

Analysis Of the Principle, Facility, And Applications of Gravitational Wave

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Abstract. When Einstein published his general relativity at the beginning of last century, gravitational wave was predicted in the study. It has been the hottest topic in astronomy, and countless astronomers have studied them. This includes the principles of gravitational wave generation, detection methods, observations, etc. This article will focus on a review of gravitational wave research, and comprehensively write down the gravitational wave research results so far from the perspectives of basic mathematical description, detection principles, detection methods and applications, and future prospects. This study gives a mathematical description of gravitational waves, including calculation formulae for the energy they carry, their amplitude, and their impact on passing objects. In addition, this article also introduces several mainstream methods for detecting gravitational waves today, and details the applications of these methods, such as MiniGRAIL, LIGO, LISA and other detection devices. Finally, the article describes expected developments in gravitational wave detection over the next decade. Based on studying and detecting gravitational waves, humans can detect many astronomical events that cannot be detected with existing methods, and gain a deeper understanding of the universe.

Keywords: Gravitational wave detection; interferometer; LIGO.

1. Introduction

Gravitational waves have become an inevitable issue in astronomy in recent years. They represent a significant general theory of relativity prediction made by Einstein. They are like tiny ripples in water made by stones; they are disruptions in the space-time's curvature brought about by the motion of enormous things in the cosmos. Gravitational waves were not directly detected until 2015, a momentous occasion that signaled a breakthrough in astronomy. Gravitational waves have given astronomers a fresh perspective on cosmic occurrences that were previously difficult to witness, such as black hole mergers and collisions of neutron stars. Since then, other gravitational wave occurrences have been found, which has increased the understanding of the composition and evolution of the universe. Gravitational wave astronomy will continue to solve cosmic riddles and broaden the knowledge of the cosmos as long as gravitational wave detection technology is developed and improved. Thus, astronomy has much to gain from the study of gravitational waves, and its importance will only grow with further investigation.

When he published General Relativity in 1916, Albert Einstein made the prediction that gravitational waves will exist. He described how to derive gravitational waves from general relativity assumptions in his paper and gave three different types of gravitational waves [1]. The existence of gravitational waves was established, but several of his theories (e.g., the three types he proposed) were shown to be incorrect. PSR B1913+16, also known as the Hulse-Taylor Binary Pulsar, was found in 1974 by Russell Hulse and Joseph Taylor, confirming Einstein's gravitational wave prediction. Since the binary system's discovery, the orbit has deteriorated, which can be explained by Albert Einstein's theory of gravitational waves and their energy loss [2]. In 2016, the LIGO and VIRGO observatories verified the gravitational wave signal resulting from the merger of two black holes in 2015 [3]. This marks the historical first direct observation of gravitational waves. It initiates a new era of gravitational wave monitoring.

The existence of gravitational waves allows astronomers to observe many cosmic phenomena that were previously impossible to observe using optical telescopes and electromagnetic telescopes, and

they can also observe the early universe because electromagnetic waves cannot penetrate the universe before recombination. This article will provide a thorough definition and mathematical explanation of gravitational waves, as well as information on detection methods, equipment, and recent detection outcomes. It will also discuss existing constraints and potential future developments. Through in-depth study of gravitational waves, one can expand the understanding of the nature of the universe, further explore the mysteries of the universe, and promote the development of the field of astronomy.

2. Basic Description

In general relativity, Einstein predicted the presence of gravitational waves, which have since been repeatedly and successfully seen by humans. In addition to carrying energy, gravitational waves are created by a gravitational field and travel through space at the speed of light. This article will briefly introduce the properties and generation of gravitational waves under the weak field approximation. Assuming that the gravitational field is weak, the Minkowski metric is used to describe the flat background space-time and the metric tensor is used to describe the disturbance propagation of gravitational waves [4]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $h_{\mu\nu} (|h_{\mu\nu}| \ll 1)$ is the metric perturbation. The time interval line element is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu + h_{\mu\nu} dx^\mu dx^\nu \quad (2)$$

Here, $h_{\mu\nu}$ is a 4×4 symmetric tensor with ten components. However, since the gravitational field equation has the properties of gauge invariance and coordinate transformation, in fact, gravitational waves only have two independent dynamic degrees of freedom. If one assumes that a certain column of gravitational waves propagates along the z-axis direction, under the transverse traceless (TT) specification, the metric of the gravitational waves is

$$h_{\mu\nu}^{TT} dx^\mu dx^\nu = h + (x^i, t)(dx^2 - dy^2) + 2h \times (x^i, t) dx dy \quad (3)$$

