2$, they stand for the resource availability under different conditions, which means that the larger the value, the richer the resources. $r_{\max} = 0.78, r_{\min} = 0.56$, they stand for the male proportions under different conditions.

The numerator of these two equations represents the increase in the male population per unit time, while the denominator represents the increase in the total population per unit time, that is, the increase in the male population plus the increase in the female population. Consider the probability of male and female births is equal, as the values of F and M are also equal per unit time.

Therefore, for the value of b_1, b_2, it can be transformed into a binary linear equation problem, and the solution is $b_1 = 0.396, b_2 = 0.327$.

By using MATLAB to solve the equation system, the predator, male, and female sea lampreys can be plotted in a single figure, and the changes in their respective populations can be obtained by taking values of different sex ratios, as shown in the following figures (Fig. 2-5). By observing the figures, certain conclusions can be drawn.

It can be observed that as the sex ratio increases, the proportion of male sea lampreys increases, the time step required for male, female sea lampreys and predator populations also increase, thus the cycle of their cycles becomes longer. This indicates that the cycle of regulating the food chain becomes longer and the stability decreases. This means that when the sex ratio is relatively balanced, people with lower gender ratios are more stable. Sex ratio affects the balance of the food chain.

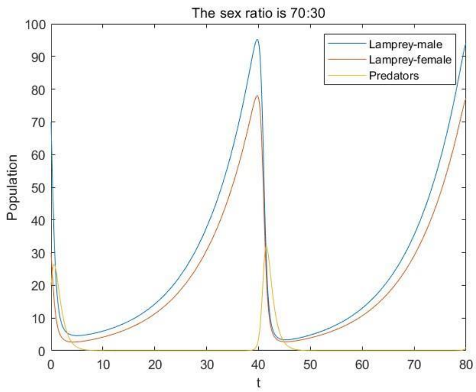


Fig. 2 Sex ratio of 70:30

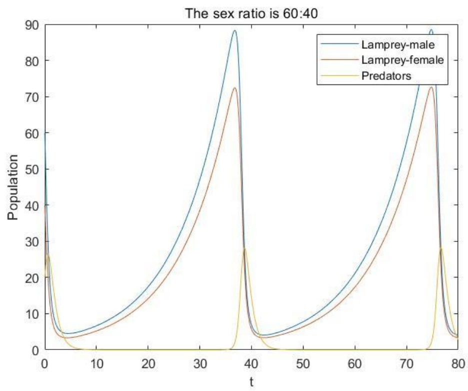


Fig. 3 Sex ratio of 60:40

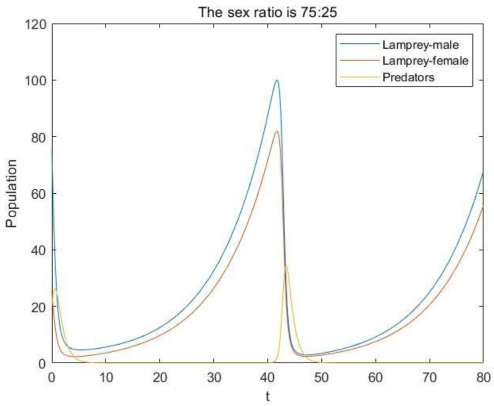


Fig. 4 Sex ratio of 75:25

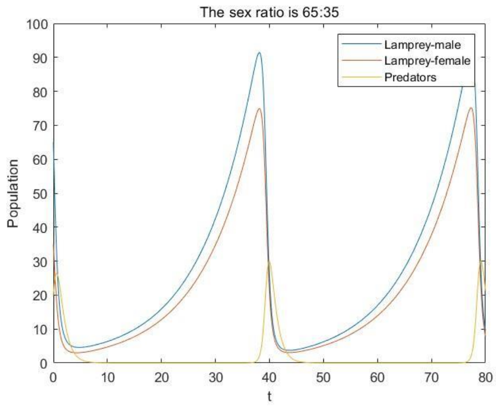


Fig. 5 Sex ratio of 65:35

2.3. Impacts of Sex Ratio on Sea Lampreys

In this problem, reference will be made to the predator model in Section 2.2 (equations (3)-(5)). By adjusting the initial sex ratio (from 56%-78% male share), the simulation of the population size for different sex ratios over the same time period will be simulated by making the overall initial number of sea lamprey constant. The population-sex ratio can be plotted over time using MATLAB as shown in the figure below (Fig. 6).

