

The Connection Between GUT and Black Hole

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Abstract. As one of the most important models of modern physics, Grand Unified theory (GUT) is not only a key to solving the mysteries of particle physics but also will help people to have a deeper understanding of the early days and the evolution of the universe with the direction of astrophysics. And black hole is one of the most mysterious objects in the universe, the study of it also helps scientists deduce many properties of the universe. There is much research on GUT and black holes at present, but much of it stays on theoretical conjecture. GUT waits for a unified theory which can include gravitation, while the confirmation of the model of black holes is still at its surface. This paper will focus on the two fields above, trying to discuss the connection between GUT and black holes through present models and conclusions, summarize the developments of the two fields and their related fields at present, and expound on the existing achievements and models. Although GUT seems unrelated to black holes, they have a mutual causal relationship in the early universe, and they can solve many questions of the other fields by their conclusions. People can have a more comprehensive overview of the universe through the research of gravitational waves based on GUT and the model of black holes, and maybe find some new particles to perfect the existing models and conclusions.

Keywords: Grand Unified Theory; Black Hole.

1. Introduction

In the process of exploring the mysteries of the universe, GUT and the black hole are two remarkable fields. GUT aims to unify four basic forces that have been concluded by scientists in a set of theories [1], while a black hole, as one of the most mysterious objects in the universe, its extreme physical environment provides a unique perspective for humans to have a deeper understanding of the universe. The investigation into these two fields can not only expand scientific knowledge but also let people have a more in-depth exploration and understanding of the essence of the universe. This paper aims to have a profound discussion of the connection between GUT and black holes and try to know more about the essence of the universe through investigation.

The origin of GUT can be traced back to the desire of scientists to unify all forces. Since the beginning of the 20th century, physicists have been dedicated to bringing electromagnetism, the weak force, the strong force, and even gravity into a unified framework [1]. The basic conception of GUT is to find out the conceptual framework above. Based on theory and experiments, scientists have found ways to unify other forces except gravitation and electromagnetic force, and gravitation which is left behind, is the key to perfecting the theory of GUT [1]. The application of GUT is a wide range, including particle physics, cosmology, and many fields of high-energy physics experiments.

The existence and formation of black holes are key subjects in astrophysics. According to Einstein's theory of general relativity, a black hole is what remains after a star has undergone complete gravitational collapse [2]. The basic characteristics of a black hole include powerful gravitation, the horizon singular point, etc. According to the different properties such as mass, angular momentum, and electric charge, black holes can be divided into different types [3]. The investigation into black holes can help people understand the behavior of matter under extreme physical conditions, it can also provide essential evidence for fields like gravitational wave detection and cosmic ray study.

The connection between GUT and black holes is mainly reflected in two aspects. On the one hand, GUT provides a new perspective on understanding the particle interaction inside black holes. Through the description of GUT's function, researchers may reveal the mechanism of interactions between particles inside black holes and consequently have a further understanding of the properties of black

holes. On the other hand, as an extreme physical environment in the universe, black holes provide a unique space for experiments to verify the correctness of GUT. By observing phenomena such as radiation and accretion, researchers can verify or optimize GUT. There are many models to find out the connection, for example, the critical state of decay black hole model may be the key to solving GUT's problems because it can provide theoretical support for unifying gravitation and electromagnetic force [4].

This paper aims to discuss models and conclusions of the connection between GUT and black holes, trying to discover underlying principles and probe into some phenomena in the universe. Although the investigation into the two fields has gained much process, there are still lots of mysteries left behind. Though seem unrelated, GUT and black holes can verify or solve questions that remain in each other's field, and provide new inspiration for promoting developments. Therefore it is necessary to investigate the connection between these two fields.

