

Submersible Position Prediction Model Based on HMM and Six Degrees of Freedom Motion

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Abstract. The mysterious deep sea has an unrivalled attraction for scientists worldwide, and the use of submersibles provides us with a clearer view of the seafloor and helps us gather information more safely. We must monitor the position information of the submersible in real time to ensure the safety of both the diver and the submersible. In this paper, we use the six-degree-of-freedom motion model and HMM to predict the motion state of the submersible, propose to replace the complex numerical simulation process of seawater motion with a stochastic process and estimate the parameter λ of the HMM by using the Baum-Welch algorithm. After the data analysis, the paper verified the model's applicability in the sea area and predicted the motion position of submersibles very well as the position prediction model of submersibles in the complex oceanic situation affected. Combined with the previous research results on the motion of the submersible, this paper provides some references for the research of the submersible motion prediction model.

Keywords: Six-degrees-of-freedom; HMM; Muti-submersible.

1. Introduction

The depths of the oceans are a mysterious and enigmatic place, characterised by dim light, infinity and many strange creatures of different shapes and sizes. This aura of the unknown has always been a source of exploration, through the use of sonar, exploring undersea landforms and mapping the clear topography of the seabed has also always been one of the pursuits of scientists [1]. Therefore, in this paper, we take the Ionian Sea as an example to collect the basic parameters of the submersible and the information on the ocean currents and water temperature in this sea area to build a model to predict the position of the submersible over time. Uncertainties affecting the prediction model are identified, the position prediction is combined with environmental factors, and a relatively flat sea is selected to calculate the equipment's initial deployment point and the search pattern, thus improving the efficiency of searching for missing submersibles.

Commonly used submersibles are ROV, HOV and AUV to ensure that they can work in hazardous conditions on the seabed [2]. In previous studies, submersibles and floating offshore platforms have moved in six degrees of freedom (DOF), which are excited by environmental forces such as waves, wind, or currents [3][4]. To solve the problem of describing the motion of a submersible on the seafloor, previous studies have typically used force analysis to model the motion [5]. Whereas, with the increasing use of deep-water floating structures, there is a growing interest in optimizing the search system using various algorithms. To improve the efficiency of the submersible search, Genetic Algorithms (GA) are popular in the optimization process of submersible search systems, to predict and learn the possible routes of motion of the submersible [6]. Moreover, since the ocean is an environment full of unknowns, the topography of the seafloor and the currents are very complex [7]. The data available through both meteorological offices and geographic surveys are minimal and can change at any time, so considering the main uncertainties affecting the position of the submersible, which are currents and circulation, seawater density, and seabed geography, is also very common in previous studies [8].

However, there are still some defects in the previous research. In this paper, the motion model of the submersible is built more completely by more detailed force analysis, considering the hydrodynamic and other factors, so that the submersible motion model applies to more situations [9].

For the genetic algorithm optimization of the ice skates, the search system needs to determine the dynamic response of the floating structure, which will lead to high computational cost as it is derived from the complex finite element time domain simulation tool. Therefore, in this paper, an implicit Markov model is used to randomly generate a stochastic sequence of unobservable states to predict and learn the submersible motion path [10]. Finally, how to take the complex environmental situations into account in the model is also something to think about in previous studies. In this paper, these factors are transformed into dynamic parameters to be considered in the model. By doing so, the original study is optimized.

2. The fundamentals of the Submersible Position Prediction Model

2.1. Modelling of submersible motion

The submarine's motion is a spatial six-degree-of-freedom motion. The attitude, depth and direction of submarine motion are controlled by manoeuvring the bow and stern elevators and rudders. In this paper, we use a fixed coordinate system, as shown in Figure 1, the ground coordinate system (referred to as static system) $E-\xi \eta \zeta$ and a motion coordinate system, the hull coordinate system (referred to as dynamic system) $O-x y z$. These two basic coordinate systems work together to describe the motion of a submarine.

