Transmission Coefficients of The Asymmetric Single and Double Barriers

Xinyu Liu [1,*] and Yinghao Lu [2]

[1] Affiliated Middle School of Henan Normal University, Xinxiang, China

[2] Changzhou No.1 Middle School, Changzhou, China

* Corresponding Author Email: yujuan01@zybank.com.cn

Abstract. The quantum tunneling effect, which is based on the quantum theory that microscopic particles behave in waves through barriers higher than their own energy, has important applications in practical technology. Microscopes such as scanning tunneling microscopes, which use quantum tunneling to generate tunnel currents, have a resolution of up to the desired level. Moreover, it can be used in biology to observe molecular structures such as macromolecules and biofilms. In this paper, the transmission coefficients of one-dimensional asymmetric unilateral barrier and asymmetric two-sided barrier under general arbitrary boundary conditions are strictly derived using the conventional method of solving the steady state Schrodinger equation by quantum mechanics and matrix. Meanwhile, the transmission coefficients of symmetric two-sided barrier are numerically studied. From the above series of methods, it is deduced that the basic working principle of resonant tunneling diode and other semiconductor electronic devices is exactly the tunnel effect. Thus, it is found that quantum tunneling effect plays an important role in life.

Keywords: Transmission coefficient; Double barrier; Tunnelling effect; Resonant tunneling diode.

1. Introduction

Quantum physics is a branch of physics that describes the behavior of particles in the microscopic world, with its theoretical foundations including quantum mechanics and quantum field theory [1,2]. In quantum physics, there are many fascinating phenomena and one of which is quantum tunneling effect. The quantum tunneling refers to the phenomenon where microscopic particles (such as electrons or atoms) can tunnel through a barrier that they would not be able to cross according to classical physics, due to the quantum mechanical tunneling effect [3]. In classical physics, particles need to have sufficient energy to overcome a barrier, otherwise they would be reflected. However, in quantum mechanics, due to the wave-particle duality, particles have a probability wave function, allowing them to tunnel through the barrier even if their energy is lower than the barrier height.

Quantum tunneling has important applications in various fields, such as nuclear fusion and scanning tunneling microscopy. In nuclear fusion [4], protons tunnel through the Coulomb barrier to initiate nuclear reactions. In scanning tunneling microscopy [5], electron tunneling is used to achieve atomic resolution imaging of sample surfaces. The theoretical basis of quantum tunneling can be explained through the Schrödinger equation and the calculation of tunneling probabilities in quantum mechanics. The tunneling probability depends on factors such as the height of the barrier, particle energy, and barrier width, and can be obtained through analytical solutions of the Schrödinger equation or numerical calculations. In summary, quantum tunneling is an important and fascinating phenomenon in quantum physics that challenges classical physics concepts and provides a key theoretical foundation for many modern technologies and applications

In this article, the models of asymmetric single barrier and symmetric double barriers are introduced firstly, using the method of constructing equations and matrix to calculate the reflection coefficient successfully. It indicates that there are a few electrons that can go through the barriers. This phenomenon verifies again the existence of quantum tunnelling effect. Two significant applications are discussed after this part. The Resonant tunneling diodes (RTDs) have higher reliability as the polarization effect exists and it can provide the highest peak to valley current ratio (PVCR). Another example is about Speaker authentication method, which improves the recognition rate and enhances new research avenues for quantum information theory and speaker authenticity identification methods.

2. Asymmetric Barriers

2.1. Asymmetric Single Barrier

In the practical application of microelectronics, under the influence of the surrounding environment, the barrier structure of the electronic device will change. For example, the symmetric square barrier will become an asymmetric structure. The model shown in Fig. 1 describes the asymmetric barrier, and the symbols A/B, and E/F respectively represent the incident or reflection amplitude.

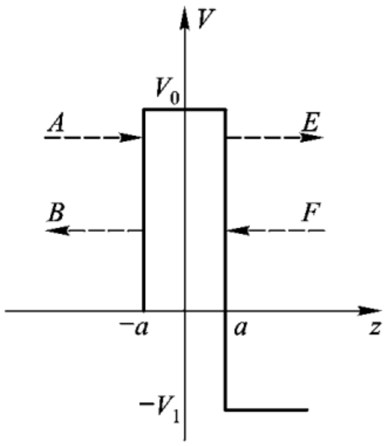

Fig. 1 Sketch of the asymmetric single barrier [3].