2. Grand Unified Theory

In the development of physics, scientists always try to unify all existing theories into one framework and find out their basic laws. GUT is evidence of this, from the unification of electromagnetic force and weak and strong force to the unification of gravitation and electromagnetic force, scientists have made a lot of attempts. After nearly a hundred of year's hard work, all forces were divided into basic forces at the end of the last century: gravitation, electromagnetic force, weak force, and strong force [1]. The four fundamental forces are divided into two kinds of forces: long-range force and short-range force. When the mass of the particle transmitting the force emitted by the matter particle is 0, forces can be exchanged over large distances, it's called long-range force [1]. Among the four fundamental forces, gravitation and electromagnetic force are long-range forces. When the mass of the transmitting particle is not 0, forces can't be exchanged over long distances, so it's called 'short-range force' [1]. Weak force and strong force are both short-range forces. According to the present investigation of the GUT, the research on the unity of the other three forces except gravity has made much theoretical progress.

2.1. The unifying of the electromagnetic and weak force

The weak force and electromagnetic force are the first and only forces that have been successfully unified. In the 1960s, Glashow, Weinberg, and Salam developed a theory of electroweak unity, using a gauge theory based on $SU(2)_L \times U(1)_Y$ to describe electroweak symmetry [5]. Through the spontaneous symmetry breaking mechanism and the Higgs mechanism, part of charged and neutral gauge bosons gain mass, and serve as 'transmission particles' for short-range forces; but the photon is still massless, it is responsible for the transmission of electromagnetic forces [5]. That is to say, the key point of unified electroweak theory lies in bosons. When a boson gains a certain of energy, its mass approaches 0, and the boson now is a gauge boson, which satisfies the gauge field theory. Bosons in this state act like photons, their interaction is long-range; while the energy of a boson is low, it possesses a large quality, so its interaction is short-range. Many subsequent experiments confirmed Glashow et al. 's unified theory of electroweak, such as the theoretically predicted $W^\pm Z^0$ gauge boson was discovered at CERN in 1983; the Higgs particle was discovered at CERN in 2012. The success of the unified theory of electroweak is the first milestone in the development of the unified theory, but there is still a long way to go from the overall goal of GUT.

2.2. Standard Model (SM)

Scientists call the model that unites the electroweak interaction with the strong interaction the Standard Model. In this model, fundamental particles are divided into two types: fermion and boson. Fermions include quarks and leptons, each quark has its corresponding antiquark, and they are involved in strong interactions; leptons mainly include electron, muon, tau, and their corresponding neutrinos, which are mainly involved in the electroweak interaction, each of which has its

corresponding antiquark [5]. Then there's the boson, which takes charge of transmitting basic interaction forces. Specifically, photon transmits electromagnetic force, gluons transmit strong force, while W and Z boson transmits weak force. Ultimately, the Standard Model also successfully predicted and discovered the Higgs particle. Higgs particle is an essential kind of particle in SM, its discovery is considered a milestone in physics. Higgs particle provides mass for other particles and is the only fundamental scalar particle in SM. Despite the great success of SM in describing known elementary particles and interactions, it still has some limitations. Initially, SM cannot explain how gravity behaves on a microscopic scale, which makes it impossible to fit perfectly with the goals of a GUT. Next, some parameters in SM need to be determined by experimental measurements rather than by theoretical derivation, which reduces the reliability of its theoretical predictions [1]. However, SM still devoted a lot to modern physics, and scientists put forward many new models according to its contribution, such as 1) deviation-standard model when $E > 200 \text{ GeV}$; 2) Superstandard models including supersymmetry, grand unification, new generation, etc. [6]. The super standard model provides a breakthrough for the realization of a grand unified theory.

