Exploring Quantum Tunneling Phenomena: Theoretical Models and Technological Applications of Multi-Barriers

Yang He *

School of Sciences, Hangzhou Dianzi University, Hangzhou 310018, China

* Corresponding Author Email: 22070814@hdu.edu.cn

Abstract. This paper focuses on the quantum tunneling effect, which describes how a particle may penetrate barriers or obstructions using the Schrödinger equation. If a particle makes a barrier tunneling, it means that a particle with low energy goes through the barrier whose energy level is much higher. It is a basis or background for the entire theory of this part. The particle transport probability is explained using both single- and multi-barrier system models. The paper studies the relationship between barrier parameters and particle transmission probability for both single- and multiple-barrier system models.The aim of the study is the provision of the theoretical devices and algorithms via a quantum operational efficiency.Particularly, the paper deals with the multipath barrier tunneling effect, and it connects the transmission coefficients with the system parameters. The goal of this article is to come up with the new strategies and procedures that one can use the quantum phenomenon of the tunneling effect in the contemporary physical th applications, cutting the gap between theoretical physics and technology.This will be done by studying and analyzing theoretical processes and mathematics of this phenomenon along with the technological applications it has.

Keywords: Tunneling effect; multi-barrier tunneling; Transmission coefficient; Semiconductor.

1. Introduction

The advancement and comprehension of quantum mechanics theory over the last few decades has brought about profound shifts in people’s understanding of the tiny universe. The quantum tunneling effect, which is undoubtedly one of the most enigmatic and incomprehensible phenomena in quantum mechanics, causes a split in people’s belief in the accepted principles that nature adheres to as per the classical physics theories. Therefore, classical physics becomes disconcerted, and this effect find great application in fields, like semiconductor physics, nanotechnology, and quantum information processing. The mere fact that a particle is recorded in an experiment to have passed through a barrier, when the barrier has much higher energies than what the particle has, is explained with the quantum tunneling effect. In addition to being of great importance in theoretical physics, this unthinkable scenario, however, is also vital to the development of modern technology.

The complexity and range of applications of the tunneling effect increase when more than one possible barrier is taken into account, or when there are numerous barriers. When the energy of the incident particles matches the quantized energy levels of a multiple potential barrier structure, a phenomenon known as resonant tunneling occurs. In this scenario, particles can almost completely tunnel through the entire structure without reflection, while particles with other energy levels are reflected back and cannot pass through. This is analogous to the concept of bound states in a potential well, and the states corresponding to resonant tunneling are referred to as quasi-bound states [1]. Applications of multi-barrier tunneling are very significant in areas like semiconductor physics, nanotechnology, and quantum information processing. For instance, the foundation of non-volatile memory systems in semiconductor devices is the electrons' capacity to tunnel [2]. Moreover, the swift advancement in the field of quantum computing also depends on a thorough comprehension and use of the multi-barrier tunneling phenomena, in which the tunneling effect is involved in both the transmission of quantum states and the manipulation of qubits. Even though multi-barrier tunneling is widely acknowledged to be important, its precise control methods and intricate mechanics continue to be hot issues and problems in current research. As a result, thorough research on the subject of multi-barrier tunneling will be an important step in the development of fundamental physics and can be with important implications for the design and application of corresponding technologies.

The purpose of this research is to establish the principles on which the Schrödinger cat thought-experiment is based, to develop corresponding mathematical models, and to explore the physical applications of multi-barrier tunneling effect in order to provide new views and approaches in understanding and utilizing this quantum phenomenon.This paper intend to theoretically back the construction of highly effective quantum hardware and algorithms to reveal the relationship between transmission coefficients and parametric parameters of the system by meaningful analysis of multi-barrier systems.

2. Theoretical Basis

2.1. Quantum Tunneling Effect

The quantum computing concept of tunneling describes the ability that is seen in quantum mechanics of microscopic items to go through energy barriers that have the height higher than the energy of the objects.The theoretical basis of the discovery of this effect is a Schrödinger equation. The Schrödinger equation stands at the heart of quantum mechanics, being the mathematical background for representing quantum systems evolvements through time. For a non-relativistic particle, the time-independent Schrödinger equation can be written as

$$-\frac{\hbar}{2m}\nabla^2\psi + V(\mathbf{r})\psi = E\psi. \tag{1}$$

Here, the reduced Planck constant $\hbar$ is involved, m represents the mass of the particle, the Laplace operator ∇^2 performs a calculation with this operator, ψ is a wave function of the particle, the potential energy function $V(\mathbf{r})$ appears and the total energy is denoted as E.

