Research on Multi Beam Sideline Model Based on Geometric Relationships

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Abstract. Marine surveying and mapping is a fundamental construction work to optimise the development of marine resources and safeguard marine rights and interests, while multibeam bathymetric system is an important tool for marine surveying and mapping engineering. This paper studies a multibeam bathymetry model based on geometric relationship. In this paper, firstly analyse the geometric relationship of measurement parameters, and get the mathematical models of coverage width and seawater depth, and the overlap rate of adjacent strips. Considering the changing relationship between the heading of the ship and the normal projection of the slope, this paper further deduces the coverage width equation in three-dimensional space on the basis of the two-dimensional multibeam slope model. It not only provides a simplified strategy for ocean mapping, but also provides a basis for solving the data coverage problem when the heading of the sounding process is not parallel to the isobath.

Keywords: Multibeam, Coverage Width, Geometric Modelling.

1. Introduction

In recent years, the development and utilisation of marine resources have gradually received attention. As a fundamental work for resource development, marine surveying and mapping is of great significance for optimising the development of marine resources and safeguarding maritime rights and interests [1]. Among many surveying and mapping technologies, multibeam bathymetric system has become an important tool for collecting seabed topographic data with its high efficiency and accuracy.

The multibeam bathymetric system transmits acoustic pulses through a transducer array installed on the survey vessel, and these pulses are reflected by the seafloor and then returned to the receiver, thus realising the measurement of water depth and generating high-resolution seafloor topographic maps [2]. However, the effectiveness of the system is highly dependent on the accuracy and coverage of the data collected, which in turn are affected by a variety of factors such as ship navigation, seabed slope and sea water conditions.

Ensuring adequate data coverage is a key challenge in multibeam mapping, especially in areas with complex seabed topography [3-5]. The coverage width, i.e., the width of the mapping area, is affected by the water depth, the overlap rate of neighbouring strips, and the geometric relationship between the ship's heading and the seabed slope. When the ship heading is not parallel to the isobath, the data coverage may be uneven or missing, which leads to inaccurate seafloor topography in the final detection [6-8].

In order to solve these problems, a multibeam bathymetry model based on geometric relationships is proposed in this study. By analysing the geometric relationship between the measured parameters, this paper aims to derive a mathematical model between the coverage width and the water depth and overlap rate. In addition, considering the changing relationship between the ship's heading and the normal projection of the slope, this paper further derives the equation for the coverage width in three-dimensional space on the basis of the two-dimensional multibeam slope model.

The significance of this study is to provide a simplified strategy for ocean mapping and to provide a basis for solving the data coverage problem when the heading is not parallel to the isobath during sounding. By providing a theoretical framework and a mathematical model, this study helps to improve the efficiency and accuracy of data collection for ocean mapping, which is of great significance for the sustainable development and utilisation of marine resources.

2. Multi-beam slope modelling

In real ocean mapping scenarios, it is often difficult to quantify the multibeam sidelobe model due to the complex environment [9]. Therefore, in this paper, it is assumed that the acoustic wave propagates in seawater in the form of uniform linear motion, and the specific detection scenario is shown in Fig.1.

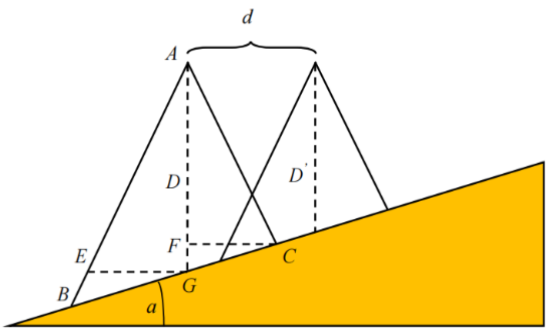


Fig. 1 multi-beam sounding in seawater

In this paper, by measuring the relative position relationship between the ship and the seafloor slope and the direction of the survey line, the actual length of the covering slope is derived according to the relevant theorem of triangles, so as to obtain the equation of seawater depth change, the equation of the covering width and the equation of the overlap rate. This research method transforms the original complex modelling problem of multibeam sounding in seawater space into a two-dimensional profile problem for the sea area. The in-Fig.1, according to the geometric relationship can be known, and according to the triangle interior angle relationship can be obtained:

$$\begin{cases} \angle ABG = \dfrac{\pi-\theta}{2} - \alpha \\ \angle ACG = \dfrac{\pi-\theta}{2} + \alpha \end{cases} \tag{1}$$