The wave functions in different regions are

$$\Psi(z) = \begin{cases} Ae^{ikz} + Be^{ikz}, & z < -a \\ Ce^{yz} + De^{-yz}, & -a < z < a \\ Ee^{ik_1 z} + Fe^{-ik_1 z}, & z > a \end{cases} \tag{1}$$

where $k = \sqrt{2mE}/\hbar$ is the left vector, γ=$\sqrt{2m(V_0 - E)}/\hbar$ is the middle barrier layer wave vector, and $k_1 = \sqrt{2m(E + V_1)}/\hbar$ is the right wave vector. According to the wave function and its derivative boundary conditions are continuously $Ae^{-ika} + Be^{ika} = Ce^{-ya} + De^{ya}$, $ik[Ae^{-ika} + Be^{ika}] = y[Ce^{-ya} - De^{ya}]$, $Ce^{ya} + De^{-ya} = Ee^{ik_1 a} + Fe^{-ik_1 a}$, and $y[Ce^{ya} - De^{-ya}] = ik[Ee^{ik_1 a} - Fe^{-ik_1 a}]$.

For the treatment of the above equations, it is convenient using matrix expression method. After calculation and finishing, one considers the above boundary condition equations as the following matrix form

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} \tag{2}$$

where $M_{22} = M_{11}^*$ and $M_{12} = M_{21}^*$. Mathematically, it is calculated that

$$M_{11} = \left[\frac{1}{2}\left(1 + \frac{k_1}{k}\right)\cosh(2\text{ya}) - \frac{i}{2}\left(\frac{kk_1 - y^2}{ky}\right)\sinh(2ya)\right]e^{i(k+k_1)a} \tag{3}$$

and

$$M_{21} = -\left[\frac{1}{2}\left(\frac{k_1}{k} - 1\right)\cosh(2\text{ya}) + \frac{i}{2}\left(\frac{kk_1 + y^2}{ky}\right)\sinh(2ya)\right]e^{-i(k-k_1)a}. \tag{4}$$

Therefore, the reflection coefficient is $\mathrm{R(E)} = \frac{k_1}{k}\frac{|A|^2}{|B|^2} = \frac{|M_{21}|^2}{|M_{11}|^2}$. The effect of tunneling electron effective mass on transmission in different regions is not considered, and the coefficient is

$$\mathrm{T(E)} = \frac{k_1}{k}\frac{|E|^2}{|A|^2} = \frac{k_1}{k}\frac{1}{|M_{11}|^2} = \frac{4k_1k/(k_1+k)^2}{1+\left(\frac{(k^2+y^2)\left({k_1}^2+y^2\right)}{y^2(k_1+k)^2}\right)\sinh(2ya)}. \quad (5)$$

As an example, the typical transmission coefficient as a function of incident energy is shown in Fig. 2. It is found that T is not monotonously varied as the energy E increases.

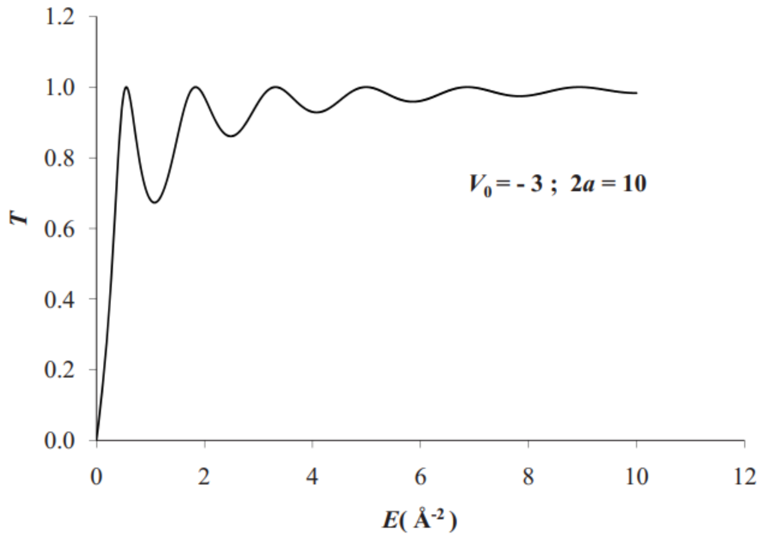

Fig. 2 transmission coefficient as a function of incident energy.

