

Predicting The Position of a Submarine Based on A Dynamic Model

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Abstract. In deep-sea exploration projects, submarines have become one of the recreational activities for people and also serve as the primary means of scientific investigation in the deep ocean. To monitor the submarine's position and status in real-time, we have developed dynamic and Monte Carlo random walk models to predict its location over some time. Due to the complexity of the underwater environment, search and rescue equipment is necessary to locate the submarine in case of malfunction or loss of contact. Therefore, we employ the VIKOR evaluation ranking model to assess and rank various search and rescue devices, ultimately determining that carrying equipment such as the Teledyne BV5000 sonar, infrared thermographic camera, and deep-sea camera is most suitable. Through our model, we have a significant impact on future submarine diving missions, enabling accurate prediction of the submarine's position and selection of appropriate search and rescue equipment, thereby enhancing safety in underwater exploration and scientific research.

Keywords: Deep-sea submersibles, dynamic model, Monte Carlo random walk model, VIKOR.

1. Introduction

With the development of manned submersibles, activities such as underwater operations, submarine exploration, scientific investigations, and search for shipwrecks in complex environments have been formed [1]. At present, China has three deep-sea manned submersibles, Jiaolong, Deep Sea Warrior, and Struggler, a breakthrough in China's manned submersibles [2]. However, manned submersibles are affected by many environmental factors in deep-sea activities, such as pressure, temperature, density and currents in seawater. Taking the Chinese Jiaolong and the American Erwin manned submersibles as examples, they both have their characteristics and when they are in the water temperature of 1°C and the pressure is 70MPa, the parameters of the submersibles are very different from those on land [3]. It is important to know that in the ocean, every 100 meters of underwater pressure increases by 10 atmospheres [4], and as the depth of the submersible increases, the pressure it is subjected to increases, the temperature becomes lower and lower, and the environment becomes more harsh, and the changes in these factors will have an important impact on the position, stability, and safety of the submersible. Therefore, the safety of submersible activities under the sea has become an important issue. In the process of performing underwater tasks, submersibles usually need to complete accurate trajectory tracking, and the complex underwater environment will affect the submersible during operation by a variety of interference factors [5], so how to predict the position of the submersible in a period needs to be solved. When the submersible loses contact, we need to use search and rescue equipment for all-round search and rescue, so we need to analyze the comprehensive evaluation of various search and rescue equipment, and comprehensively sort each piece of equipment according to the weight ratio of each index, to easily derive their advantages and disadvantages, and then determine the search and rescue equipment that needs to be carried, to minimize the loss and find the location of the submersible as quickly as possible.

2. The Establishment of model

2.1. Dynamics model and Monte Carlo random walk model

A dynamic model is a mathematical model that describes the dynamic behavior of a system or process. In physics, dynamical models are often based on the laws of classical mechanics, particularly Newton's laws of motion. These models can describe the relationship between an object's position, velocity, acceleration, and the forces acting on the object.

When the submersible is at a neutral buoyancy point, gravity equals buoyancy.

$$F_{buoyancy} = G_{submersible} \quad (1)$$

The buoyancy is related to the density of seawater, the volume of water displaced by the submersible, and the gravity is related to the mass of the submersible [6]. The density of seawater is strongly related to the depth of the location of the submersible, so when the density of seawater is known, the depth of the location of the submersible can be calculated.

$$\rho gV = Mg \quad (2)$$

where: ρ represents the density of seawater, g denotes the acceleration due to gravity, V represents the volume of the submarine, and M represents the mass of the submarine.

After analyzing before establishing the model, it can be obtained that the density and depth of seawater significantly change, and the density of seawater is mainly related to the temperature and depth. The salinity of seawater is a relatively fixed value, only a small change, so you can ignore the effect of salinity on seawater density. First of all, this question first analyzes the depth, pressure and density, and the specific model is established as follows:

Depth and pressure relationships in seawater

Through literature search, we found the relationship between depth and pressure in seawater with the equation [7]:

$$z_s = (1 - c_1)p - c_2 p^2 \quad (3)$$

where: z_s represents the depth, and p denotes the pressure of seawater, $c_2 = 2.21 \times 10^{-6} mdb^{-2}$
 $c_1 = (5.92 + 5.25 \sin^2 \phi) \times 10^{-3} [mdb^{-1}]$.

