

From Electromagnetic Wave to Special Relativity: Insight into Modern Physics

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Abstract. The aim of this work is to apply the Maxwell's theorems on electromagnetism, and it also include the proofs and functions of these equations. Therefore, special relativity that developed at the beginning of last century by a group of successful physicists such as Elbert Einstein, entirely refreshes people's mind about the surrounding world will be discussed. This work will justify how special relativity refresh people's mind and the significance of special relativity before the establishment of general relativity. Firstly, there is a short introduction to this article, including the background of theorems that is about to be discussed, along with significance of them and how they dramatically change the classical physics and the contributions of these great theories. Throughout the paper, a series of equations and concepts will be given as the fundament of electromagnetism in which relativity is temporarily negligible. Furthermore, it also provides information about several very basic theorems in special relativity, ending with Lorentz transformation and mass-energy equation.

Keywords: Maxwell's equations, Electromagnetic fields, Special relativity, Mass-energy equation.

1. Introduction

This paper mainly includes the basic information about the progressing electromagnetic waves starting with several fundamental theorems that are related to the electromagnetic wave, to demonstrate the major identity such as the components, mathematic laws and the progression of the electromagnetic wave. By giving explanations around Maxwell's equations by James Clerk Maxwell in Edinburgh and Cambridge in which the properties of electromagnetic fields will be discussed and the mutual excitation of electric and magnetic fields [1].

After that is the special relativity developed in the last two centuries. From the fixed ether theory to the mass-energy equation, a part of this paper demonstrates the evolution and limitation of special relativity. At low speed, an observer can rarely realise the effect of Lorentz contraction of space and time, which contributes to the generation of Galilean invariance that all of objects in space share an absolute time, by differentiate the positions of objects in frame of references to achieve a common acceleration [2]. Nevertheless, they undergo different times as they are moving relative to each other. Thus, there are two postulates of special relativity in order to formulate the contraction. In this article, there will be more detailed information.

The contribution of those new developed theories in early last century is significant. The Maxwell's equations formulate almost all phenomena in electromagnetism by using ∇ operator, stating the fundamental patterns of rest and varying electromagnetic fields. Moreover, special relativity also alters the views in classical physics and lays a solid foundation for modern relativity. In section 2, further information will be provided on electromagnetism. After section 2, special relativity will be discussed in section 3. The last section is for conclusion.

2. Electromagnetism

2.1. Basic Concepts of Electromagnetic Wave

The electromagnetic wave has two components, which are varying electric field and varying magnetic field. It has frequency, period, wavelength, and phase difference, which highlight the wave properties such as refraction, diffraction, interference, Doppler effect and so on. By the end of the

19th century, numerous scientists also stated that every material has wave-particle duality. In terms of that, electromagnetic waves can behave like particles and a series of experiments has proved these phenomena (photoelectric effect and so on) [3]. This essay only indicates the wave property of the electromagnetic wave by giving words about electromagnetism, based on Maxwell's equations. It explains the special relativity which is strongly corresponding to the electromagnetic waves.

2.2. Maxwell's Equations

The propagation of electromagnetic waves relies on varying electromagnetic fields, which were formulated by Maxwell in 1865. The so-called Maxwell's equations are given by [4]

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad (1)$$

$$\nabla \cdot B = 0 \quad (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

$$\nabla \times B = \mu_0 j_D + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}. \quad (4)$$

The propagation of the electromagnetic waves is mainly related to the Eq. (3) and Eq. (4). They state the curl of the non-electrostatic field and the vibrating magnetic field. For the first two equations, the divergence of the electrostatic field is the charge density of the enclosed surface divided by the permeability in free space, mathematically supported by Gauss's Law

$$\oiint E \cdot dA = \iiint (\nabla \cdot E) dV. \quad (5)$$

To differentiate this equation (the left-hand side is the electric flux through the enclosed surface), the Maxwell's equation is obtained. In this way, the electric field strength can be easily calculated by this equation via defining an enclosed surface and finding the net charge inside the surface. In terms of the second equation of Maxwell's, the divergence of the stable magnetic field is zero because it is not able to form a monopole, which means there is no magnetic charge in this universe.