Among them, h_+ and h_\times are the gravitational waves' two independent degrees of freedom, which are called the radiation gravitational field's two polarization degrees of freedom. When gravitational waves pass through a detection instrument, the length of the detection instrument will cause a stretching effect. If the original length of the instrument is L , when the gravitational wave h_{jk} passes by, the change in the length of the instrument is

$$\delta L = \frac{1}{2} L h_{jk} \hat{n}^j n \hat{n}^k \quad (4)$$

The h_+ component can cause periodic oscillations in the length of the x- and y-directions of the detection instrument, while the h_\times component can cause periodic changes in the length of the detection instrument in the angular direction of the x-axis and y-axis.

The mechanism of gravitational wave radiation differs from the generation mechanism of electromagnetic fields. Since gravitational waves are quadrupole radiation, the energy and momentum tensor of the gravitational wave source's quadrupole moment must vary with time. Assume that the distance between the detector and the gravitational wave source is r , and the received gravitational wave signal is (taken as a first-order small quantity):

$$h_{ij}^{TT}(t, r) = \frac{2Gd^2}{rc^5 dt^2} Q_{ij}^{TT}(t - r) \quad (5)$$

Among them, Q_{ij}^{TT} is the quadrupole moment of the energy and momentum tensor of the gravitational wave source under the TT specification. If it is assumed that the two polarization

components of gravitational waves are equivalent, the energy flow density of gravitational waves is of the order

$$I = \frac{\pi c^3}{4G} h^2 v^2 \quad (6)$$

The energy radiated by a wave source r from the earth in time $T \approx \frac{1}{v}$ is $\Delta E = 4E = \frac{4\pi^2 r^2 I}{v}$, and the gravitational wave's amplitude is

$$h \approx \frac{1}{r} \left(\frac{G}{\pi^2 c^3} \right) \frac{\Delta E^{\frac{1}{2}}}{v} \quad (7)$$

If the frequency v of the gravitational wave and the energy loss of the gravitational source are known, the gravitational wave's amplitude can be estimated. The gravitational radiation produced by higher polar moments is very weak and is usually ignored.

3. Principles of Searching and Facilities

Physicists like Russell Hulse and Joseph Hulls have theoretically created the groundwork for the presence of gravitational waves since Einstein released general relativity in 1916 and predicted their existence. Taylor discovered the Hulse-Taylor pulsar pair in 1974. But the first direct observation of gravitational waves did not occur until 2015. This is due to the fact that most celestial events release gravitational waves, which are severely attenuated due to their extraordinarily lengthy journey to Earth. The detection of gravitational waves requires very high measurement accuracy. Since the introduction of gravitational waves, astronomers have proposed a variety of ways to detect gravitational waves. This article will introduce some of the most widely used methods.

3.1. Weber Bar

Joseph Weber proposed the original Weber Bar in the 1960s. It was a device that could observe gravitational waves by using a property that could slightly change the size of an object. A Weber Bar is a slender metal rod that vibrates at a preset resonant frequency. When gravitational waves pass through Weber Bars, changes in the gravitational field distort space, thereby changing the length of the Weber Bars. This slight change in length can cause the resonant frequency of the Weber Bar to change. Precision measuring instruments such as piezoelectric sensors can measure changes in the resonance frequency, thereby inferring various data on passing gravitational waves [5].

3.2. Laser Interferometry

Weber Bars were not sensitive enough to detect weaker gravitational waves, so another laser interferometry method was proposed for measuring gravitational waves. The measurement accuracy of this method is higher than that of Weber Bars, and it is easier to detect some weak gravitational wave signals. The core of laser interference technology is the Michelson interferometer, which consists of a light source, two reflectors, and a beam splitter. After the light emitted by the light source hits the beam splitter, it will be divided into two beams, which are directed to two light storage arms several kilometers long. After reaching the other end, they will be reflected and then returned to the beam splitter for recombination. Due to the interference properties of light, the final optical path difference between the two beams of light will lead to a phase difference, so the combined light will produce interference fringes at the intersection of light and dark [6]. When gravitational waves pass through, the spatial disturbance generated will cause the length of the light storage arm to change, thereby affecting the optical path difference between the final two beams of light, and thus affecting the interference fringes. Precision photodetectors and other instruments can measure changes in interference fringes to calculate the physical properties of gravitational waves. A sketch is shown in Fig. 1 [7].