The pressure equation can be derived from equation (3):

$$p = \frac{(1 - c_1) - \sqrt{(1 - c_1)^2 - 4c_2 z}}{2c_2} \quad (4)$$

Based on TEOS-10 density calculation model

TEOS-10 Density Calculation Model is a model for seawater density calculation, the TEOS-10 Density Calculation Equation describes the relationship between seawater density and pressure, temperature, salinity, and the Gibbs function, where the density calculation formulas are [8].

$$\rho = \rho(S_A, T, p) = (g_p)^{-1} = \left(\frac{\partial g}{\partial p} \Big|_{S_A, T} \right)^{-1} \quad (5)$$

where: ρ is the density of seawater, T denotes temperature, S_A denotes salinity, and g_p is the partial derivative of the Gibbs function for pressure over salinity and temperature. is the partial derivative of the Gibbs function of pressure with respect to salinity and temperature.

The Gibbs function is a very important quantity in thermodynamics and it can be expressed in the TEOS-10 equation as g^s and g^w the sum of which the formula expression is [8]:

$$g(S_A, T, p) = g^w(T, p) + g^s(S_A, T, p) \quad (6)$$

where: g^w denotes the pure water term in seawater and g^s is the brine term.

The Monte Carlo Random Walk model is a computational method based on probability and statistical theory that approximates the solution of mathematical problems or physical phenomena by simulating a large number of random experiments.

Ocean currents refer to the three-dimensional, regular flow of water in the ocean, both horizontal and straight [9]. The speed of water in the horizontal direction is usually about a few tens of times that in the vertical direction. It can be seen that the influence of ocean currents on the movement of submersibles is mainly in the horizontal direction. Therefore, only the influence of ocean currents on the horizontal direction of the submersible is considered.

The horizontal motion of the submersible was analyzed using the Monte Carlo random walk model, and the flow data collected from January 2024 and February to December 2023 are shown in Table 1.

Table 1: Current direction

Date	Jan-24	Feb-23	Mar-23	Apr-23
Direction	northwest	northwest	north	north
Date	May-23	Jun-23	Jul-23	Aug-23
Direction	north	south	southeast	southeast
Date	Sep-23	Oct-23	Nov-23	Dec-23
Direction	northwest	north	north	northwest

2.2. VIKOR evaluation ranking model based on independence weight coefficient method

The core idea of VIKOR's method is to find a compromise between decision-makers considering all decision criteria, which assumes that the decision-makers have an ideal optimal solution that is the best on all criteria. In reality, however, it is difficult to find a solution that is optimal in all aspects, and VIKOR tries to find a compromise solution that is as good as possible on the most important criteria while not being too bad in other aspects.

The equipment on the main ship needs to be able to accurately search the location of the submersible. Relevant data were found, including the interventional underwater vehicle, Teledyne BV5000 mechanical sonar, infrared thermal imager, side scan sonar, sea sweeping light, magnetometer and other equipment. The depth of their work, search and rescue scope, price, reliability, etc. as a measure, combined with environmental conditions, and finally calculate the most cost-effective equipment. The data are shown in Table 2:

Table 2: Performance of each product

Product name	Depth (m)	Range (m)	Cost (\$)	Reliable line
Underwater robot	1500	5000	80000	0.9
Machine sonar	4000	9000	15000	0.7
Thermal imager	2500	10000	10000	0.5
Side-scan sonar	300	16000	8000	0.7
Sea lamp	10	25000	9000	0.3
Magnetometer	1000	10000	10000	0.5
Deep-sea cameras	3000	1000	20000	0.9
Marine LiDAR	800	10000	15000	0.7

To determine the weights of each indicator, this paper employs the independent rights coefficient method for analysis. This method considers the linear correlation between various indicators to obtain their respective weights [10]. Subsequently, we can use the VIKOR model to comprehensively rank the various equipment, thereby determining their levels of superiority or inferiority [11]. Such ranking can assist the company in making the final decision.

In this paper, four indicators are represented by x_1, x_2, x_3, x_4 , which correspond to depth, search range, price, and reliability, respectively. According to the independent rights coefficient method, we have the formula for the complex correlation coefficient [12]:

$$R_j = \frac{\sum_{j=1}^4 (x_j - \tilde{x})(\tilde{x} - \bar{x})}{\sqrt{\sum_{j=1}^4 (x_j - \bar{x})^2 \sum_{j=1}^4 (\tilde{x} - \bar{x})^2}} \quad (j = 1, 2, 3, 4) \quad (7)$$

where: R_j represents the complex correlation coefficient of equipment j , x_j represents the j -th indicator, $\tilde{x}$ is the residual matrix obtained by removing x_j from x , $\bar{x}$ represents the average value of the indicators.