2.2. Single-Barrier System Model

In quantum mechanics, the single-barrier system is one of the simplest ones and is known as the model where the quantum tunneling effect is being explored. This model is a system constituted of two infinite-sized-platforms and a potential barrier height of finite-size.It can also be viewed as an obstacle, that free particles will have difficulties crossing where the height and width will determine the probability of particle crossing.

In the single-barrier model, the potential energy function $V(\mathbf{x})$ can be expressed as:

$$V(\mathbf{x}) = \begin{cases} 0 & , \quad (x < a, x > b) \\ V_0 & , \quad (a < x < b) \end{cases}. \tag{2}$$

Here, in this case, $D = b - a$ is the obstacle's width, V_0 is the height of the obstacle, and x is the position coordinate of the particle. The stumble block is in the Fig. 1.

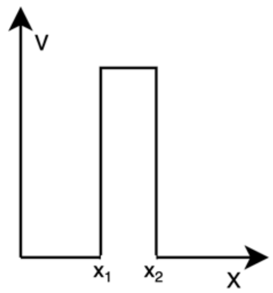

Fig. 1 Illustration of the single-barrier model.

In accordance with the way quantum mechanics views things, even if the energy E of a particle is lower the cause of the barrier V_0 there is still some probability that the particle can go through the barrier using the tunneling effect.This is an issue that is related to some physical properties of the wave function, which may survive and exist within the barrier region, thus giving the particle to appear on the right side of the barrier. By solving the boundary conditions of the Schrödinger equation, the transmission coefficient is obtained [3]

$$T = \exp\left(-\frac{2}{\hbar}\sqrt{2m(V_0 - E)}D\right). \tag{3}$$

2.3. Multi-Barrier System Model

2.3.1 Theoretical Analysis

A multi-barrier system consists of two or more barriers, which can be identical or different (for an illustration, see Fig. 2). In quantum mechanics, multi-barrier structures can be described by a series of potential energy functions $V(\mathbf{x})$, where each barrier and potential well has specific width and height [4]. These structures are very important for studying the quantum tunneling effect because they allow particles to tunnel through multiple barriers, thereby exhibiting complex quantum interference effects.

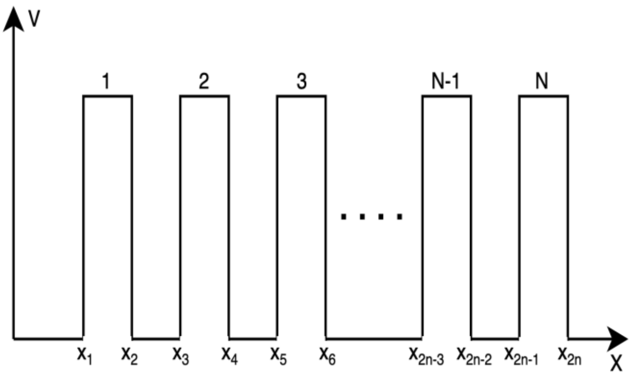

Fig. 2 Illustration of the multi-barrier model.

The one-dimensional Schrödinger equation for each region translates as [5]

$$\frac{\mathbf{d}^2\psi_j}{\mathbf{d}x^2} + k_j^2\psi_j = 0, \tag{4}$$

in which

$$k_j = \begin{cases} \dfrac{2\pi\sqrt{2m_jE}}{h} , & (j = 1,3,5,\dots) \\ \dfrac{2\pi\sqrt{2m_j(E - V_0)}}{h} , & (j = 2,4,6,\dots) \end{cases} \tag{5}$$

Solving the general solution of the differential equation, it is found that

$$\psi_j(x) = C_{2j-1}\exp(\mathrm{i}k_jx) + C_{2j}\exp(-\mathrm{i}k_jx) \tag{6}$$

where C_{2j-1},C_{2j} is the coefficient to be determined. The first term represents the transmitted wave of the particle, and C_{2j-1} is the transmitted amplitude. The second term represents the reflected wave of the particle, and C_{2j} is the reflected amplitude [6].