The Eq. (3) is originated from the Faraday's Law of electromagnetic induction. The change in magnetic field is responsible for the electromotive force, generating a potential difference in a loop

$$\oint E \cdot dl = - \iint \frac{\partial B}{\partial t} \cdot dS. \quad (6)$$

According to the definition, the potential difference is equal to the linear integral of the electric field, equivalent to the minus changing rate of magnetic flux, defined by the change in magnetic flux divided by the change in time. The left-hand side is equal to the integral of the curl of non-electrostatic field and the right-hand side is minus change in magnetic flux. Then differentiate both sides of the equations, the curl of the electric field equal to minus the changing rate of the magnetic flux density. This is how the varying magnetic field produces the non-electrostatic field in free space.

To find the consequences of a non-electrostatic field, Ampere's law gives some clues. Based on Kelvin-Stoke's formula, it converts the linear integral into an area integral

$$\oint E \cdot d\Gamma = \iint \nabla \times E \cdot d\Sigma. \quad (7)$$

The magnetic field has been able to be calculated by Ampere's Law. The linear integral of magnetic flux density on the enclosed curve in the space is equivalent to the curl of the magnetic field, which equals the permeability in free space times the current enclosed by the curve. Similarly, differentiating both sides of the equation, the curl of the magnetic field is equal to the permeability times the charge density. The first term of Ampere's Law states that current induces a magnetic field around the current, while Maxwell's Displacement current postulate gives opinions that the changing electric field behaves like a current. The postulate assumes that there is an additional term after the first term in

Ampere's Law (permeability times the charge density), this term originated from the conservation of the charge.

By solving simultaneous equations of conservation of charge and the divergence of the electrostatic field [5],

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0, \quad (8)$$

partial charge density/partial time equal to partial divergence of electric field /partial time times permeability in free space, $\nabla \cdot \nabla \times B = \nabla \cdot [\mu_0(j + j_D)] = 0$. Then substitute in conservation of charge, $\nabla \cdot j + \nabla \cdot j_D = 0$. By solving a simultaneous equation, with $\nabla \cdot E = \frac{\rho}{\epsilon_0}$, so $\nabla \cdot j_D = \frac{\partial(\epsilon_0 \nabla \cdot E)}{\partial t}$, then cancelling the nabla operator for each side, $j_D = \epsilon_0 \frac{\partial E}{\partial t}$. This equation highlights that a current or a varying electric field contributes to the generation of the magnetic field, which is one of the main causes of the propagation of electromagnetic waves.

3. Special Relativity

3.1. An Introduction to Special Relativity

Until the last century, academic people supposed that the surrounding was stuffed with a continuous elastic medium named "ether", as the medium of many rays and radiations, sounding like the pressure waves in air. Ether is elastic, and the speed of light should be higher to someone on a spacecraft displacing towards it, and slower on another spacecraft travelling in the same direction with light. A series of experiments tended to argue with this idea, the most typical and rigorous experiment was the Michelson-Morley experiment hold by Albert Michelson and Edward Morley at the Case School of Applied Science in Cleveland, Ohio in 1887. By comparisons between two beams of light and they were perpendicular to each other, the apparatus was moving through the ether with variable velocity due to the spin and the revolution of the earth (the motion of the solar system, Milky Way and so on are taken into account as well because nothing is absolute stationary in the universe), but Michelson and Morley had found no daily or annual differences between these two beams of light, no matter how quick and directions they were moving [6].

A new idea was developed by the Irish physicist George FitzGerald and the Dutch physicist Hendrik Lorentz. The bodies moving through the ether would contract and the clock slows down that will be mentioned in Sec. 3.2. People measure the same speed of light. In June 1905, Albert Einstein pointed out that the laws of physics should appear to all freely travelling observers, which is independent of the speed of light, so everyone has a personal time rather than sharing a universal time only except when they are at rest concerning each other. An experiment was carried out to confirm this idea: two aeroplanes travel around the earth with only a difference in direction. The clock in the aircraft flying towards the west records more time than travelling in the opposite direction.