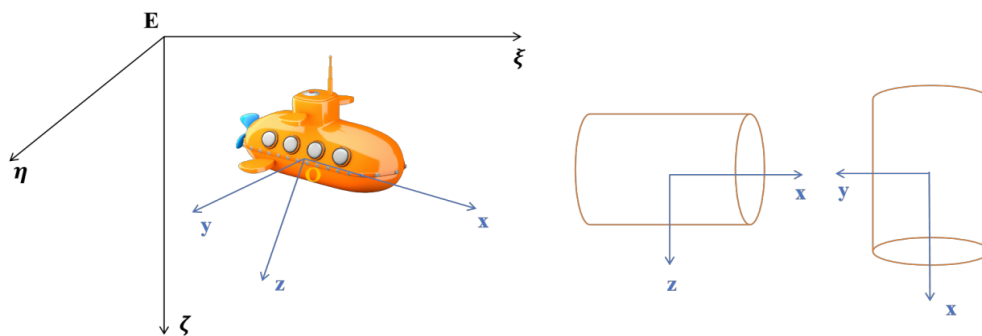


Figure 1. Ground and hull coordinate systems
(Left) Side and top views of the hull coordinate system (Right).

The hydrodynamic force of submarine movement in seawater is closely related to the direction of submarine movement. The hull coordinate system can reflect the submarine motion changes in simple terms. The submarine motion constructed in this paper is a six-degree-of-freedom motion, as shown in Figure 2

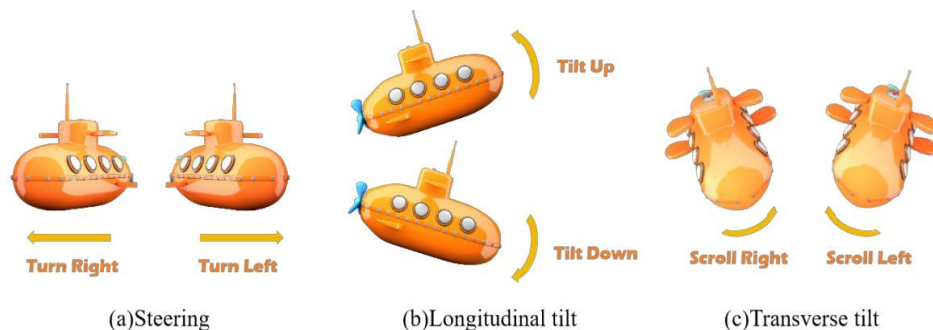


Figure 2. Three directions of swing.

The forces to which a submersible is subjected during its movement in the deep sea can be broadly categorized into four types. These are hydrodynamic forces F_H , the submersible's thrust F_T , hydrostatic forces F_R , and disturbance forces F_E caused by environmental perturbation. Construct a differential equation modelling the motion of the submersible in conjunction with Newton's second law.

$$F = F_H + F_R + F_T + F_E \quad (1)$$

$$F = (m + \Delta m)a \quad (2)$$

Where, F indicates the force on the submersible, a is the acceleration of the submersible, m is the total mass of the submersible, and tourists, Δm is the quality of seawater storage.

F_H Stands for hydrodynamics, which can be broadly divided into two categories: inertial hydrodynamics and viscous hydrodynamics F_L . The hydrodynamic equation can be listed as:

$$F_H = F_I + F_L \quad (3)$$

$$F_L = f_0 + \frac{\delta f}{\delta V}(V - V_{water}) + \frac{\partial^2 f}{\partial V^2}(V - V_{water})^2 \quad (4)$$

Where, V refers to the speed of the submersible, V_{water} refers to the speed of seawater flow, both are vectors. f Is a function of viscous hydrodynamics and submersible velocity?

F_T Represents the thrust of the submersible itself.