2.3. Grand Unified Theory

None of the unified theories mentioned above include gravitation, however, how to unify it with the other three forces is the main question of GUT at present. This is because the gravitational field according to the usual field quantization approach will encounter the difficulty of renormalization, after all, general relativity has fundamentally changed the view of time and space, so its quantization should be fundamentally different from the quantum field theory method in direct spacetime [5]. However, the unification of gravity is not completely without breach, for example, gravity and electromagnetic force, although they are not the same from the macro performance and micro mechanism, also have very similar places. Initially, particles with an electromagnetic force on each other are usually thought to have an electric charge. Therefore it can emit or receive virtual photons, and create an electromagnetic force [1]. Apply this concept to all particles, then they can be thought to have gravitational charge. As they have gravitational charge they can receive or emit graviton and produce gravitational effects. Next, charged particles carry charges and move at variable speeds to excite electromagnetic waves. Einstein believed that matter particles carrying gravitational charges in variable velocities would also generate gravitational waves, and the concept of gravitational waves has been proven [1].

So far, scientists are trying to complete GUT in two ways, one is to unify Quantum mechanics and gravitation, and the other is to find new particles that can help to complete the model of GUT. For the first method, the existing theory of quantum gravity is based on one common ground between relativity and quantum mechanics: a basic principle of special relativity is the constant speed of light in a vacuum, and the first physical particle introduced in quantum mechanics is the quantum of light. To a certain extent, both can be considered to be based on photons [7]. Although this theory has some opposition and deviation from basic concepts and actual conjectures, scientists still got some inspiration in the process and obtained some theoretical results such as antiparticles.

The second method, finding new particles which are sometimes called superstandard particles. It is known that in the description of the Standard Model, the mass of particles derived from the gauge symmetry equation should be 0, but this is at odds with the fact. Scientists don't want to give up SM, so they put forward Higgs particles and the Higgs mechanism. The basic idea of the Higgs mechanism is to hypothesize that there is a kind of ubiquitous Higgs field in the universe when it interacts with other gauge particles, spontaneous symmetry breaking occurs because the vacuum state of the Higgs field is non-zero and the gauge particles gain mass and produced a Higgs boson which mass is not 0 [8]. After a series of experiments and research by scientists at CERN's Large Hadron Collider between 2011 and 2013, the Higgs boson was officially announced on March 14, 2013 [8]. This finding not only perfects the description of SM but also solves the question of symmetry breaking. According to the Higgs mechanism, whether the symmetry of the Higgs field is broken depends on the temperature. In the early Big Bang, when the temperature was high enough, the Higgs mechanism failed, and the

particles met the symmetry and did not have mass; subsequently when the temperature dropped, the Higgs field gave some particles mass, as can be observed today. Although the Higgs mechanism cannot fully explain GUT at present, it solves many contradictions and opposites and provides clues for future research based on improving existing theories.

In GUT, although the unity of the four fundamental forces is still unresolved, the creation and research of this theory have promoted the development of many branches of science and solved many problems that scientists could not explain before. At the same time, the ensuing research also brought many new questions and discoveries, constantly enriching human understanding of the universe, but also promoting a lot of follow-up research.

3. Black Hole

3.1. The Origin and Concept of Black Hole

After Einstein published the general theory of relativity, German astronomer Schwarzschild derived a critical radius of $r_g = 2GM/c^2$ (where G is Newton's gravitational constant, and c is the speed of light) based on Einstein's theory of the effect of gravity on light, which is called Schwarzschild radius [9]. Since then, Schwarzschild predicted a black hole: when the radius of the celestial body is smaller than r_g , nothing can escape from it including light [9]. In the 1930s, scientists Oppenheimer and Snyder pointed out: that when the nuclear fuel of a massive star is depleted and completely cooled down, without repulsive force produced by thermal expansion and radiation to balance the gravitation of its own, the body would collapse endlessly under the great power of gravitation. When it collapses to a critical size, it forms a closed horizon. Objects and radiations outside can enter the horizon, but things inside it cannot escape. In the end, it will turn into a 'black body' [9]. Later scientist Wheeler named this 'object that collapses under the power of gravitation' a black hole. Research shows that three mechanisms of reality may form a black hole. Initially, in the early age of the universe, high-density media can form small black holes with a mass of about 10^{12}kg due to density fluctuations. Next are black holes that are mentioned above, which collapse due to gravitation. Ultimately, supernovae, star clusters, and galactic nuclei may collapse under gravity into massive black holes with a mass of $10^4 \sim 10^9 M_{\odot}$ [10].