The boundary of the Schrödinger equation should be continuous and derivable. Namely,

$$\begin{cases} \psi_j(x_n) = \psi_{j+1}(x_n) \\ \left.\dfrac{\mathbf{d}\psi_j}{m_j\mathbf{d}x}\right|_{x=x_n} = \left.\dfrac{\mathbf{d}\psi_{j+1}}{m_j\mathbf{d}x}\right|_{x=x_n} \end{cases} \qquad j,n = 1,2,3,\ldots 2\mathrm{N} \tag{7}$$

Bring them in separately to get $4N$ formulas [7]

$$\begin{cases} \exp(\mathrm{i}k_1x_1)C_1 + \exp(-\mathrm{i}k_1x_1)C_2 - \exp(\mathrm{i}k_2x_1)C_3 - \exp(-\mathrm{i}k_2x_1)C_4 = 0 \\ \exp(\mathrm{i}k_2x_2)C_3 + \exp(-\mathrm{i}k_2x_2)C_4 - \exp(\mathrm{i}k_3x_2)C_5 - \exp(-\mathrm{i}k_3x_2)C_6 = 0 \\ \vdots \\ \exp(\mathrm{i}k_{2n}x_{2n})C_{4n-1} + \exp(-\mathrm{i}k_{2n}x_{2n})C_{4n} - \exp(\mathrm{i}k_{2n+1}x_{2n})C_{4n+1} = 0 \\ \dfrac{k_1\exp(\mathrm{i}k_1x_1)}{m_1}C_1 - \dfrac{k_1\exp(-\mathrm{i}k_1x_1)}{m_1}C_2 - \dfrac{k_2\exp(\mathrm{i}k_2x_1)}{m_2}C_3 + \dfrac{k_2\exp(-\mathrm{i}k_2x_1)}{m_2}C_4 = 0 \\ \dfrac{k_2\exp(\mathrm{i}k_2x_2)}{m_2}C_3 - \dfrac{k_2\exp(-\mathrm{i}k_2x_2)}{m_2}C_4 - \dfrac{k_3\exp(\mathrm{i}k_3x_2)}{m_3}C_5 + \dfrac{k_3\exp(-\mathrm{i}k_3x_2)}{m_3}C_6 = 0 \\ \vdots \\ \dfrac{k_n\exp(\mathrm{i}k_{2n}x_{2n})}{m_n}C_{4n-1} - \dfrac{k_n\exp(-\mathrm{i}k_{2n}x_{2n})}{m_n}C_{4n} - \dfrac{k_{n+1}\exp(\mathrm{i}k_{2N+1}x_{2n})}{m_{n+1}}C_{4n+1} = 0 \end{cases} \tag{8}$$

Since there is no reflected wave at the last boundary, there is no reflected wave term in the last equation.