F_R Represents the hydrostatic force, which is determined by the gravitational force of the submersible and the buoyant force to which it is subjected, leading to the equation:

$$F_R = W - B \quad (5)$$

Where, acting on the submersible include gravity W , and buoyancy B . The gravitational force of the submersible is the sum of its weight, the weight of the tourists and the weight of the stored seawater.

$$W = (m + \Delta m)g \quad (6)$$

$$B = \rho V_s g \quad (7)$$

F_E Stands for Environmental Disturbance Force, which is a small probability event. So, this paper uses a random variable to represent what it does to the submersible.

$$F_E = \varepsilon_E(t) \quad (8)$$

Where, $\varepsilon_E(t)$ is defined as a Brownian motion stochastic process, conforming to the following equation.

$$f(x, t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{x^2}{2\sigma^2 t}} \quad (9)$$

Supplementarily, due to the different flow rates of seawater at different locations, this paper introduces a stochastic process based on Brownian motion to simulate seawater flow, which is more in line with the characteristics of seawater as a chaotic system.

$$V_{water}(t+1) = V_{water}(t) + \varepsilon_{water}(t) \quad (10)$$

As for the density of seawater, this paper uses the International Equation of State of Seawater (TEOS-10) published by the International Union of Oceanography (UNESCO) to calculate the density of seawater:

$$\rho = \rho_0 + \Delta\rho(T, S, P) \quad (11)$$

Where, ρ is the density of seawater (kg / m^3), ρ_0 is the reference density (about $1000kg / m^3$). $\Delta\rho(T, S, P)$ Is the effect of temperature T (degrees Celsius), salinity S (PSU) and pressure P (pascals) on density.

2.2. Constructing HMM-based prediction models

The search route is formulated in advance to maximize the probability of finding the submersible. The probability of finding a submersible is not only related to the search time but also to the current search results. HMM (Hidden Markov Models) are well suited to deal with this problem.

HMM describes a process of randomly generating a random sequence of unobservable states from a hidden Markov chain and then generating each observation from the individual states thus producing a random sequence of observations. Observations can be used to backtrack on the hidden causes that produced them.

This paper set up the dive path as a sequence of observations and the search path as a sequence of states. The sequence of states is expected to be inferred from the observations at each moment in time. Figure 3 shows a conceptual diagram of the implicit Markov model.

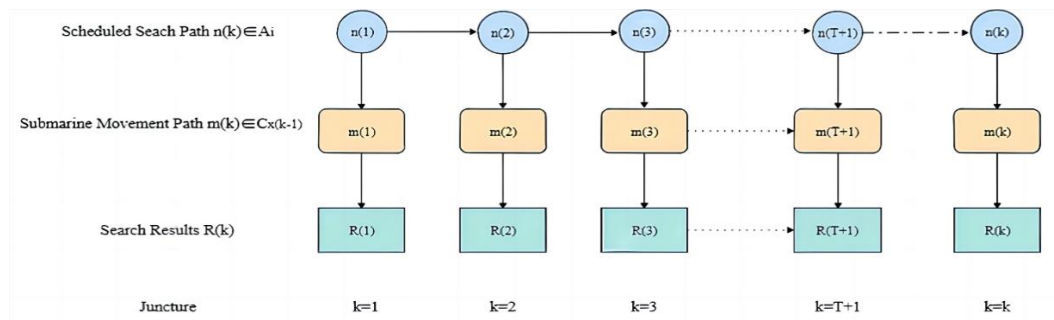


Figure 3. Markov Chain of searching submersible.

The above figure, $n(k)$ denotes the position of the searcher at moment k , $C_x(k-1)$ denotes the set of possible positions that the submersible could be at moment $k-1$, and $R(k)$ denotes the observation starting at moment k .

The specific algorithm is shown in Figure 4:

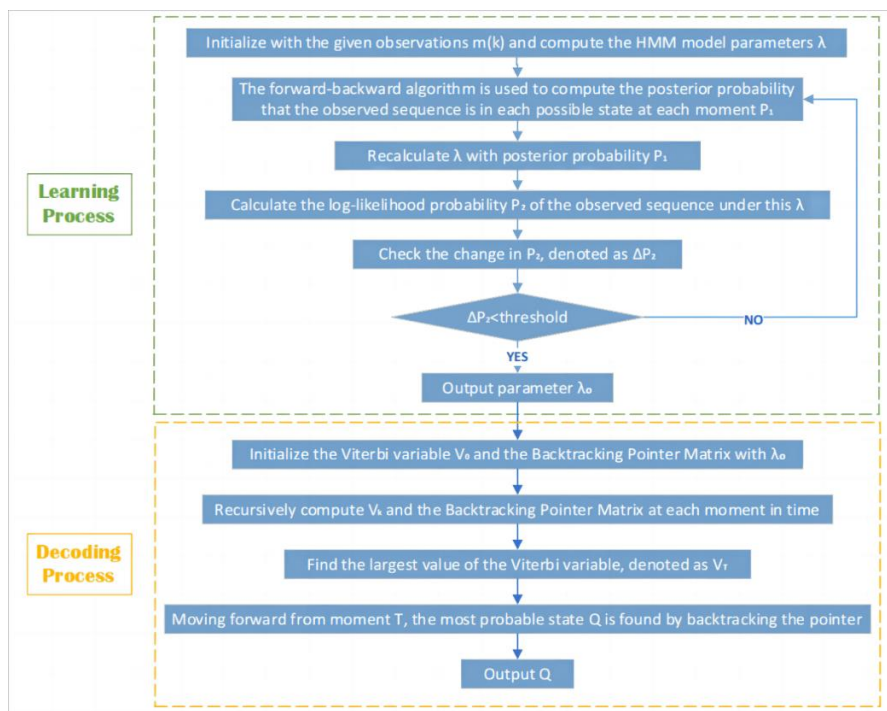


Figure 4. Algorithm steps.

The mathematical representation of the HMM is:

$$\left\{ \begin{array}{l} \lambda = (\pi, A, B) \\ \pi_i = \frac{P(C, i_1 = i | \bar{\lambda})}{P(C | \bar{\lambda})} \\ a_{ij} = \frac{\sum_{t=1}^{T-1} P(C, i_t = i, i_{t+1} = j | \bar{\lambda})}{\sum_{t=1}^{T-1} P(C, i_t = i | \bar{\lambda})} \\ b_i(k) = \frac{\sum_{t=1}^T P(C, i_t = i | \bar{\lambda}) I}{\sum_{t=1}^T P(C, i_t = i | \bar{\lambda})} \end{array} \right. \quad (12)$$

Where, π is the initial state probability vector? A Is the state transfer matrix, denoting the probability of transferring to state at moment $k+1$, conditional on being in state $m(k)$ at moment k . B Is the observation probability matrix, which represents the probability of generating observation $n(k)$, conditional on being in state $m(k)$ at the time k .

In this paper, an underwater robotic ROV is used as a submersible search tool to search along a route where the search area can be approximated as part of a cylinder. To expand the search range, this article proposes a spiral motion search pattern. The spiral search is the blue range, and the linear search is the orange range. The blue range is much larger than the orange range. The described search pattern is illustrated in Figure 5.

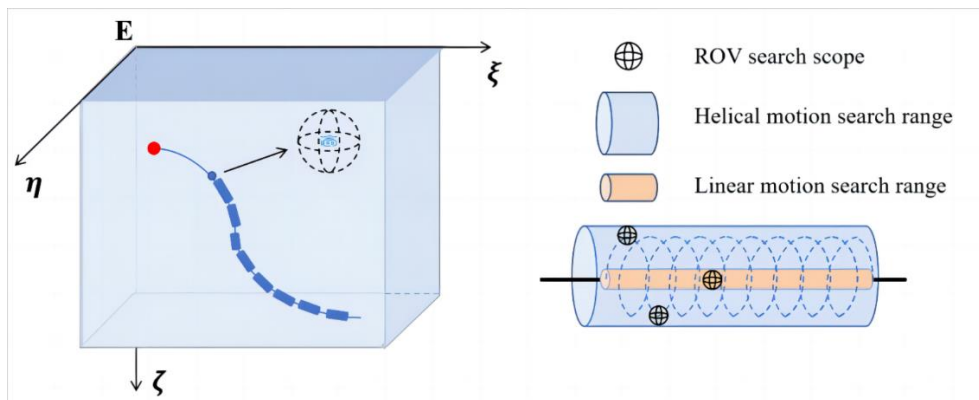


Figure 5. Linear and Helical Search.