Where α represents the slope angle, θ represents the opening angle of the multibeam transducer, and the value of θ is the same as the size of $\angle BAG$ hand $\angle CAG$. The depth of the water body is determined by the relationship between the corresponding angle and the corresponding side of the triangle. In the actual bathymetric process only need to determine the depth of the water body where the ship is located *D*, according to the relationship between the corresponding angle and the corresponding side of the triangle can be obtained:

$$\begin{cases} \dfrac{D}{\sin\left(\dfrac{\pi-\theta}{2} - \alpha\right)} = \dfrac{BG}{\sin\dfrac{\theta}{2}} \\ \dfrac{D}{\sin\left(\dfrac{\pi-\theta}{2} + \alpha\right)} = \dfrac{CG}{\sin\dfrac{\theta}{2}} \end{cases} \tag{2}$$

Eq.(2) can be obtained by further derivation of Eq.(2) based on the induction formula of trigonometric functions:

$$\begin{cases} \dfrac{D}{\cos\left(\dfrac{\theta}{2}+\alpha\right)} = \dfrac{BG}{\sin\dfrac{\theta}{2}} \\ \dfrac{D}{\cos\left(\alpha-\dfrac{\theta}{2}\right)} = \dfrac{CG}{\sin\dfrac{\theta}{2}} \end{cases} \tag{3}$$

The final multibeam coverage width *BG* and the length of *CG* on both sides of the sailing vessel can be obtained as:

$$\begin{cases} BG = \dfrac{D}{\cos\left(\dfrac{\theta}{2}+\alpha\right)} \cdot \sin\dfrac{\theta}{2} \\ CG = \dfrac{D}{\cos\left(\alpha-\dfrac{\theta}{2}\right)} \cdot \sin\dfrac{\theta}{2} \end{cases} \tag{4}$$

Based on the geometric relationships in Fig.1, the equation for the depth of the water column at a distance d between the sailing vessel and the centre point *A* of the initially probed sea area can be found:

$$D' = D - d \cdot \tan\alpha \tag{5}$$

Therefore, as the detection position of the vessel changes, at a certain point the position of the vessel is at a distance *d* from the centre *A* of the initially detected sea area, at which point the length of the coverage on both sides is:

$$\begin{cases} BG' = \dfrac{(1-\tan\alpha)D}{\cos\left(\dfrac{\theta}{2}+\alpha\right)} \cdot \sin\dfrac{\theta}{2} \\ CG' = \dfrac{(1-\tan\alpha)D}{\cos\left(\alpha-\dfrac{\theta}{2}\right)} \cdot \sin\dfrac{\theta}{2} \end{cases} \tag{6}$$

The equation for the width of coverage varying with the distance of the sailing vessel from the centre of the sea can be obtained by adding the lengths of coverage on both sides:

$$W = D' \cdot \sin\frac{\theta}{2} \cdot \cos\alpha \cdot \left[\frac{1}{\cos\left(\alpha-\dfrac{\theta}{2}\right)} + \frac{1}{\cos\left(\alpha+\dfrac{\theta}{2}\right)} \right] \tag{7}$$

Since there are overlapping regions in the detection area covered by multiple beams in the detection process, this paper introduces the overlap rate to measure the reasonableness of the wiring in the detection process. If the overlap rate is high, it indicates that there are more overlapping detection areas in the actual detection process.

$$\eta = 1 - \frac{d}{(BG' + CG) \cdot \cos\alpha} \tag{8}$$

3. Multi-beam three-dimensional modelling

There is an angle between the horizontal projection of the normal direction of the seabed slope and the level of the ship's bow in the track direction, which is called the track direction inclination [10]. In the actual sea area to be measured, there is a slope of the seabed, and the top view of the acoustic wave irradiated by the multibeam detection system on the slope is shown in Fig.2. At this time, the specific situation of the multibeam detection process is shown in Fig.3.

