

The Flatness of The Universe Through Theory and Observations

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Abstract. Understanding the universe's shape is crucial for cosmology. Whether the shape is flat not only influences many other studies based on the assumption that the universe is flat but also helps people to build a better model to explain the evolution of the universe. This paper introduces several available methods to determine its flatness, focusing on the basic ideas and related theories. The paper mainly introduces the method of using the CMB's angular power spectrum to determine the universe's shape and gives the conclusion of a flat universe. Utilizing the lambda-CDM model, which suggests a nearly flat universe. Then the paper shows other ways of using other features like redshift and geometrical properties of different types of space (open, flat, or closed) directly to measure the curvature of the universe. These methods compared to using the angular power spectrum of the CMB are less independent of the correctness of theories and models, providing a different aspect to study the shape of the universe. Although there are debates, contemporary research leans towards the conclusion of a flat universe.

Keywords: Universe's shape; The flatness of the universe; Theory and observations.

1. Introduction

Studying the shape of the universe is a topic of widespread interest because it not only involves the understanding of the cosmic structure and evolution but also holds academic significance for the fields of astronomy and cosmology.

Back in 600 BC, human beings started to study and observe the universe, coming up with the geocentric theory, which said that the Earth was the center of the universe and that other celestial bodies orbit around it. In the 16th century, Nicholas Copernicus proposed the heliocentric theory. After that, with the explosion of technology, scientists develop more and more advanced theories with the help of complicated tools, such as various telescopes and supercomputer. Nowadays, people have a much deeper and broader view toward the universe. Supported by huge data and observations, more and more advanced models and theories come up to describe the universe.

The observation technology plays an important role in learning the universe. People develop different kinds of telescopes to capture radiation in different wavelengths, using them to observe distant stars and galaxies, as well as helping people to understand the history and future of the universe.

Among all of the cosmological research, the universe's shape is a very interesting one. It not only provides an overview of what the universe is like but also tightly related to many other properties of the universe. For instance, some observations show that the universe is almost flat, but there is not enough matter and energy to make it flat according to the relativity theory. This contradiction inspired people to rethink the Big Bang model and develop the cosmology constant Ω_A , which represents the density of vacuum energy. With the participation of matter, energy, and vacuum energy, it reaches an identical density to make the universe flat [1].

Back to this paper, will summarize several ways that determine the shape of the universe done by other previous researchers, and conclude some primary debates and ideas. According to the data, the lambda-CDM model agrees with most of the research, indicating an almost zero curvature.

2. Shapes of Different Universes

The shape of the observable part of the universe can be described primarily with the spatial curvature. The curvature can be greater, smaller, or equal to zero. Imaging a plane, if it has a positive curvature, it will be bent like a sphere. If it has a negative curvature, it looks like a saddle. If the curvature is zero, it will be just like a flat sheet of paper.

Likewise, a universe with positive curvature is finite but also lacks endpoints. In the situation with positive curvature, the mass is larger and exceeds the threshold necessary to halt its current expansion. Expansion will eventually stop and transition into contraction. Consequently, galaxies and stars will start to approach each other, and the universe will finally collapse. This is a closed universe.

A universe with negative curvature lacks the boundary and will continue expanding with insufficient mass to halt its expansion, constituting as an open universe.

Lastly, a universe with flat curvature, in which the mass present is precisely enough to eventually halt expansion. Thus, while the universe also lacks boundaries and expands perpetually, the rate of expansion keeps decreasing. This configuration is termed a flat universe or a Euclidean universe [2].

3. Flatness of the Universe

Geometrically, the sum of all angles of a triangle in the space can judge the spatial curvature. If the summation equals 180° , the space is flat. If the summation is greater than 180° , the space is closed. If the summation is smaller than 180° , the space is opened [3].

The basic idea of studying the universe's curvature is to calculate the density parameter Ω . It contains three parts: the mass density of ordinary matter and dark matter, the effective mass density of relativistic particles, and the effective mass density of dark matter. The value of the density parameter Ω is the summation of these three densities divided by the critical density required for the universe to be exactly flat. According to the general relativity theory, spacetime is bent by the presence of mass and energy, and space is flattened only if the average density of all matter and energy is equal to the critical density $\rho_{\text{critical}} = 9.47 \times 10^{-27} \text{ kg m}^{-3}$. Let the real density be divided by this critical density, then get the density parameter Ω . If the value of the density parameter is one, the real density equals the critical density, and the universe would be flat. If it is greater than one, then the curvature is greater than zero and the universe will be closed. If it is smaller than one, then the curvature is smaller than zero, resulting in an open universe [3].

One common way to determine these density values is by studying the cosmic microwave background (CMB). The cosmic microwave background is the microwave radiation that comes from any direction in the sky, which consists of a map of the background radiation in the sky called the cosmic microwave background. Compared to the light of stars, these radiations are very subtle, and people can only use optical telescopes to observe. They are very faint and almost uniform all over the universe, yet small variations (0.04% wavelength difference between the peak strength) can be still found with the most sensitive optical equipment [4]. These microwave radiations are originally from the recombination and cooling period of the universe. It is an imprint from the early universe and comparing it with the present universe provides a way to test if the theory indicates the same evolution. Thus, studying the cosmic microwave background is crucial to understanding the fundamental processes that shaped the universe's early stages.