Instead of considering a black hole as a celestial body, it's more like a piece of special space in the universe. This space has a closed sphere, and its boundary is called the "event horizon". Everything can enter the horizon but nothing can escape from it. According to the black hole's features such as massive and horizon, scientists have long built a complete model of it, and its model is also corroborated by later "pictures" of black holes. People are familiar with the appearance and broad concepts of the black hole, but the research towards other deeper properties remains theoretical. According to theoretical calculations, the closer the black hole's event horizon, the slower time will pass, until finally, the clock stops. Scientist Stephen Hawking has made outstanding contributions to the study of the intrinsic properties of black holes, such as the entropy and temperature of black holes, the event horizon, and axial symmetry [9]. According to the concept of black holes and the symmetry of physics, scientists also proposed that there should be objects in the universe where objects can only go out white holes exist, and the concept and model of "wormholes" have been developed, constantly enriching the theoretical research centered on black holes. Although many studies on black holes and their derivative concepts are still theoretical, however, after scientists' continuously detailed research, these mysteries will be explained soon in the future.

3.2. Classification of Black Holes

At present, there are two kinds of classification of black holes, one is to classify according to different properties of gravitational sources under general relativity, and the other is to classify according to the mass of black holes. Initially, for the first kind, it divided black holes into four types. Schwarzschild's solution to Einstein's gravitational field equation assumes that the sources of gravity are spherically symmetric, it has a mass of M , it's stationary, not spinning, and not charged. The

gravitational radius is r_s , and black holes corresponding to such gravitational sources are called Schwarzschild black holes; then there is another class of solutions to Einstein's gravitational field equations by Leissner and Norstrom, unlike the former, these black holes carry electric charge Q , its gravitational radius is

$$rR = \frac{1}{c^2} [GM + \sqrt{G^2M^2 - GQ^2}] \quad (1)$$

this kind of black hole is called Leissner black hole; then the other is from Kerr's finding of the axisymmetric solution to Einstein's gravitational field equations. Different from the first type of black hole, this kind of black hole would spin, it has angular momentum and its horizon is

$$r_k = \frac{1}{c^2} [GM + \sqrt{G^2M^2 - a^2}] \quad (2)$$

(a is angular momentum per unit mass), this solution refers to Kerr black holes; last is the new theory discovered at present: stable black holes are axisymmetric, the gravitational sources of ordinary black holes are also axisymmetric, the three parameters that characterize black holes are mass M , charge Q , and angular momentum a and this kind of black hole is called Kerr-Newman [10]. The last kind of black hole refers to those kinds of black holes mentioned above according to different values of the parameters.

The second way of classifying black holes also divides them into four types: Stellar black holes, massive black holes in galactic nuclei, intermediate-mass black holes, and primary black holes (PHB) [11]. Stellar black holes are black holes formed late in the evolution of massive stars, they are the most easily inferred black holes. Giant black holes form when supermassive stars, galaxies, and even clusters collapse. The cause of the formation of intermediate black holes is not clear, its mass lies between stellar black holes and massive black holes. The last type is formed in the early universe by density fluctuations in the dense medium. Compared with the first three types of black holes, their mass is so small that it is completely impossible to form a new and stable existence in the current universe. Even if they do form, they will quickly evaporate and annihilate.

4. The Connection between GUT and Black Hole

4.1. Formation Mechanism of Primordial Black Holes Under D-parity Breaking Mechanism

As mentioned above, under the theory framework of GUT, the universe probably underwent a series of symmetry-breaking processes at an early age, then it formed the structure that could be observed today. The PBH mentioned above, which is formed at the early age of the universe, among these kinds of black holes, those with a mass less than $10^{15}g$ would evaporate completely in the current era, while the PBH with a mass of about $10^{15}g$ would evaporate in the current era, thus forming the cosmic gamma-ray background, galactic gamma-ray background, gamma-ray bursts, and radio bursts and some of them may gather in the galactic halo. Those PBHs with a mass of more than $10^{15}g$ still exist until the present time and they can be detected by gravitational effects. Since they originated in radiation-dominated eras, such PBH can also be used as non-baryonic dark matter candidates, and they can also act as seeds for supermassive black holes at the center of the galaxy [12].