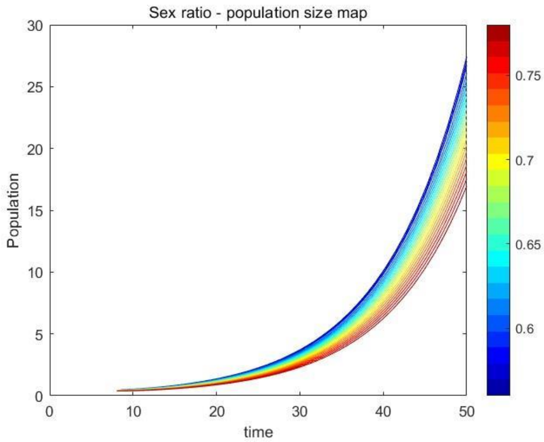


Fig. 6 Population changes over different sex ratios

The color bar on the right side of the graph is the sex ratio, with darker colors indicating a higher proportion of males. It can be intuitively observed that in the predator model, the darker the color in the image, the slower the population grows in areas with a higher proportion of males. That is, the higher the proportion of males at the beginning, the slower the population growth over time.

It also makes common sense that populations with a more balanced ratio of males to females are able to have more pairs of males mating with females in order to reproduce, leading to faster population growth. Therefore, changing the sex ratio can affect the level of reproduction of sea lampreys, which in turn affects population growth.

3. Algae-Sea Lamprey Model

The Lotka-Volterra equation for species with predator-prey relationships is obtained as follows [5].

$$\frac{dA}{dt} = \beta_1 A - \gamma_1 AL = f_1 \tag{8}$$

$$\frac{dL}{dt} = \delta_1 AL - \epsilon_1 L = g_1 \tag{9}$$

Where A stands for algae population, L stands for sea lamprey population. β_1 is the birth rate of Algae, γ_1 is the proportion of algae eaten by sea lamprey, δ_1 is the conversion relationship between the number of algae eaten and the number of newborn sea lamprey, ϵ_1 is the morality rate of sea lamprey.

The following will be examined by solving for the stable point and the Jacobian determinant eigenvalues. By making the above equations (8) and (9) zero, it follows that:

$$(A,L)=\left(\frac{\epsilon_1}{\delta_1},\frac{\beta_1}{\gamma_1}\right) \tag{10}$$

In equations (8) and (9), the partial derivatives are obtained for A, L respectively:

$$\begin{aligned}\frac{\partial f_1}{\partial A}&=\beta_1-\gamma_1 L\ \frac{\partial f_1}{\partial L}=-\gamma_1 A\\ \frac{\partial g_1}{\partial A}&=\delta_1 L\ \frac{\partial g_1}{\partial L}=\delta_1 A-\epsilon_1\end{aligned} \tag{11}$$

Thus, the Jacobian determinant is:

$$J=\begin{pmatrix}\beta_1-\gamma_1 L & -\gamma_1 A\\ \delta_1 L & \delta_1 A-\epsilon_1\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix} \tag{12}$$

The eigenvalues could be solved:

$$|\lambda E-J|=\begin{vmatrix}\lambda-a & -b\\ -c & \lambda-d\end{vmatrix}=\lambda^2-(a+d)\lambda+ad-bc=0 \tag{13}$$

Thus λ could be obtained:

$$\lambda=\frac{(a+d)\pm\sqrt{(a-d)^2-4bc}}{2} \tag{14}$$

For the form in (10), let eigenvalue $\lambda=s+\ ti$, the solution of equation (8) and (9) is $f(x)=e^{sx}(Acost\ \ x+Bsint\ \ x)$. The solution of the equation converges only if $s<0$.

Using MATLAB plot the population change curves for the maximum and minimum male proportions, as shown in the following figures (Fig. 7 and Fig. 8). These figures show that the population change curve has a clear periodicity, and it is preliminarily speculated to be stable.