To establish the spiral motion equation:

$$\left\{ \begin{array}{l} \xi(t) = A \cos(Kt) \\ \eta(t) = A \sin(Kt) \\ \zeta(t) = C e^{-Dt} \end{array} \right. \quad (13)$$

3. Results

3.1. The analysis of results from the submersible motion model

The main function of this model is to simulate the travelling path of the submersible during its movement. A basic premise and basis for solving the submersible kinematic model to facilitate the learning of the later learning paths.

The solution is carried out using the Lunge-Kuta method. This method provides solutions with higher accuracy, especially the fourth-order Lunge-Kuta method (RK4). The accuracy of the calculation is improved by combining the slopes at several different points.

This article randomly selected an area near the coast for our calculations. The seafloor in this area is relatively flat, and most of it is about 2000 meters deep. If the submersible dives down to 500 meters from the seabed for exploration. Using the seawater density formula this paper finds that the density of seawater changes very little, increasing by only about $0.05g/cm^3$ per kilometre of descent. And the depth of the area we chose does not vary much. So, the calculation can be simplified by approximating the density of the region as equal.

At about 40h, the submersible suddenly started to sink. As can be seen in the figure, the colour of the seawater changed. Therefore, we deduce that the sudden change in the depth of the seabed and the temperature change caused by the ocean currents together affected the density of the seawater at this time. The submersible begins to sink to reach the point of neutral buoyancy. The results are shown in Figure 6.

Observe the position of the submersible and the direction of the seawater flow, the direction of movement of the submersible was consistent.

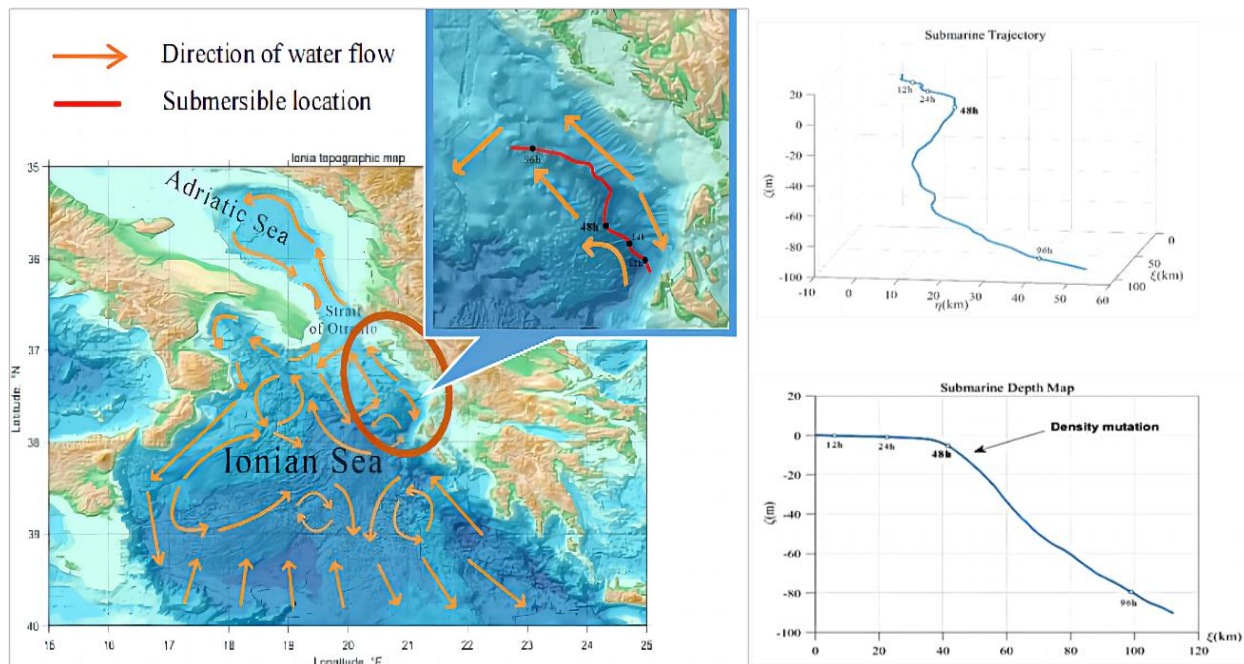


Figure 6. Predicted position of the submersible.