3.2. The Theories About Special Relativity

The postulates of special relativity are the following. One is that the laws of physics are invariant in all inertial frames of reference, while the other is that the speed of light in a vacuum is the same for all observers, regardless of the motion of the light source of the observer.

3.2.1 Galilean Invariance

To estimate the differences between two reference frames of relative motion to each other, Galileo Galilei was the first one demonstrated this principle in 1632. If there are two reference frames in the space. The laws of motion are the same in all inertial frames of reference, and Axioms from Newton's Theories must be true that firstly, absolute space exists, where Newton's Laws are true. An inertial frame is a reference frame in relative uniform motion to absolute space, following that all inertial frames share a universal time (theoretically).

In two inertial frames S and S' , events can be denoted by (x, y, z) and (x', y', z') , t is considered as the shared time in these two frames of reference. Suppose S' is in relative uniform motion to S with velocity v , a point object whose position is given by functions $r'(t)$ in S' and $r(t)$ in S . In the primed frame of reference, an event in un-primed frame is $r'(t) = r(t) - vt$. In mechanics, velocity is the differentiation of position in respect of time, so $v'(t) = \frac{d}{dt}r'(t) = \frac{d}{dt}r(t) - v = u(t) - v$ is the observed velocity of the event in S . Similarly, the acceleration of this event: $a'(t) = \frac{d}{dt}u'(t) = \frac{d}{dt}u(t) - 0 = a(t)$. Assuming mass is invariant in all inertial frames, the above equation shows Newton's Laws of mechanics $a(t) = a'(t)$.

3.2.2 Lorentz Transformation

As the above-shown formulas are only feasible at low speeds, the accordance of special relativity is still insufficient since in most situations in relativity, the speed of the object seems very significant. The Lorentz transformation solved this problem by introducing a new parameter γ , to find the relationship between the universal time and the recorded time in each reference frame.

In Lorentz transformation, a parameter group of linear transformations from a coordinates frame in space-time to another frame that moves at a constant velocity relative to the former. In the transformation, the primed frame moves in the x direction as shown in the following equations [7]:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \tag{9}$$

$$x' = \gamma(x - vt) \tag{10}$$

$$y' = y \tag{11}$$

$$z' = z \tag{12}$$

The (t, x, y, z) and (t', x', y', z') are coordinates of an event in two frames with the origin coinciding at $t=t'=0$, and the velocity of the primed frame is v along the x -axis relative to the unprimed frame $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}} \right)^{-1}$ is the Lorentz factor.

To prove it, four scenarios must be considered in Lorentz transformation. In the first scenario, two inertial reference frames have different times t and t' , and the primed frame travels away from the unprimed frame in the x -direction at a constant velocity v .

In the scenario 1, there is an event that happens at some point in the coordinates system. $x' = Ax + Bt$ and $t' = Dx + Et$ are the observed position and time. In terms of the primed coordinates system, the event is denoted by $x'=0$, so in the other coordinates system, it will be $x=vt$. (Parallelogram). After that is simple algebra, substituting the values of x and x' , $0 = vt + Bt$, so the following equation is the first relationship between four undefined constants.

$$B = -Av \tag{13}$$

In the scenario 2, another event happens in the space. For this time $x = 0$, then $x' = -vt'$, and substitute the equation gained in scenario 1. Substitute $x' = A(x - vt)$ and $t' = Dx + Et$ in $x' = Ax - Avt = A(x - vt)$, the following relationship is obtained

$$A = E \tag{14}$$

In the scenario 3, by considering the situation of high speed, a light impulse is introduced into the model (the speed of light is supposed to be the same in every inertial frame of reference) the light impulse in two frames of reference is described differently, $x = ct, x' = ct'$. Substitute in the equation that came out from scenario 1 and 2 respectively, $ct' = A(ct - vt)$, $c(Dct + At) = A(ct - vt)$, as a common factor t in both sides of the equation, they are cancelled out, so $D = -\frac{Av}{c^2}$.