2.2. Asymmetric Double Barrier

In actual microelectronic devices, it is often composed of double or multi-layer barriers. Fig. 3 is a schematic diagram of the asymmetric double barrier structure. The width of the left and right barriers is $2a_1$ and $2a_R$, respectively. Other parameters are shown in Fig. 3. The two barriers divide the space into five regions: I, II , III, IV and V, corresponding wave vectors are k, y, k_1, y_1 and k_2 respectively, where $\mathrm{k} = \sqrt{2\mathrm{mE}}/\hbar$, $\mathrm{y} = \sqrt{2\mathrm{m}(\mathrm{V_0} - \mathrm{E})}/\hbar$, $\mathrm{k_1} = \sqrt{2\mathrm{m}(\mathrm{E} + \mathrm{V_1})}/\hbar$, $\mathrm{y_1} = \sqrt{2\mathrm{m}(\mathrm{V_{01}} + \mathrm{E})}/\hbar$and $\mathrm{k_2} = \sqrt{2\mathrm{m}(\mathrm{V_2} + \mathrm{E})}/\hbar$. For the treatment of a single barrier, the phase of an electron tunneling between the left and right barriers is connected by the following relation $\mathrm{A}' = \mathrm{Ee^{ikb}}$ and $\mathrm{B}' = \mathrm{Ee^{-ikb}}$, where b is the distance between the left and right barriers, so the amplitude A', B' and E, F is related to each other by the following matrix:

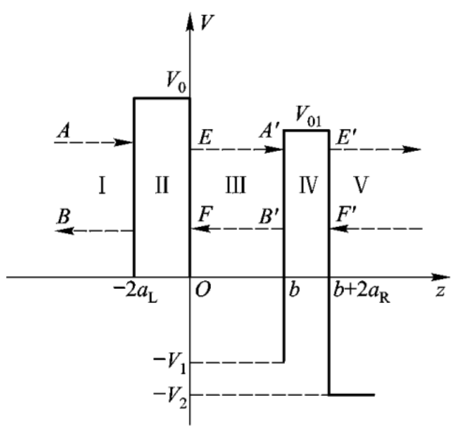

Fig. 3 Sketch of the asymmetric double barriers [3].

$$\begin{bmatrix} E \\ F \end{bmatrix} = \begin{bmatrix} e^{-ikb} & 0 \\ 0 & e^{ikb} \end{bmatrix} \begin{bmatrix} A' \\ B' \end{bmatrix} = M_W \begin{bmatrix} A' \\ B' \end{bmatrix}. \quad (6)$$

Here, M_w represents the transition matrix between the two barriers. In this way, the relationship between the incident amplitude and the outgoing amplitude can be obtained by matrix operation:

$$\begin{bmatrix} A \\ B \end{bmatrix} = M_w M_L M_R \begin{bmatrix} E' \\ F' \end{bmatrix} = M_T \begin{bmatrix} E' \\ F' \end{bmatrix} \quad (7)$$

where M_L and M_R are the tunneling matrix of the left and right barriers respectively, M_T represents the double-junction matrix after inclusion. Eq. (7) can also be specifically expressed as

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} M_{T11} & M_{T12} \\ M_{T21} & M_{T22} \end{bmatrix} \begin{bmatrix} E' \\ F' \end{bmatrix} \quad (8)$$

where $M_{T11} = M_{T11} M_{R11} e^{-ikb} + M_{L12} M_{R21} e^{ikb}$. In the above formula, M_{L12} and M_{R21} are left and right barrier tunneling matrix elements respectively. To simplify the expression, a single barrier tunneling matrix element M_{11} can be marked with the following form $M_{11} = m_{11} e^{i\Theta}$.