3.2. Analysis of the results of the prediction model

Based on the results of the motion modelling of the first problem, the optimal search path can be calculated by HMM. However, the position of the submersible is subject to random perturbations. In this paper, multiple possible travelling paths are generated for the submersible by adding this random perturbation. The probability of finding a diver is defined as the ratio of the number of diver paths that intersect the search path to the total number of diver paths.

The Markov model should satisfy the following two assumptions:

The state at any moment depends only on its previous moment state, independent of the states and observations at other moments.

Observations at any moment depend only on the state at that moment, the independent of observations and states at other moments.

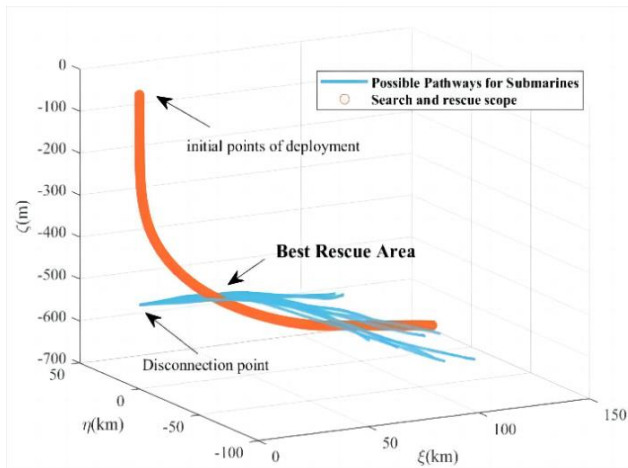


Figure 7. Search Path.

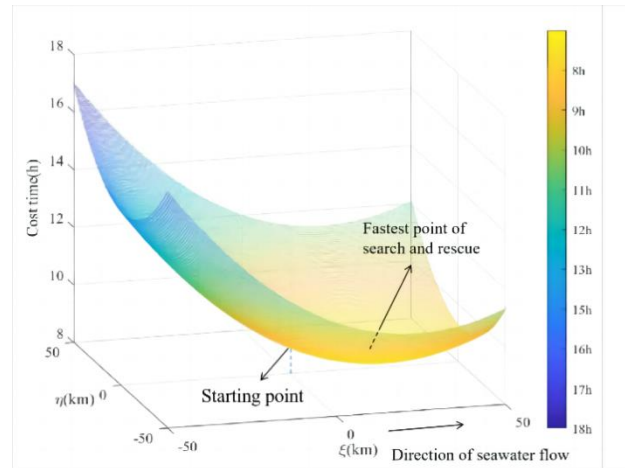


Figure 8. Initial deployment point.

Figure 7 shows the search path points calculated using the predicted points as the initial deployment. The initial deployment point location is then changed and the relationship between the deployment point and the search time is calculated.

Using the predicted position point as the origin, an area of $100\text{km} \times 100\text{km}$ was chosen. Divided into $0.1\text{km} \times 0.1\text{km}$ grids. For each grid, calculate the corresponding search time for the submersible. Obtain a plot of the relationship between deployment point and search time. As shown in Figure 8.

Getting results: The initial deployment point was 24 kilometres along the ξ axis in the forward direction (direction of the current) and 1.5 kilometres in the reverse direction of the η axis. The minimum search time is obtained as 8.5h.

The article then calculates the relationship between the single search time the cumulative number of searches and the probability of finding the submersible. The results are shown in Figure 9.