So far, there are two equations is gained, $x' = A(x - vt), t' = A \left(-\frac{vx}{c^2} + t \right)$.

Furthermore, the last is the scenario 4. If the light impulse travels in y-direction. Besides, $y = y', x = 0$. Then $(x')^2 + (y')^2 = (ct')^2$. In the process of calculation, $x = 0$ is substituted in the equation, and t squared are all cancelled out as a common factor of both sides of the equation

$$A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{15}$$

in which A is known as the constant γ .

3.2.3 Mass-Energy Equation

If research has been carried out on a street in a village that how much percentage of people know the equation of $E = mc^2$, almost 100% of them know about it. However, when the researchers ask for the application of this equation, just a few villagers can give a response, and have no idea about how to prove this equation.

This equation can be divided into two completely diverse parts: the kinetic energy and the rest mass (or energy) of an object. As widely accepted, the formula for kinetic energy is one over two times mass times velocity squared, so some people believe that the kinetic energy of this object equals the rest energy of it, but unfortunately, these people need to improve their thinking, because the term “c” represents the speed of light. For example, if a particle travels at a speed of over 2 million metres per second, the kinetic energy of this particle must be less than its rest energy as its speed can only less than the speed of light. Thus, the rest energy is more than kinetic energy. (The same for macroscopic objects) [8].

The other way to calculate the particles’ energy is to calculate the Broglie wavelength of the particle by momentum is equal to the Planck constant divided by wavelength and use the formula $E=hf$ to calculate the energy of the particle (due to the wave-particle duality). Similarly, $E=mc^2$ can be used to estimate the “mass” of a photon as it’s relatively easy to find the wavelength of the photon. To prove the equation $E = mc^2$, energy can be expressed in work function $E = W = F \cdot s$. More precisely, $E = \int F \cdot ds$. The expression of force is the derivative of momentum is accepted in special relativity, so $F = \frac{dp}{dt}$ (p is momentum), which can be written as $F = \frac{d(mv)}{dt} = \frac{dm}{dt}v + \frac{dv}{dt}m$ according to product rule. Energy $E = \int v^2 dm + mv dv$.

For a mass m , the relationship between it and its rest mass m_0 is [9]

$$m = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0. \tag{16}$$

Differentiate the mass about velocity, $\frac{m}{dv} = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \left(-\frac{1}{2}\right) \left(-\frac{v}{c^2}\right) = \frac{m_0 v}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{v}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{-1} = \frac{mv}{c^2 - v^2}$. As a result, $c^2 dm = v^2 dm + mv dv$. Ultimately, substitute in the expression of energy, $E = \int c^2 dm = mc^2$.

This equation is used to calculate the rest energy of a stationary object, so this equation is not allowed under the condition that the material is moving. Fortunately, the mass-energy equation has been improved over and over again in general relativity. For instance, if materials with no rest mass, there is an additional term after mc^2 : $E^2 = (mc^2)^2 + (pc)^2$, in which p is the momentum of material, calculated by $p = \frac{h}{\lambda}$.

3.2.4 Limitation of Special Relativity

There is a paradox that oppose the traditional theory in Galilean invariance called Twins Paradox. This is a theory that each observer has personal time led to the twin’s paradox. If there is a pair of twins on the ground (they are in same age), one of them (1) leaves the earth by taking the spacecraft

that travels close to the speed of light, while the other one (2) stays on the earth. Due to (1)'s speed, the time recorded by him is relatively longer than his brother's, so (1) will realise that (2) seems to be much older than himself because (1) experiences shorter time. Thus, many experiments have shown that the traveller is expected to be younger. This problem was solved in Lorentz transformation as a rest object undergoes more time relative to a moving object due to Lorentz contraction on time.