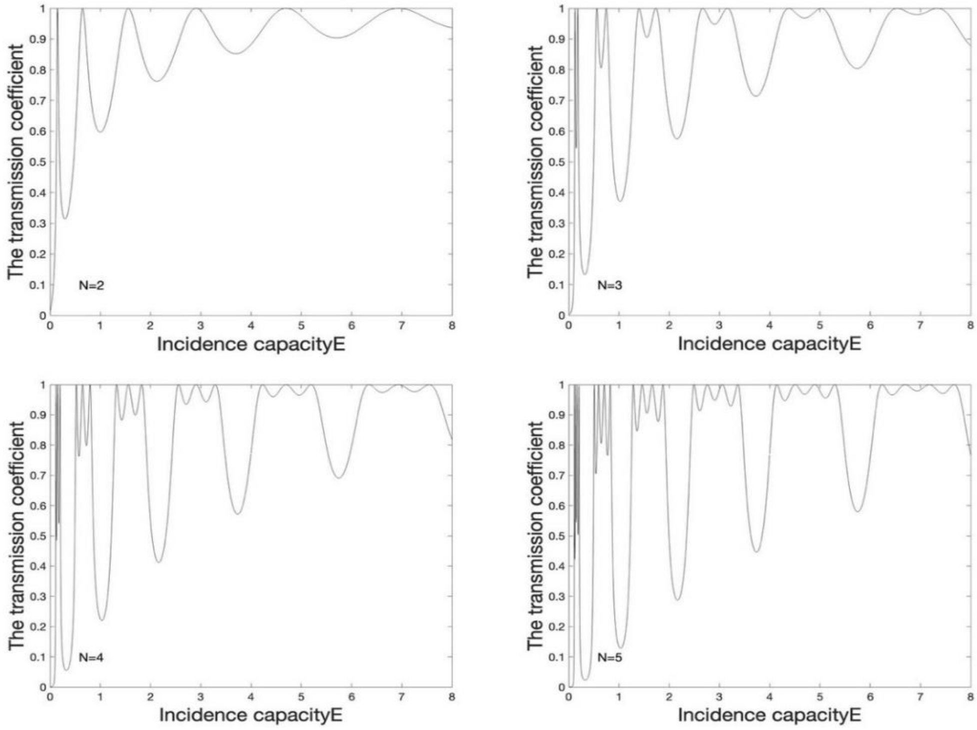

Fig. 3 When $V = 4$ and $N = 2,3,4,5$, the transmission coefficient varies with the incident energy.

The transmission coefficient is the ratio of the flow of particles out of the multi-barrier system to the flow of particles into the multi-barrier system.Therefore, the transmission coefficient T can be expressed as follows [8]

$$T = \frac{|C_{4n+1}|^2}{|C_1|^2}. \tag{9}$$

In the same way, the resulting reflection coefficient R is $R = |C_2|^2/|C_1|^2$, and the two need to meet the following relationships $T + R = 1$.

2.3.2 Numerical Simulation

In Fig. 3, it is possible to see approximately the differing spectrum of different numbers of barriers, under the same conditions (interval, width, height of the faction, the same input energy). The overall trend is roughly the same, with the increase of the number of barriers. Moreover, the resonance state is divided into several energy levels, and the energy level width is gradually widened as the energy increases [4].

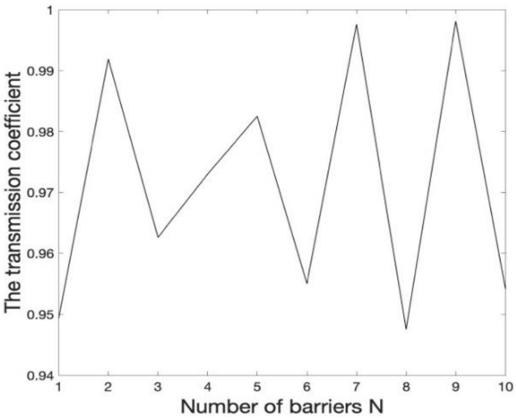

Fig. 4 When $E = 3$ and $V = 4$, the transmission coefficient varies with the number of barriers.

In Fig. 4, when the energy of the incident particles (E) is less than the height of the potential barrier (V), the transmission probability exhibits an oscillatory distribution for different numbers of equally high square barriers. This indicates that, for an odd (even) number of barriers, the resonance energy levels are approximately the same, suggesting the presence of certain parity properties in multiple barriers tunneling.

3. Applications

There has been remarkable progress in engineering and engineering quantum-size effects of $\mathrm{MI^2M}$ tunneling kernels and quantum dots in the recent years, which have shown the promise in advanced optoelectronics and and nanoelectronics technologies. The studies on quantum dots have demonstrated that the electronic features from semiconductor materials can be largely enhanced by selecting the right nanometer size and composition for the quantum dots. It has been shown that the electronic transport performance of the quantum dots can be exceptionally improved through the utilization of the resonant tunneling phenomenon and will present exciting new prospects for the development of effective photoelectric conversion and quantum computing [9].

Moreover, $\mathrm{MI^2M}$ junction studies likewise demonstrated that resonant tunneling plays an essential role in tuning electrical properties. The study in $\mathrm{Ni/NiO/Al_2O_3/Cr/Au}$ form of current diodes shows that the impedance and the responsiveness of a device can be well controlled by manipulating the thickness of the NiO layer. The article reports on a brand-new design outline of multi-barrier resonant tunneling mechanism and applies the frequency concept to optoelectronic devices next time [10].