The weight formula can be expressed as:

$$R = \left[\frac{1}{R_1}, \frac{2}{R_2}, \frac{3}{R_3}, \frac{4}{R_4} \right] \quad (8)$$

$$W_j = \frac{\frac{1}{R_j}}{\sum_{j=1}^4 \frac{1}{R_j}} \quad (9)$$

where: R represents the complex correlation coefficient, and W_j represents the weight of the i -th indicator.

The VIKOR method is a multi-attribute decision-making method proposed by Opricovic in 1998. By determining the positive ideal solution and the negative ideal solution, the evaluation objects are evaluated and ranked according to the difference between the ideal index value and the merits.

The formula for the positive ideal solution is as follows:

$$S_i = \sum_{j=1}^n W_j \frac{f_j^+ - r_{ij}}{f_j^+ - f_j^-} \quad i = 1, 2, \dots, j = 1, 2, \dots \quad (10)$$

$$R_i^+ = \max_j W_j \frac{f_j^+ - r_{ij}}{f_j^+ - f_j^-} \quad i = 1, 2, \dots, j = 1, 2, \dots \quad (11)$$

where: r_{ij} represents the normalized data of the j -th indicator in the i -th device, S_i^+ and R_i^+ represent the utility values of the positive ideal group and individual regret values, f_j^+ represents the positive ideal solution for the j -th indicator, and f_j^- represents the negative ideal solution for the j -th indicator.

The negative ideal solution formula is:

$$S_i^- = \sum_{j=1}^n W_j \frac{r_{ij} - f_j^-}{f_j^+ - f_j^-} \quad i = 1, 2, \dots, j = 1, 2, \dots \quad (12)$$

$$R_i^- = \min_j W_j \frac{r_{ij} - f_j^-}{f_j^+ - f_j^-} \quad i = 1, 2, \dots, j = 1, 2, \dots \quad (13)$$

where: S_i^- and R_i^- are the negative ideal group utility value and individual regret value respectively.

The group benefit and individual regret value can be obtained through formula (10) (11) (12) (13):

$$S_i = \frac{S_i^+}{S_i^-} \quad i = 1, 2, \dots \quad (14)$$

$$R_i = \frac{R_i^+}{R_i^-} \quad i = 1, 2, \dots \quad (15)$$

where: S_i and R_i represent the group benefit and individual regret value of the i -th equipment respectively.

Calculate benefit ratio and rank:

$$Q_i = v \frac{S_i - S_{i\min}}{S_{i\max} - S_{i\min}} + (1 - v) \frac{R_i - R_{i\min}}{R_{i\max} - R_{i\min}} \quad (16)$$

where: $R_{i\min} = \min R_i$, $R_{i\max} = \max R_i$, $S_{i\max} = \max S_i$, $S_{i\min} = \min S_i$, Q_i is the benefit ratio, and v is the decision-making mechanism coefficient.

3. The solution of the model

3.1. The solution of dynamic model and random walk model

We employ the dynamic model and random walk model to predict the position of the submersible. By judging its depth and horizontal position, the position of the submersible in the deep sea can be determined. The result is shown in Figure 1:

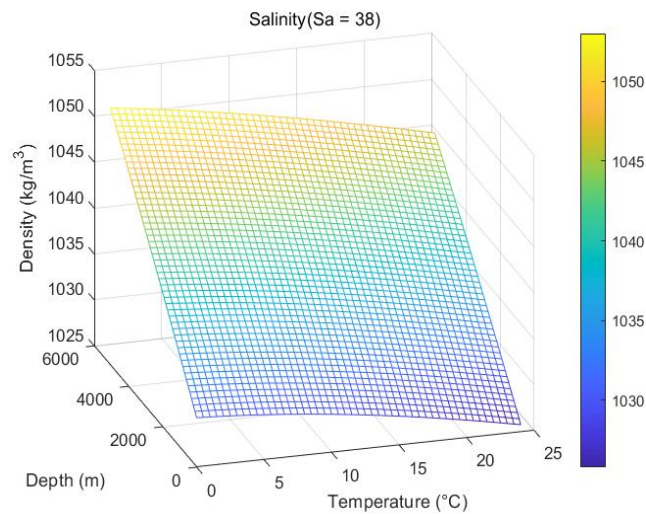


Figure 1: Relationship between density, temperature, and depth

By observing Figure 1, it is evident that temperature and depth significantly impact seawater density. As the temperature increases, seawater density tends to decrease, indicating an inverse relationship between the two. Conversely, as the depth increases, seawater density shows an upward trend. Utilizing this characteristic of density variation, we can roughly estimate the depth position of a submersible.