In one study, researchers have proposed a new model of the formation of primordial black holes, in which they refer to the concept of 'Domain-wall (DW) evolution'. DW was formed by D-parity breaking in the early universe, and DW evolution inflects the local energy density in the universe by changing the distribution and variation of domain wall energy, further affecting the properties of quantum fluctuations and density fluctuations, thus potentially forming black holes [12]. Researchers used numerical simulation to simulate the dynamic evolution of DW in the universe, including the formation, growth, collision, and decay of DW, explored the effect of the DW on the energy density distribution of the universe and the threshold of forming black holes, thus study the properties and distributions of these black holes. During the simulation, researchers found that the supersymmetric

GUT, which satisfies the proton decay constraint and is stable for its hierarchy at the electroweak scale, is suitable for the study of the process of DW black holes. In particular, the model ADAPTS to the right-handed neutrino state, making the neutrino mass natural, while also including the accidental symmetric D-parity, which can exchange left-handed and right-handed [12]. Later, by modifying different parameters in GUT's model, researchers may be able to gain a deeper understanding of how these PBH forms.

4.2. The Search for Super Standard Particles

In the process of completing SM and GUT, scientists often fail to find the desired particle because the collider cannot produce energy that high enough. Black holes just have the extreme conditions that suit the requirements, thus they become a breakthrough to find new theories and create new models.

According to the nature that energy can be converted from a black hole, scientists Press and Teukolsky put forward the concept of a 'black hole bomb' [13]. Then scientists studied particles carrying the energy from the black hole's incident band of superradiation. As these particles can be brought out of a black hole's energy layer by reflected waves, their mass should be really small, therefore they're called ultralight bosons. Using Bayesian estimates, the variation of the spin of the black hole shows that the mass of the particle falls roughly in two intervals:

$$-12.16 \leq \lg \frac{\mu}{eV} \leq -12.05; \quad -11.83 \leq \lg \frac{\mu}{eV} \leq -11.76. \quad (3)$$

When the ultra-light boson at the Compton wavelength corresponds to the gravitational radius of the parent black hole, it would not escape from the gravitational range of a black hole due to the attachment of reflected waves, however, they would be captured by the black hole's gravity again after leaving the black hole's energy layer. Then they become part of the black hole's accretion disk and form a 'cloud' that has a high density, called an 'ultralight Boson cloud' [13]. Physicists use the K-G function under curved spacetime to describe this kind of cloud, then they found their structure is very similar to an atom, so they call the system of a central parent black hole and a boson cloud outside the black hole called 'gravitational atom'. To observe the gravitation atom, scientists introduced a companion star system into this system because in binary gravity atoms are 'stimulated by radiation', there will be a lot of ultra-light boson transitions released, and the energy they release would be stronger than the gravitational wave of a single particle, so it would be easier for observation.

As a type of super standard particle, an ultralight boson is no wonder a key to solving the mystery of GUT. Though this particle still cannot be observed through the methods mentioned above, the results of the research greatly reduced the difficulty of observation, and it also made the way of observing superstandard particles and gravitational waves clearer.