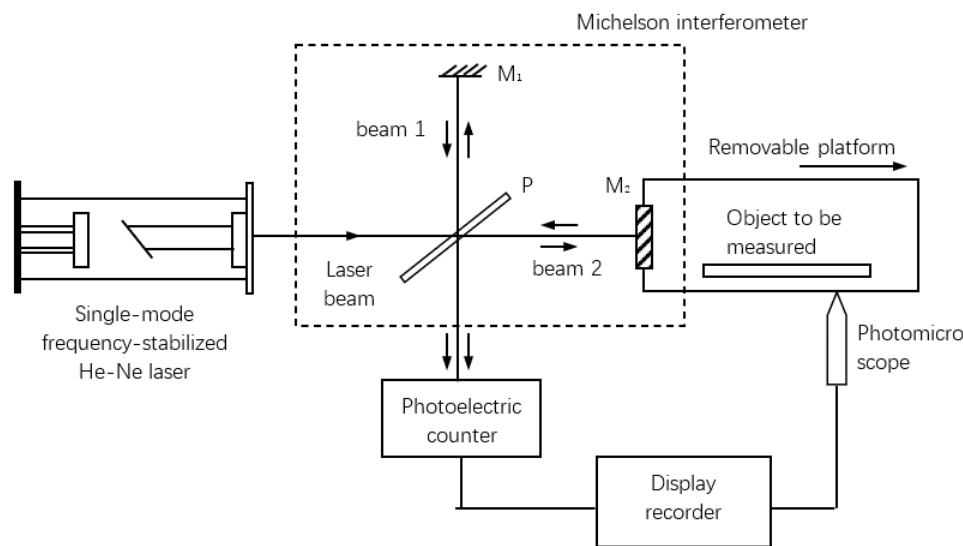


Fig. 1 Laser Interferometry [7].

3.3. Pulsa Timing Array

Compared with Weber Bar and laser interferometry, the pulsar timing array measurement method is a little different. It uses the properties of pulsars to detect the existence of gravitational waves. A pulsar is a rapidly rotating star that emits electromagnetic radiation at regular intervals. Therefore, one can determine whether gravitational waves are present by monitoring the time intervals between multiple pulses arriving at Earth because gravitational waves can affect the propagation of electromagnetic radiation [8].

4. Applications

Based on the three different gravitational wave detection methods mentioned above, many different projects around the world have deployed their detectors, and some have successfully observed multiple gravitational waves.

4.1. Weber Bar

When the original Weber Bars were made, their measurements were simply not accurate enough to detect gravitational waves. After years of improvements, Leiden University has deployed a gravitational wave detector called MiniGRAIL. MiniGRAIL is different from conventional Weber Bars in that it is spherical rather than cylindrical. The advantage of this is that it can detect gravitational waves from all directions [9]. However, its accuracy was still insufficient to accurately detect gravitational waves, and the MiniGRAIL project was eventually terminated in 2005.

4.2. Laser Interferometers

There are currently many different laser interferometer projects in the world. Among them, the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the Virgo Interferometer (VIRGO) are the two most sensitive laser interferometers [10]. Currently, LIGO and VIRGO are responsible for the majority of gravitational wave observations that have been recorded. Furthermore, gravitational waves were detected by LIGO for the first time in 2015. About 1.3 billion light-years away, two enormous black holes merged to produce this gravitational wave signal. The first observation of gravitational wave signals from a binary neutron star system merging was reported by LIGO and VIRGO in 2017.

Meanwhile, there is another project to build a laser interferometer in space called the Laser Interferometer Space Antenna (LISA). This interferometer consists of three spacecraft, each carrying

two sets of interferometers. Lasers would be emitted from one spacecraft and eventually reach another spacecraft. The advantage of LISA compared to LIGO and VIRGO is that it can isolate many noise interferences on the earth in space and achieve clearer gravitational wave detection [11]. At present, LISA is still in the preparation stage and has not yet started construction. One of the sketches is presented in Fig. 2 [12].