The corresponding relationship between the amplitude and in the above formula, m_{11} is the amplitude of the matrix element and Θ is the phase of the matrix element. Then the formula phase of each matrix element and the corresponding wave vector are

$$m_{L11} = \sqrt{\frac{1}{4}\left(1 + \frac{k_1}{k}\right)^2 \cosh^2(2ya_L) + \frac{1}{4}\left(\frac{kk_1 - y^2}{ky}\right)^2 \sinh^2(2ya_L)} \quad (9)$$

$$m_{L12} = \sqrt{\frac{1}{4}\left(1 - \frac{k_1}{k}\right)^2 \cosh^2(2ya_L) + \frac{1}{4}\left(\frac{kk_1 + y^2}{ky}\right)^2 \sinh^2(2ya_L)} \quad (10)$$

$$m_{R11} = \sqrt{\frac{1}{4}\left(1 + \frac{k_2}{k_1}\right)^2 \cosh^2(2y_1 a_R) + \frac{1}{4}\left(\frac{k_1 k_2 - {y_1}^2}{k_1 y_1}\right)^2 \sinh^2(2y_1 a_R)} \quad (11)$$

and

$$m_{R21} = \sqrt{\frac{1}{4}\left(1 - \frac{k_2}{k_1}\right)^2 \cosh^2(2y_1 a_R) + \frac{1}{4}\left(\frac{k_1 k_2 + {y_1}^2}{k_1 y_1}\right)^2 \sinh^2(2y_1 a_R)}. \quad (12)$$

Here, $\Theta_{L11} = -\text{atan}\left[\frac{kk_1 - y^2}{(k+k_1)y}\tanh(2ya_L)\right] + (k + k_1)a_L$, $\Theta_{L12} = -\text{atan}\left[\frac{kk_1 + y^2}{(k-k_1)y}\tanh(2ya_L)\right] + \pi + (k - k_1)a_L$, $\Theta_{R11} = -\text{atan}\left[\frac{k_1 k_2 - {y_1}^2}{(k_1+k_2)y_1}\tanh(2y_1 a_R)\right] - (k_1 + k_2)a_R$, as well as $\Theta_{R21} = \text{atan}\left[\frac{k_1 k_2 + {y_1}^2}{(k_2-k_1)y_1}\tanh(2y_1 a_R)\right] + \pi + (k_1 - k_2)a_R$. In addition, m_L and m_a in the above are the amplitudes of the tunneling matrix elements of the left and right barriers respectively, and Θ_L and Θ_R are the phases of the tunneling matrix elements of the left and right barriers respectively. The above amplitudes and phase relations are obtained by substituting them into the relation

$$|M_{T11}|^2 = (m_{L11} m_{R11} - m_{L12} m_{R21})^2$$
$$+4 m_{L11} m_{R11} m_{L12} m_{R21} \cos^2\left(k_1 b + \frac{\Theta_{L12} + \Theta_{R21} - \Theta_{L11} - \Theta_{R11}}{2}\right). \quad (13)$$

Then, the total transmission coefficient of the double junction is

$$\mathrm{T(E)} = \frac{\mathrm{k_2}}{\mathrm{k}} \frac{1}{|\mathrm{M_{T11}}|^2}. \quad (14)$$

3. Applications

This part is going to talk about the applications which are associated with quantum tunneling.

3.1. Resonant Tunneling Diode

Since Esaki first researched and discovered RTD [6], these diodes have been widely studied. RTD consists of III-V compounds semiconductor materials, such as GaAs, GaAs, InP etc. These materials have good electron transport performance and quantum effect, which are suitable for making efficient RTD devices. Fig. 4 shows the main structure of the RTDs. The RTDs are semiconductor quantum well structure nano devices with finite rectangular barriers which have negative differential resistance in their $I - V$ characteristics.