In terms of Lorentz transformation, the Newton's second law $F = ma$ is not feasible in special relativity, which states that the mass of an object contracts as well. As a result, the rest energy is calculated, but still, limited because this can only calculate the rest energy of this object, that means when the velocity becomes non-zero, mass-energy equation does have a defect.

4. Einstein's Electrodynamics of Moving Objects

4.1. Special Relativity and Maxwell's Equations

The transformations of several basic quantities have been worked out in this paper, including position and time for a moving object in a moving frame of reference. Yet, there is still a mystery in the proof of mass-energy equation, that is, $m = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0$. In Lorentz transformation, this can only

be applied to change in position in x direction but not mass. Thus, there is a relatively rigorous method to introduce special relativity to dynamics. As the topic of this section, electromagnetism is the best way to do this, which chosen by Albert Einstein, showing the connection between electromagnetism and mechanism based on the results that have come out before it.

For this connection, Newton's Laws is partially rejected initially, but not the axioms of mechanics. For instance, acceleration is the differentiation of velocity on time, and force is the differentiation of momentum on time due to momentum theorem: $\int F \cdot dt = \Delta p$, which is accepted in special relativity. For the velocity that is seen by an observer in unprimed frame of reference, $u'_x = \frac{u_x - v}{1 - \frac{v}{c^2} u_x}$.

In x direction, unprimed acceleration is equal to the changing rate of unprimed velocity $a_x = \frac{du_x}{dt}$. Mathematically, that is equal to $\frac{du_x}{du'_x} \frac{dt'}{dt} a_x$. For the term $\frac{du_x}{du'_x}$, applying a quotient rule to gain a equation in which there are only unknown velocities and constants, and work out $\frac{dt'}{dt}$. Then

substituting into the overall equation, $a_x = \left[\frac{\sqrt{1-\beta^2}}{1 + \frac{v}{c^2} u_x} \right]^3 a_x$.

4.2. Maxwell's Equations in a Moving Frame of Reference

As mentioned in the last paragraph, $F = \frac{dp}{dt}$ is accepted in special relativity so investigating an elastic collision might be an easy way to justify some relationships between p and p' , or something else. However, this is not that rigorous because elastic collision is almost generated from Newton's classical mechanics, which are rejected by relativity. In contrast, electromagnetic forces are more basic and more comprehensive so that better than just simply writing down some equations on the paper and see what happens after the collisions of two ideal objects. Einstein finally chose to apply Maxwell's equation, as a resultant force of electromagnetic forces can be expressed as $F = q(E + v \times B)$ (E , v , B are vectors). People also argue that in relativity, the charge may be no longer q , but there is a fact that when a particle accelerator fires a neutral particle, there will be no partial charge. Maxwell's equations have been explained in Section 1, but here only two of them is necessary: $\nabla \cdot B = 0$ and $\nabla \cdot E = -\frac{\partial B}{\partial t}$.

In expansion, the Maxwell's equations are [10]:

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad (17)$$

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -\frac{\partial B_x}{\partial t} - \frac{\partial B_y}{\partial t} - \frac{\partial B_z}{\partial t}. \quad (18)$$

In these equations, divergence and curl of magnetic and electric fields are denoted respectively. To find the relationships between two frames of reference, total differential is used. $\frac{\partial}{\partial x} = \frac{\partial}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial}{\partial t'} \frac{\partial t'}{\partial x}$, $\frac{\partial}{\partial y} = \frac{\partial}{\partial y'} \frac{\partial y'}{\partial y} + \frac{\partial}{\partial t'} \frac{\partial t'}{\partial y}$ and $\frac{\partial}{\partial z} = \frac{\partial}{\partial z'} \frac{\partial z'}{\partial z} + \frac{\partial}{\partial t'} \frac{\partial t'}{\partial z}$. So those equations gained in Lorentz transformation works here, to find the expression for partial x coordinates and time. Then substitute into the Maxwell's equations above, there will be four extremely complicated equations. Nonetheless, the power of human's intelligence, or rather Einstein's is very enough to simplify those four equations and find out how every single quantity be like in the new Maxwell's equations: $E_x = E'_x$, $B_x = B'_x$, $E'_y = \frac{E_y - vB_z}{\sqrt{1-\beta^2}}$ and so on.