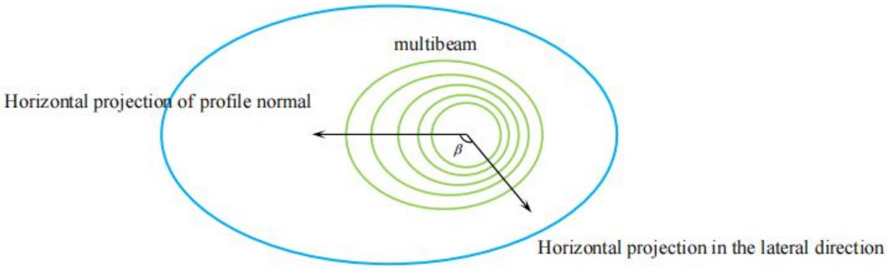


Fig. 2 Top view of slope with multibeam

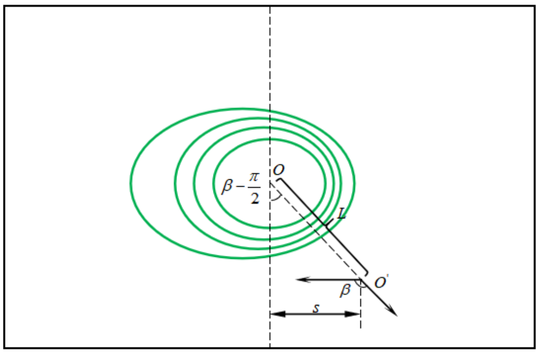
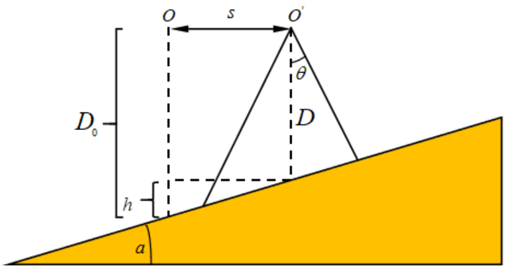


Fig. 3 Distance relationship to the orthographic projection of the centre of the sea area



Cross section along the slope at point O'

Fig. 4 Beam profiles on sea slopes

The equation for the vertical distance between the sailing ship and the horizontal line in the north-south direction at this point is:

$$s = L \cdot \sin\left(\beta - \frac{\pi}{2}\right) \tag{9}$$

Where L represents the distance between the ship and the centre of the sea area, and the difference between the track direction inclination β and $\frac{\pi}{2}$ is expressed as the horizontal angle between the track line and the north-south direction. Since the effective irradiation area area of the multibeam detection system on the sea slope presents an elliptical shape and the direction of the long axis of the ellipse is parallel to the boundary line of the sea area in the east-west direction, in order to simplify the model, the width of the coverage where the long axis is located is taken as the study in this paper. According to the above vertical distance expression and the geometric relationship existing on the sea slope irradiated by the multibeam detection sea area in Fig.4 can be obtained:

$$h = s \cdot \tan\alpha \tag{10}$$

An equation for the change in seawater depth for a change in distance relative to the centre of the sea can then be derived:

$$D = D_0 - s \cdot \tan\alpha \tag{11}$$

Where D_0 represents the depth of seawater in the centre of the sea. According to the schematic situation of acoustic wave irradiation on the sea slope, as shown in Fig.4, it can be seen that the expression of the coverage width of the multibeam in three-dimensional space is similar to the derivation process of Eq.(7), so we can get the equation of the coverage width of the multibeam in three-dimensional space by substituting D into Eq.(7):

$$W = (D_0 - s \cdot \tan\alpha) \cdot \left[\frac{1}{\cos(\alpha - \frac{\theta}{2})} + \frac{1}{\cos(\alpha + \frac{\theta}{2})} \right] \cdot \sin\frac{\theta}{2} \cdot \cos\alpha \tag{12}$$

Bringing Eq. (9) into the equation for the coverage width on the multibeam 3D space can be finally obtained:

$$W = (D_0 - L \cdot \sin(\beta - \frac{\pi}{2}) \cdot \tan\alpha) \cdot \left[\frac{1}{\cos(\alpha - \frac{\theta}{2})} + \frac{1}{\cos(\alpha + \frac{\theta}{2})} \right] \cdot \sin\frac{\theta}{2} \cdot \cos\alpha \tag{13}$$