To study the pattern of these microwave backgrounds, scientists apply Wien's displacement law, which tells that the temperature of the blackbody surface is inversely proportional to the wavelength emitted by the surface [5]. Since the early universe emitted photons very good to regarded as blackbody radiation, the temperature of this radiation is around 2.725 K by applying Wien's displacement law. However, the fluctuations of temperature are too small to observe, further processes are needed to enhance the fluctuations and wipe off interference from other phenomena. The dipole patterns on the graph are pairs of hot poles and cold poles, caused by the Doppler Effect when the solar system is actually in motion relative to the observable universe. After subtracting the

average temperature, removing all these dipole patterns, and multiplying each point with a big constant, the remaining temperature fluctuations can be seen on the graph [6].

From the temperature fluctuations map, which is also called the cosmic microwave background anisotropy, people can obtain the angular power spectrum with mathematical tools. The peak in the angular power spectrum of the CMB represents the point at which there is a maximum level of power or amplitude associated with temperature fluctuations across different multipoles. The peak position is dependent on the value of the total density of the universe. In a flat universe, theoretical predictions from the cold dark matter model (known as the Λ -CDM model) in a flat universe suggest that the first peak in the angular power spectrum should occur at around $\ell \approx 200$. This prediction is based on the idea that sound waves in the early universe, known as acoustic oscillations, leave characteristic imprints on the CMB temperature fluctuations, and the scale of these oscillations corresponds to the size of the observable universe at the time of recombination. And one of the research projects showing the same ℓ with an accuracy of 10%, indicates an almost flat universe [7].

Besides studying the cosmic microwave background, the geometry ways are also able to check the flatness of the universe. There are three methods: luminosity, Scale length, and numbers [8].

The luminosity method uses standard candles, which are objects of known intrinsic brightness. By observing these standard candles, such as the brightest quasars or certain types of supernovae, cosmologists can measure the brightness and determine their distance. By studying these objects out to high redshifts, scientists can infer the geometry of the universe. If the universe is flat, the relationship between luminosity and distance should match theoretical predictions.

In the method of the scale length, scientists use standard rulers, such as the size of the largest galaxies or galaxy clusters, to measure distances. By comparing the observed sizes of these standard rulers to their expected sizes based on theoretical models, scientists can infer the curvature of space.

The last method is to just count the number of galaxies within different distances. Since the universe is proposed to be isometry, the number of galaxies should increase linearly as the distance increases in a flat universe, while different in a universe with curvature not equal to zero.

However, one of the problems is the accuracy. For example, when testing the summation of angles of a triangle, the triangle needs to be giant and the measurement of each angle needs to be very accurate to get a precise result, which is pretty hard. On the other hand, some researchers are trying to improve the accuracy, but this may cause much difficult calculations in the meantime.

There are some ideas to improve this problem. One is to create a function that could show the relationship between the distribution of galaxies and redshift. Different from comparing data by counting the galaxies and redshift and other properties, presenting them as curves on the graph can be more accurate in making comparisons [9].

4. Discussion

Even though determining the universe curvature from the angular power spectrum of the CMB is becoming more and more very accurate, there are still some ideas thinking that the way of using the angular power spectrum of the CMB may be ambiguous. If the curvature parameter's value is less than approximately 10^{-4} , any study currently will enable us to accurately determine the true geometry of the Universe. In other words, if the curvature is close to zero, then it needs extremely accurate data and methods to judge if it is exactly zero or none zero, and hard to figure out the real topology shape of the universe [10].

In addition, there are some considerations regarding whether the theoretical model used to determine the curvature is correct. Among all models, the Λ -CDM model with six parameters performed the best. It outputs an almost zero curvature using different observation data in most research. the Planck team 2013 improved the Λ -CDM model, the results show that using the data from Planck and combining it with several other data as assistance comes out with the Ω_k close to zero with uncertainty smaller than 0.01 [11].

However, the debate still exists. Some original result from the Plank shows a negative Ω_k between -0.095 and -0.007, while after combining with the observation data from BAO and supernovas, the team adjusted the value to 0.0007 ± 0.0019 , which is closer to a flat universe. In addition, the results do not fit with the temperature power spectrum at low multipoles. In conclusion, though people already have a common agreement and accurate result that the Ω_k is close to zero, it still can tell if the universe is exactly flat. Moreover, there are a lot of inconsistencies in the results from different research using different methods which may cause by unknown problems in the models and theories [12].

Recently, one different research provided a more direct way to calculate the universe Ω_k . The main idea of the method is to directly measure cosmic spatial curvature from observations without relying on theoretical assumptions or complex modeling.

The team calculated the Ω_k using a single triangle formed by the observer, lens, and source. Through careful observation of distances between these points, particularly via time delays in lensed images, they derived it without relying on the Friedmann equations. In general, they tried their best to deduce the result with minimum uncertainty and as independent as possible from models which may include some problems. The findings suggest that the Ω_k is very close to zero with a precision of approximately 6×10^{-3} [13].

5. Conclusion

This paper provides an overview of the basic ideas and research done in the past 20 years, showing the ways to determine the shape of the universe according to the geometry and angular power spectrum of the CMB. From all of the research, one can conclude that the universe is almost flat. It seems that 20 years ago, scientists already determined the shape is almost flat based on the cosmic models and observation data. In the following decades, cosmologists not only developed models and used more accurate data but also tried different ways to study the shape of the universe. This overview also shows that scientist almost trying their best to reach the most accurate curvature they can, and it might not be such meaningful to continue measuring the curvature without any giant changes in the cosmic models. A guess is that the universe is just perfectly globally flat, which fits the inflation theories and other cosmic theories. Checking if the universe is flat is still important since many theories are developed based on the hypothesis of a flat universe, and dark energy is the reason that makes it flat.

In general, this paper as an overview, shows readers an idea of what is the common opinions toward the shape of the universe in the cosmology field. It also reveals some conflicts when using different data and methods to measure the curvature. These could raise awareness to develop a better model to better fit all the observations and methods.

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