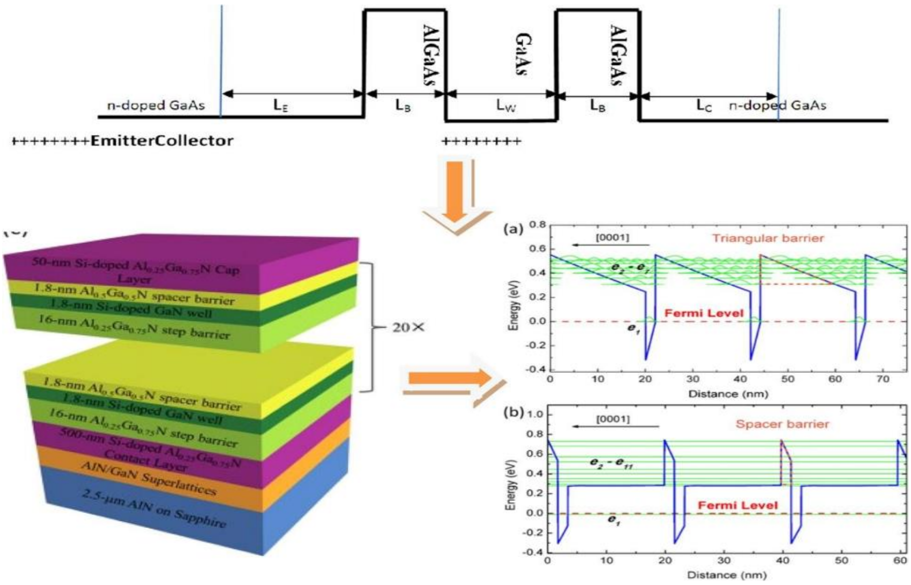

Fig. 4 RTD’s main structure: resonant tunneling structure and control layer [7].

The dominant theory of RTD is based on tunnelling effect from quantum mechanics

$$-\frac{\bar{h}^2}{2\mu}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x,t)\Psi(x,t) = \frac{i\bar{h}\big(\partial\Psi(x,t)\big)}{\partial x}. \tag{15}$$

The Schrödinger equation can be used, by solving this equation, one result can be obtained that when a quantum wave encounters a "potential barrier", although its amplitude will exponentially decrease, there is a certain probability that the amplitude on the other side of the "potential barrier" will not be zero. This means that microscopic particles have a certain probability of directly "tunneling through" the barrier, which is the theoretical basis of quantum tunneling. The resonant tunneling diodes commonly consist of two ultra-thin semiconductor layers with a thin insulating layer between them [7]. When a voltage is applied, electrons can go through the insulating layer to the other side due to quantum mechanical effect. When the voltage reaches a certain value, the electrons form a resonant state, causing a sharp increase in current, this phenomenon is known as resonant tunneling effect.

According to their unique exhibition that is negative differential resistance in their current-voltage characteristic, this type of diode can be used in various electronic circuits [8]. In addition, there is an important effect happens in the RTD, which is the polarization effect [9]. When the electric field is applied on it, the internal electrons and the hole distribution will change. It will affect the transmission property of electrons and holes in the material, including drift velocity of the charge carriers, effective mass, and energy band structure. For example, the output characteristic curve of the extracted RTD device with polarization effect displays significant non-symmetry [10], so the polarization effect is one of the most significant factors that affect the output characteristic of devices.

Another example is that it can use the polarization effect to determine the optimum device parameters [11], which is the optimum spacer layer is found to be half of the de-Broglie wavelength associated with the bound state of the corresponding finite quantum well. The proposed relations for the optimum parameters can be used to design RTD based on any two appropriate materials to attain the highest PVCR. This experiment and discovery improve the availability of RTD to further ensure reliability.

3.2. Identification the Authenticity of Speakers

A method which can identify the authenticity of the speaker was proposed based on the quantum tunnelling effect in physics [12]. With the increasing influence of digital multimedia technologies represented by speech signals on modern society, speaker verification has become one of the current research hotspots. This model is based on the method which is invented by Reynolds who used Gaussian Mixture Model to propose a method of identify the status of speaker. First of all, the quantum characteristics of speech signal framing can be studied by considering each speech signal frame as a quantum state. By implementing the quantization of the algorithm, one can explore how these frames behave within a quantum context. After that, a potential barrier, which is displayed on Fig. 5, was employed to segregate the energy features. The barrier ensemble was established to isolate the energy spectrum attributes of the signal and utilize them as the defining parameter.

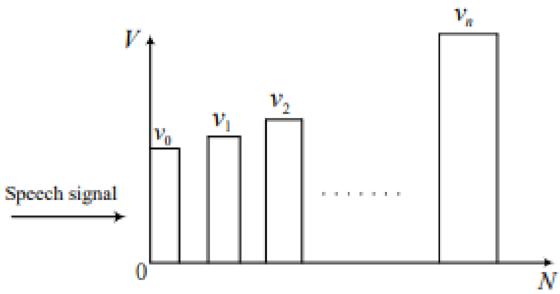

Fig. 5 multiple quantum barriers.