4. Experimental analysis

The seabed slope α of the sea area measured in this paper is 15°, the seawater depth d at the centre of the sea area is 70*cm*, and the transducer opening angle θ of the sailing vessel is 60°. Thus, the results of seawater depth, coverage width, and overlap rate with the previous line can be obtained at different distances between the sideline and the centre point, as shown in Table 1. On the left side of the centre of the sea, the depth of the sea water, the coverage width of the multibeam sounding, and the overlap rate with the previous line increase as the sideline is further away from the centre point; on the right side of the centre of the sea, the opposite results are shown as the sideline is further away from the centre point. At 600m and 800m from the centre, the overlap with the previous line is negative, which is due to the fact that the spacing between the two lines is too large during the detection process, resulting in an area where there is no overlap in the width of coverage, i.e., an area of missed detection.

Table 1. Beam slope sidelining results

Distance of the lateral line from the centre / m	-800	-600	-400	-200	0	200	400	600	800
Depth of sea / m	90.9	85.7	80.5	75.2	70	64.8	59.5	54.3	49.1
Coverage width / m	315.7	297.5	279.3	261.2	243.0	224.8	206.6	188.4	170.3
Overlap with previous sideline / %	——	0.35	0.31	0.26	0.21	0.14	0.07	-0.01	-0.12

For the multibeam three-dimensional model of sea area detection, the value of the distance of the measuring vessel from the centre of the sea area and the value of the angle in the direction of the measuring line are substituted into the multibeam three-dimensional coverage width equation to be solved, as shown in Table 2. According to the results in Table 2, it can be seen that the coverage width is unchanged between 90°and 270°in the direction of the survey line, which is due to the fact that the depth of the sea water is always kept constant in the process of the survey because the trajectory is parallel to the isobath of the seabed slope. The multibeam three-dimensional model effectively solves the coverage width measurement problem brought about by the non-parallelism between the ship's detection direction and the isobath direction in the process of sea area detection and provides a basis for the line routing of sonar detection in the complex seabed terrain, thus avoiding the redundancy of detection data due to the high overlap rate of the coverage in the process of detection.

Table 2. Multi-beam three-dimensional modelling results

Coverage width/m		Measurement of the distance of the ship from the centre of the sea area							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Side line direction angle/°	0	416.5	467.1	517.6	568.1	618.6	669.1	719.1	770.1
	45	416.5	452.3	488.0	523.7	559.4	595.1	630.8	666.5
	90	416.5	416.5	416.5	416.5	416.5	416.5	416.5	416.5
	135	416.5	380.8	345.1	309.4	273.7	238.0	202.3	166.6
	180	416.5	366.0	315.5	265.4	214.5	164.0	113.5	63.0
	225	416.5	380.8	345.1	309.4	273.7	238.0	202.3	166.6
	270	416.5	416.5	416.5	416.5	416.5	416.5	416.5	416.5
	315	416.5	452.3	488.0	523.7	559.4	595.1	630.8	666.5

5. Conclusions

This paper proposes a multibeam bathymetry model based on geometric relationships, which derives equations for the coverage width and its overlap rate by analysing the geometric relationship between the multibeam sounding sounder line and the seafloor slope. At the same time, this paper takes into account the non-parallel relationship between the heading and the isobath during the sounding process, so as to construct a three-dimensional multibeam sidetrack model in any heading. The two detection models provide a simple calculation scheme for the detection of complex seabed topography, which effectively simplifies the problem of detecting the complex topography of the sea area. However, due to the idealisation of the multibeam underwater propagation process and the neglect of the influence of seawater on the multibeam acoustic wave propagation route, the applicability of the models may be reduced in the face of more complex seawater conditions. Nonetheless, the analytical ideas in this paper still provide an effective solution for multibeam ocean sounding. Future research can further consider the influence of seawater conditions on acoustic wave propagation to improve the applicability and accuracy of the model, so as to better serve the ocean mapping and resource development work.

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