Not only that, but tech ingenuity is also displayed in these two works through the advanced technologies involved in the study and use of quantum phenomena. This was achieved by developing and optimizing a feature of resonant tunneling called transfer matrix approach using a number of sophisticated quantum mechanical simulation tools, such as the transfer matrix approach. The exploitation of the Friedel tunneling effect in the domains of optoelectronics and high-speed electronics is yet more than the only point that the audience gets to appreciate the quantum behaviour of materials, but it is also a more than essential part of the design of next-generation high-performance devices.

Finally, the influence of the resonant tunneling effect in the nanoscience and quantum technology fields has achieved a significant deal of progress. Such breakthroughs have not only enhanced people's understanding of the quantum mechanism, but they have also plunged us into new perceptions and innovations that will be used towards technology advancement. The exploration of the resonant tunneling effect as the foundation in many systems of physics should yield application strides in nanotechnology.

4. Conclusion

In order to conduct theoretical and quantitative evaluation of the multi-barrier model, this study uses the transitional matrix method for jointly solving a number of sherptonic equations. The study shows that, under the same conditions, the width of the energy levels grows with the growth of incident energy, and the energy levels of the resonance states undergo splitting as the number of barriers increases. The number of divisions decreased by the number of factions by 1. It is worth noting that in the chart, the increase in the number of barriers to the resonance energy of the transmission rate shows a sawn-shaped distribution, which indicates that there is a certain equal relationship between the rate of transmission and the number of obstacles to resonant energy. Remarkably, there is a certain parity relationship between transmittance and the number of obstacles at the resonance energy. These results are essential for understanding the resonant tunneling phenomenon. Finally, this paper explores the application of multiple barriers in different areas in recent years.

References

[1] Paul Harrison, Alex Valavani. Quantum Wells, Wires and Dots: Theoretical and Computational Physics of Semiconductor Nanostructures. John Wiley & Sons, Ltd, 2016.

[2] Devese Sam, Pot Catherine, Natali Franck, Granville Simon, Plank Natalie, Ruck Ben J, Joe Trodahl H, Holmes-Hewett William. Non-volatile memory storage in tri-layer structures using the intrinsically ferromagnetic semiconductors GdN and DyN. Nano Express, 2022, 3(4):1-7.

[3] Fujia Yang, Atomic Physics. Beijing: Chinese high education press, 2008.

[4] Xu Huai-zhe, Wang Yin-yue, Zhang Fang-qing, Chen Guang-hua. Theoretical Studies on Resonant Tunneling in Multi-barrier Structures. Acta Physica Sinica, 1992, (09):1493-1498.

[5] Tang Li-bin, Wang Yi-feng, Yue Qing. MATLAB Analysis and Calculation of Quasi-Bound State in One-dimensional Multi-barrier Structure. Infrared Technology, 2011, 33(02):85-91.

[6] Tang Li-bin, Wang Yi-feng. A Simple Method for Solution of One-dimensional Schrödinger Equation with Transfer Matrix and MATLAB. Infrared Technology, 2010, 32(03):177-180.

[7] Hongmei Wang, Huaizhe Xu, Yafei Zhang. Indispensable factors influence the quasi-bound levels of biased multi-barrier quantum well structures.Physics Letters A, 2005, 340(1):347-354.

[8] Zhang Hong-mei, Liu De. Transfer matrix method in the study of quantum transmission for rectangular barrier. Journal of Hebei University of Science and Technology, 2006, 32(03):196-199+213.

[9] Liu Mingxiu,Wei Jingxuan,Qi Liujian,An Junru,Liu Xingsi,Li Yahui,Shi Zhiming,Li Dabing,Novoselov Kostya S,Qiu ChengWei,Li Shaojuan.Photogating-assisted tunneling boosts the responsivity and speed of heterogeneous WSe2Ta2NiSe5 photodetectors. Nature communications, 2024, 15(1):141-141.

[10] Belkadi Amina,Weerakkody Ayendra,Moddel Garret.Demonstration of resonant tunneling effects in metal-double-insulator-metal (MI2M) diodes. Nature communications, 2021, 12(1):2925-2925.