4.3. The Structure and Evolution of Black Hole

During the process of black hole decay, the four fundamental forces in GUT are involved, it also produces all kinds of particles. So the research into this progress can help to understand the composition and structure inside the black hole, and may further solve the question of the GUT. Based on the study of the graviton model of black holes, the decay of black holes and the internal structure of black holes are deeply discussed. In this model, the critical state transitions upward into a black hole by absorbing gravitons; Gravitons are critical states between classical black holes with minimum energies and quantum fields with maximum energies just below M_{Planck} , and it's the borderless remnant of a hot terminal radiating black hole, it's either stable or has a certain very short life span [13]. Through research into the gravitons model of black holes, the conclusion researchers came along is that: The GUT theory, based on $SO(10)$, provides everything needed to describe the universe from the Big Bang to the collapse of matter into black holes, the decay of a black hole will produce a bouncing phenomenon on all length scales, and later a GUT based on $SO(10)$ can be used to deal with the entire life cycle of particles in the (bouncing) universe [14].

The graviton model of black holes fills the blank of the study of the structure and behavior of Planck black holes, it surpasses the SM of particles, towards a GUT model of Planck-scale phenomena including gravity. Researchers also say that follow-up studies may lead to the discovery of new quantum particles.

4.4. About the Gravitation Wave

The research of the early universe not only includes models built from DW mentioned above, there are other scientists put forward new opinions from another perspective. Researchers unified inflationary dynamics with GUT, and provide an SU(5) inflation single-field inflection-point model. The inflation is a gauge-singlet of SU(5) which links to the SU(5) Higgs responsible for the Higgs-portal mixed quartic coupling that induces SU(5)-symmetry breaking. The SU(5) symmetry is similarly disrupted when the inflation approaches its potential minimum, resulting in a one-to-one connection between the GUT scale and the scale of the inflection point via the inflation VEV [15]. The researchers believe that this inflation pattern would have created many gravitational waves and PBH. The main object of study in this model is scalar-induced gravitational waves, different from primary gravitation waves that formed directly due to inflation, they may be caused by tensor perturbations of the second order through nonlinear effects, thus they're called scalar-induced gravitational waves. This kind of gravitation wave may be observed by some existing probes such as LIGO, VIRGO, KAGRA, and space probe LISA, and many scientists pay attention to it due to they carry a lot of information about the early universe. In addition, scalar-induced gravitational waves also have an important impact on the formation of primary black holes, dark matter and other cutting-edge issues in cosmology. Using the inflection point model described above, the researchers simulated the approximate amplitude and frequency of scalar-induced gravitational waves: the amplitude is $\Omega_{\text{GWh}}^2 \sim 5 \times 10^{-10} - 10^{-8}$, the peak frequency is $f_{\text{peak}} \sim (0.1 - 300)\text{Hz}$ [15]. In this model, the researchers believe that follow-up studies could advance humanity's understanding of primordial black holes. Also formed early in the universe, scalar-induced gravitational waves caused by second-order perturbations are now much easier to observe than primordial gravitational waves caused by first-order fluctuations.

In addition, black holes also form gravitational waves when they form, annihilate, or merge, this allows humans to observe, learn, and even confirm more information about black holes. Gravitational waves will become the starting point for subsequent observations and studies of the universe, and it is expected that they will lead to a better understanding of the universe and its early structure and appearance.

5. Conclusion

This article introduces the basic concept of GUT and black holes, it covers the details of the research that has been done in the Grand Unified Theory and the classification of black holes. After this, the relationship between the Grand Unified theory and black holes is summarized, and various models involved in related research are introduced. The connection between GUT and black holes exists in the early universe, it is closely related to some properties of symmetry of GUT. According to the theoretical model of symmetry breaking in the model, people can deduce a formation mechanism of the PBH, discuss the energy distribution in the early universe and some properties of this primordial universe; gravitational waves may have been generated during the formation of PBH, and these gravitational waves carry important information about the formation of black holes and conditions in the early universe. The connection between GUT and black holes also exists in modern times, mainly through the internal of black holes and the surrounding gravity of the atoms in search of new particles and trying to complete GUT. At present, many of the above research results are still in the theoretical stage or the conclusions are not very accurate, but they point the way for future research, believing that many of the results will produced in the future to help humans understand the mysteries of the universe further.

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