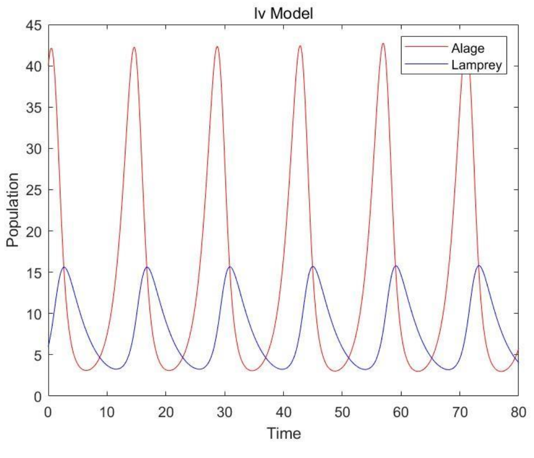


Fig. 7 Population under 0.56 male proportion

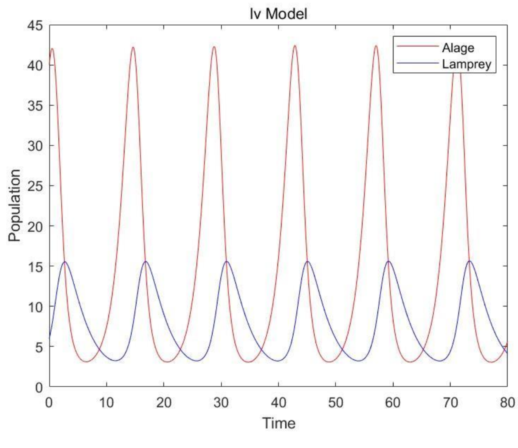
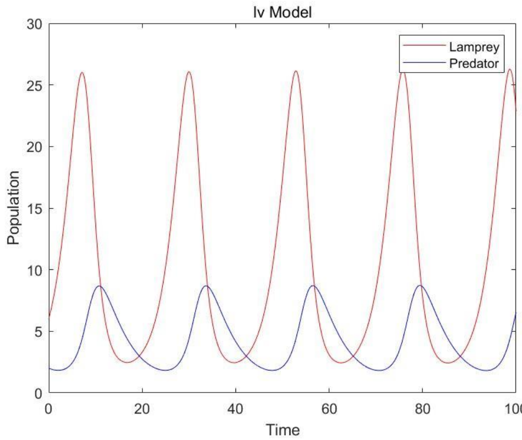


Fig. 8 Population under 0.78 male proportion

4. Sea Lamprey-Predator Model

The same as Section 3 , the relevant parameters were first selected, i.e., the natural growth rate of the sea lamprey was set to be $1-r$, the probability of being preyed upon to be 0.1 , and the natural mortality rate of the predator to be 0.2 versus the success rate of predation to be 0.02 , for each 0.01 step in the interval of the sex ratio r from 0.56 to 0.78 .

Using MATLAB, the figures of population over different sex ratios could be plotted as Fig 9 and Fig. 10 below.



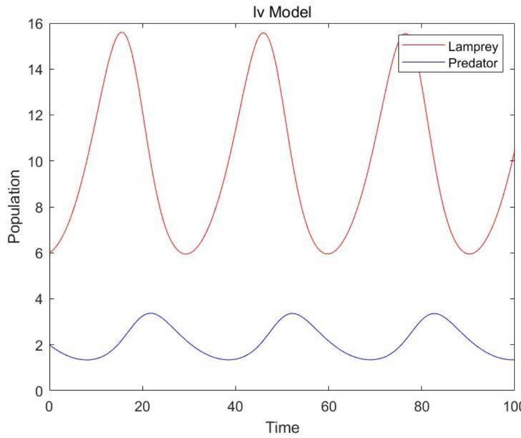


Fig. 9 Population under 0.56 male proportion **Fig. 10** Population under 0.78 male proportion

Similarly, it can be verified mathematically, i.e., for equations:

$$\frac{dL'}{dt} = \beta_2 L' - \gamma_1 L'P' = f_2 \tag{15}$$

$$\frac{dP'}{dt} = \delta_2 L'P' - \epsilon_2 P' = g_2 \tag{16}$$

Where L' stands for sea lamprey population, P' stands for sea lamprey population. β_2 is the birth rate of sea lamprey, γ_2 is the proportion of sea lamprey preyed by predator, δ_2 is the conversion relationship between the number of sea lamprey being preyed and the number of newborn predators, ϵ_2 is the morality rate of predator.