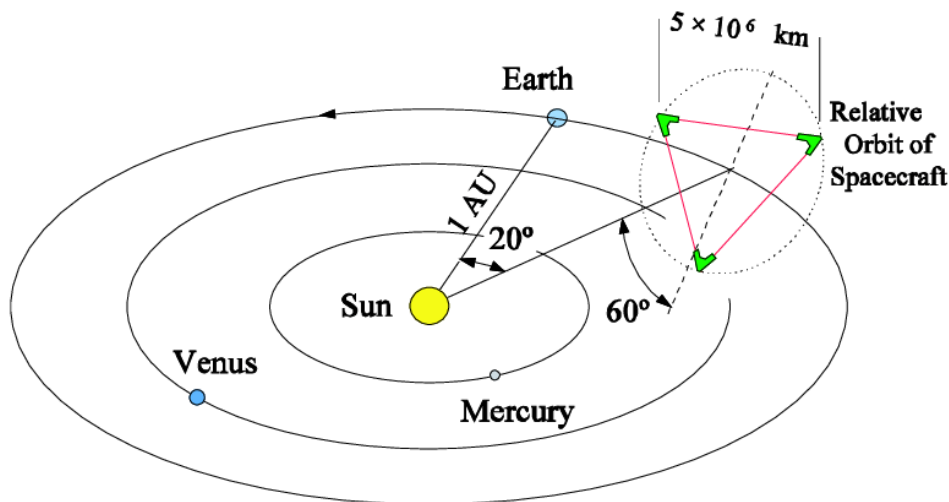


Fig. 2 Laser Interferometer Space Antenna [12].

5. Limitations and Future Outlooks

At present, all gravitational wave detectors have a common serious problem, which is the problem of sensitivity. Most gravitational wave observatories are still unable to detect weak signals from very distant astronomical events. Second, existing gravitational wave detectors are only sensitive to gravitational waves in certain frequency ranges. Gravitational waves with low frequency generated by events such as the merger of massive black holes or high-frequency gravitational waves emitted by rapidly rotating neutron stars are difficult to detect on the ground. Detector detected. At the same time, there are many noise sources on the earth, such as seismic activity, which will affect the detection of gravitational waves.

Currently, both LIGO and VIRGO are preparing upgrade plans, called Advanced LIGO and Advanced VIRGO respectively [13]. The goal is to more than double the existing detection sensitivity. The main methods include improving the reflector to lower thermal noise, better compressed light performance, etc. At the same time, the LIGO project will also build a high-sensitivity laser interferometer with the same configuration in India in the future to detect more gravitational wave signals. In addition to the upgrade plans of these two existing gravitational wave observatories, there are two other gravitational wave detectors under preparation and construction, namely the ET in Europe and the CE in the United State [14]. The design of ET is similar to LISA. It also uses a triangular structure to deploy three interferometers, but it will be built underground, and the interferometer arm length will be increased to 10 kilometers. Being built underground can effectively reduce noise during the detection process and obtain better gravitational wave detection results. The design of CE is similar to LIGO and others. It is also two light storage arms arranged in an L shape, but the arm length will be increased to 40 kilometers. The arm lengths of current gravitational wave observatories are only 3-4 kilometers, so this can greatly improve the accuracy of detection. Meanwhile, preparations for the space-based observatory LISA are also underway. Gravitational wave measurements in space can also isolate a lot of noise, and it is easier to build longer arms in space than on the ground or underground. Therefore, these new generation gravitational wave

observatories that are being prepared to be built will well solve the current problem of insufficient sensitivity in gravitational wave detection and detect more and more precise gravitational wave signals for mankind.

6. Conclusion

In conclusion, the analysis of the principles, facilities, and applications of gravitational waves underscores their profound impact on the understanding of the universe. Through the groundbreaking work of researchers and the development of sophisticated detection facilities such as LIGO and Virgo, scholars have unlocked a new window into the cosmos. Gravitational waves not only confirm Einstein's general theory of relativity but also provide unprecedented insights into the most energetic and violent cosmic events, e.g., collisions of neutron stars. Moreover, the potential applications of gravitational wave research extend beyond astrophysics, with implications for fundamental physics, cosmology, and even technological innovation. With continuing to refine the techniques and expand the capabilities in detecting and interpreting gravitational waves, one stands poised to unravel even more mysteries of the cosmos and usher in a new era of discovery.

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