4.3. Dynamics and Advanced Mass-Energy Equation

After defining the quantities in a moving frame of reference, they can be substituted in that basic equation, that is, $F = q(E + v \times B)$. Again, doing oceans of algebra, the expression between forces is established: $F'_x = \frac{F_x - \frac{v}{c^2}u_y F}{1 - \frac{v}{c^2}u_x}$, $F'_y = \frac{\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x} F_y$, $F'_z = \frac{\sqrt{1-\beta^2}}{1 - \frac{v}{c^2}u_x} F_z$.

As soon as forces are done, momentum is much easier to be explained. As $F = \frac{\partial p}{\partial t}$, $F'_x = \frac{\partial p'_x}{\partial t'}$. Rearranging this equation, $dp'_x = F'_x dt'$ and substitute the results that came out from above deduction from forces, the expression for the momentum of an object moving in another frame of reference has been algebraically calculated, $p'_x = \frac{p_x - \frac{v}{c^2}E}{\sqrt{1-\beta^2}}$, in which E is the energy of the object. For the reason why $p_y = p'_y$, $p_z = p'_z$, is because there is no motion in y and z direction in the scenario of Lorentz transformation and it is not concerned due to the complexity of algebra if do so.

In order to estimate the mass contraction in special relativity, the formula for momentum, velocity and mass is used in y direction (easier to calculate). $m' = \frac{p_y'}{u_y'} = \frac{p_y}{u_y} \frac{1 - \frac{v}{c^2}u_x}{\sqrt{1-\beta^2}} = \frac{1 - \frac{v}{c^2}u_x}{\sqrt{1-\beta^2}} m$. If $u_x = 0$, m is stationary relative to observation, which means the object is stationary in observer's point of view, so $m' = \frac{m}{\sqrt{1-\beta^2}}$.

If the object is not stationary, instead, moving. The energy of this object is the combination of kinetic energy and rest energy, in the way of Pythagorean theorem $E^2 = (mc^2)^2 + (pc)^2$. Let m_0 and m are the mass of the object before and after Lorentz expansion, kinetic energy is $mc^2 - m_0c^2$, $E = mc^2$. So $E\sqrt{1-\beta^2} = m_0c^2$, $E^2 = (m_0c^2)^2 + E^2 \frac{u^2}{c^2} = (mc^2)^2 + (pc)^2$. This formula is clearly more comprehensive and accurate, even the energy of light can be calculated because photons are massless, but this is not the final stage of this equation, as many physicists believe.

5. Conclusion

In conclusion, this paper mainly gives information about the structure and several properties of electromagnetic waves, especially when the wavelengths of the waves are in visible range. However, there is no word related to fundamental properties about electromagnetic waves, such as reflection, refraction, diffraction, double slits interference, or Doppler effect. Moreover, more details are needed for the propagation of electromagnetic waves, and the author has attempted to tell the main story of electromagnetic wave. For example, general relativity, and the applications of electromagnetic

radiation are expected to appear in the passages as well, such as a very appealing fact that people are living in a universe filled with background radiation originated from the Big Bang of numerous years ago. The new horned universe is still in the Planck period and the strong nuclear force is the major way that particles 'communicate'. This article briefly outlines the development of modern physics, which is entirely diverse from classical physics that people think during old days, represented by Galilean invariance and Newton's motion. There are still many theories that are significant but not mentioned in this article, such as Einstein's electromagnetic mechanism in special relativity. This tells stories that how special relativity works in Maxwell's equations, and a combination of mechanics and electromagnetism, which makes good connection between two topics in this essay. Moreover, this paper also introduced how the Lorentz transformation is applied to the contraction and expansion of several physics quantities and outlined the energy of a moving object.

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