The stability of the model can be inferred by finding the eigenvalues λ of the Jacobian determinant and determining the positivity or negativity of its real part [6]. That is, it is stable when the real part is negative. Similarly, for each sex ratio in the interval, the real part of the eigenvalues obtained is negative, i.e., the model is stable.

Brady (2024) developed a population dynamics model to simulate changes in the number of species in a food web, and the algorithm in this paper has similar results to that algorithm. By using their algorithm for validation, the image can be plotted as Fig. 11. Where the black line is the predator, the green line is the algae, and the pink line is the sea lamprey. By observing the above image, it is found to be similar to the image obtained in this paper. Therefore, the simulation results are considered to be more realistic.

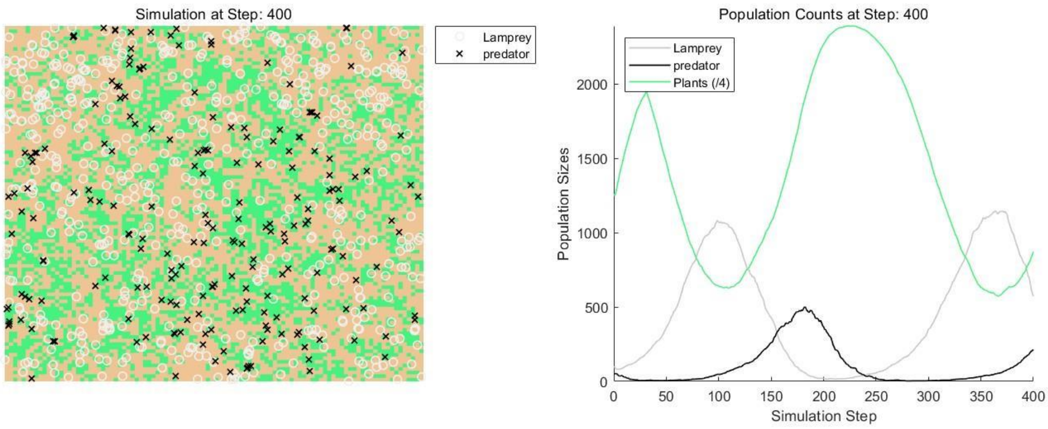


Fig. 11 Population distribution & population changes

That is to say, changing the sex ratio of the sea lamprey does not affect the stability of the ecosystem.

5. Summary

Through this study, we used logistic population modeling and Lotka-Volterra predator modeling to delve into the ecosystem impacts of changing sex ratios in the sevengill. The logistic population model revealed the effect of sex ratio on population growth, and the Lotka-Volterra model simulated the effect of changing sex ratio on sea lamprey species and found that sex ratio was directly related to predator scarcity. Further improving the model, we investigated Jacobi determinants under different sex ratios and showed that sex ratio changes do not destabilize the ecosystem. Taken together, our approach provides strong support for understanding the impacts of changing sex ratios on the ecosystem of the sea lamprey, and provides an important reference for future ecosystem management and conservation.

References

[1] Johnson N S, Buchinger T J, Li W. Reproductive ecology of lampreys[J]. Lampreys: Biology, Conservation and Control: Volume 1, 2015: 265-303.

[2] Arino J, Wang L, Wolkowicz G S K. An alternative formulation for a delayed logistic equation[J]. Journal of theoretical biology, 2006, 241(1): 109-119.

[3] Hansen, M. J., Madenjian, C. P., Slade, J. W., Steeves, T. B., Almeida, P. R., & Quintella, B. R. (2016). Population ecology of the sea lamprey (Petromyzon marinus) as an invasive species in the Laurentian Great Lakes and an imperiled species in Europe. Reviews in Fish Biology and Fisheries, 26(3), 509-535.

[4] Chipman, A., & Morrison, E. R. (2013). The impact of sex ratio and economic status on local birth rates. Biology Letters, 9(2), 20130027.

[5] Ghasemabadi A, Rahmani Doust M H. Investigating the dynamics of Lotka $-$ Volterra model with disease in the prey and predator species[J]. International Journal of Nonlinear Analysis and Applications, 2021, 12(1): 633-648.

[6] Soares A L O, Bassanezi R C. Stability analysis of epidemiological models incorporating heterogeneous infectivity[J]. Computational and Applied Mathematics, 2020, 39: 1-20.