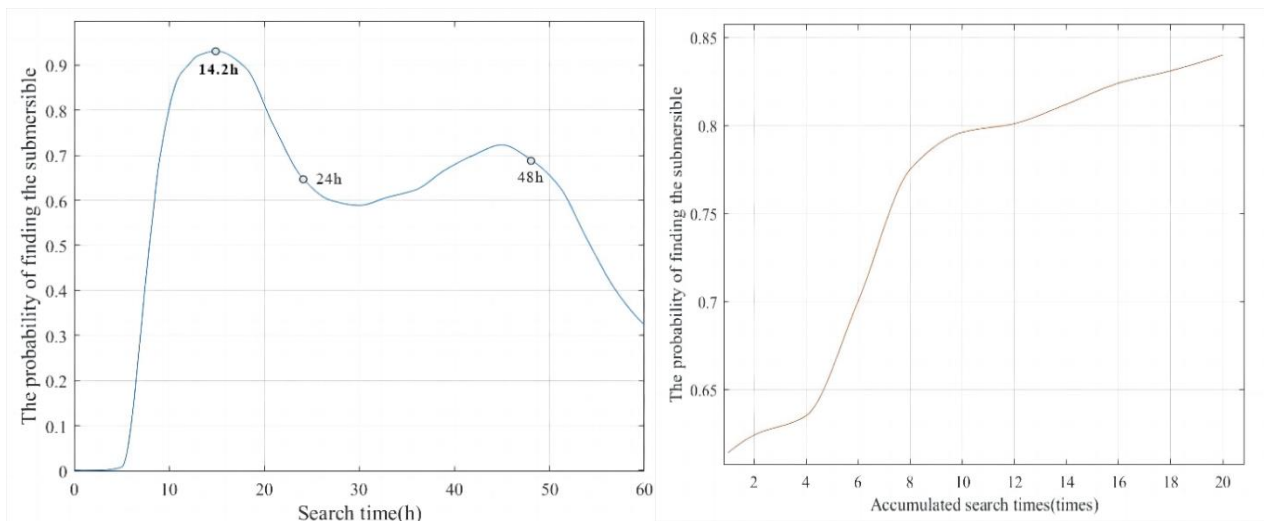


Figure 9. Search time and accumulated search times versus probability

Search time: The trend of the curve is consistent with the location of the intersection point in Figure 6. In the beginning, there is no intersection point, and the probability of finding a submersible is low. At 14.2h the path of many submersibles intersects with the search path, at which point the probability reaches the maximum. Then the probability decreases with time, rising gradually between 30h-48h and dropping sharply after 48h.

Accumulated search times: As the number of searches increases, the probability of finding the submersible gradually rises. Rapid growth between 4 times and 8 times. Increase in probability levels after 10 times.

4. Conclusions and outlooks

In the middle of this paper, the prediction of the position of the submersible is successfully addressed. The position prediction model applies to different ocean conditions and different sea areas. The results are generalizable based on validation. In this paper, the final submersible position prediction model is derived by combining the submersible motion model and the predictive learning model. Regarding the submersible motion model, in this paper, the six-degree-of-freedom model is used as the motion model of the submersible by establishing differential equations, which are combined with Newton's second law and solved by the Lunge-Kutta method. To improve the calculation accuracy, the case of different thruster positions and operating modes is also considered. As for the predictive learning model, this paper adopts the HMM algorithm, which can quickly update and optimize the preset results, and the spiral forward mode, which can better expand the search range and thus improve the probability of a successful search. The model can better provide help for marine scientific research and improve the safety of undersea research. At the same time, it can be extended to assist exploration work on offshore operating platforms and other aspects.

Of course, there are some limitations to the research in this paper. For example, the set of state spaces defined by the HMM may not contain all cases. To reduce the computational complexity and achieve ideal modelling results, this paper selects the state with the highest probability for simulation. Compared with the traditional numerical simulation based on the hydrodynamic model, there is still a gap in computational accuracy. Therefore, during future research, the state space set of the model is increased to include all the motion states of the submersible as much as possible. Alternatively, use HMM as the basis and add hidden states to form a deep neural network. The accuracy of the search path model is improved, and it is possible to combine hydrodynamics and stochastic processes in an attempt to build a model with high computational efficiency and accuracy.

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