The speech signal modeling was ultimately conducted using the Gaussian Mixture Model to finalize the speaker authentication process. The simulation outcomes indicate that employing quantum tunneling theory for speaker identification can effectively decrease algorithm complexity, enhance discrimination, and offer a novel avenue for both speaker authentication and quantum information theory, as compared to conventional methods.This model perfectly showed the widely and thoroughly usage of quantum tunnelling effect in various occasions. In addition, this model provided a new method for clustering unstructured data.

4. Conclusion

To conclude, this paper starts from introducing the concept of quantum tunneling effect. Next, it deduces the transmission coefficient by establishing matrix, Schrodinger equation, wave function and some pictures. In addition, the third and last part mainly introduces the application of some quantum tunneling effect. For example, the application of resonant tunneling diode, as well as the impact of resonant tunneling diode on people's daily life, etc. However, for the multiple (more than double barrier) potential barriers, the derivations are still slightly lacking, while for the quantum tunneling effect the transmission coefficient is affected by temperature changes. In addition, there are many applications of quantum tunneling effect that have not been involved in this article, such as scanning tunneling microscope, the use of quantum tunneling effect to produce tunnel current microscope, its resolution can reach the willing level, in biology, used to observe molecular structures such as macromolecules and biofilms. Devices of this kind are not covered. It is hoped that the influence of quantum tunneling effect under different conditions can be calculated by more detailed derivation in the future.

Authors Contribution

All the authors contributed equally, and their names were listed in alphabetical order.

References

[1] Wang D., Su J., Tan W., et al. Research progress and application of THZ resonant tunneling diode detector. Terahert Journal of Science and Electronic Information, 2022, 20(10):1-3.

[2] Li Haifeng, Wang Xinmao, et al. Study and numerical simulation of quantum tunneling for one-dimensional double square potential barriers. College Phsyics, 2022, 41(1):4-5.

[3] Yang Jun, Chen Lei, Chen Zhi-li, Xiao Xue-wang. Study and numerical simulation for quantum tunneling characteristics in asymmetric potential barrier. College Phsyics, 2011, 30(10): 7-10+29.

[4] Li Ming, Wang Hongzhang, Study on the quantum effect of the cold fusion in metal electrodes. Chinese Journal of Nuclear Science and Engineering, 2005, 25(4): 12-16.

[5] Liu Shuyi, Wolf Martin, Kumagai Takashi. Plasmon-Assisted Resonant Electron Tunneling in a Scanning Tunneling Microscope Junction. Physical Review Letters, 2018, 121(22): 226802.

[6] Hughes J F, van Dam A, Foley J D, et al. Computer Graphics: Principles and Practice. Upper Saddle River, NJ: Pearson Education, 2013: 1047-1048.

[7] Yahyaoui N., Sfina N., Said M., et al. Electron transport through cubic InGaN/AlGaN resonant tunneling diodes. Computer physics communication, 2014, 185(12): 3119-3126.

[8] Ali Rezaei, Patryk Maciazek, Vihar P. Georgiev, et al. Statistical device simulations of III-V nanowire resonant tunneling diodes as physical unclonable functions source. Solid -State electronics, 2022, 194:108339.

[9] Bommalingaiah B., Narayan Gaonkar, R. G. Vaidya. Effect of spontaneous polarization field on diffusion thermopower in AlGaN/GaN heterostructures. Chemical Physics Impact, 2023, 7: 100251.

[10] Rong Tao-tao. Study of polarization effect in gallium nitride-based resonant tunnelling diodes. Journal of Baoji University of Arts and Sciences (Natural Science), 2022, 42(2):65-70.

[11] Sushree Ipsita, P. K. Mahapatra, P. Panchadhyayee. Optimum device parameters to attain the highest peak to valley current ratio (PVCR) in resonant tunneling diodes (RTD). Physica B: Condensed Matter, 2021, 661:412788.

[12] Huang Liang, Pan Ping, Zhou Chao. The speech signal modeling was ultimately conducted using the Gaussian Mixture Model (GMM) to finalize the speaker authentication process. Journal of Computer Applications, 2017, 37(9): 2617-2620.