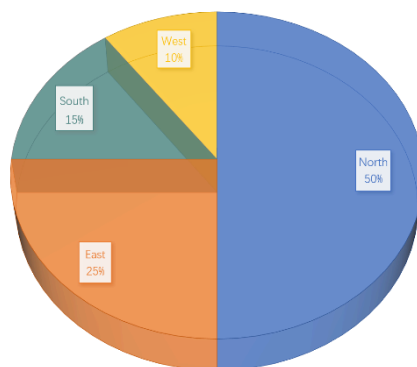


Figure 2: Probability Direction Chart

Figure 2 clearly shows the probability of ocean currents flowing in different directions within a year. The overall probability of eastward flow is approximately 25%, the overall probability of westward flow is approximately 10%, and the probability of northward flow is the highest, reaching about 50%. , while the probability of southward flow is only about 15%.

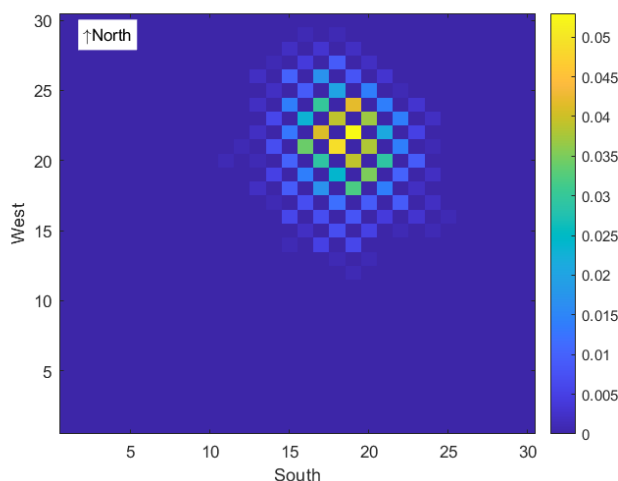


Figure 3: Regional Probability Chart

Figure 3 shows the results. Different colors are used in the figure to represent different probability intervals, with yellow representing the highest probability. Based on this analysis, we can determine the coordinates of the position with the highest probability of submersible presence in this area.

3.2. VIKOR evaluation ranking model based on independence weight coefficient method

First, calculate the weight of each indicator according to the independence weight coefficient method:

Table 3: The weight of each indicator

Index	R	1/R	Weight(%)
Depth	0.723	1.383	26.207
Range	0.877	1.14	21.61
Cost	0.647	1.547	29.312
Reliable line	0.829	1.207	22.872

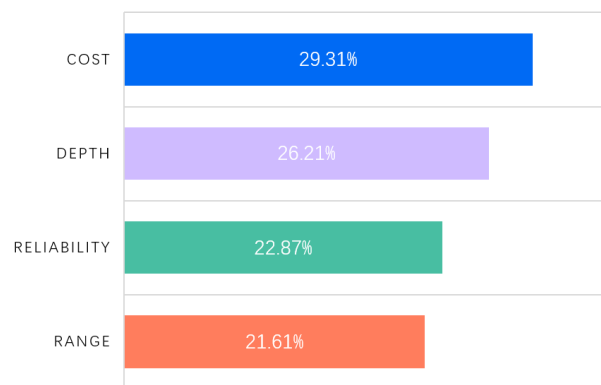


Figure 4: Indicator importance histogram

From the calculation results in Table 3 and Figure 4, we can see that the weight of depth is 26.207%, the weight of the search range is 21.61%, the weight of price is 29.312%, and the weight of reliability is 22.872. Although there is not much difference in the overall indicators, price is still more prominent as the indicator with the largest weight.

Table 4: Optimal ordering of equipment

Name	S	R	Q	Sequence
Underwater robot	0.6279	0.2931	1	8
Machine sonar	0.2299	0.1252	0	1
Thermal imager	0.3729	0.1525	0.2609	2
Side-scan sonar	0.3647	0.2430	0.5203	6
Sea lamp	0.4948	0.2620	0.7403	7
Magnetometer	0.4714	0.1970	0.5174	5
Deep-sea cameras	0.3370	0.2162	0.3976	3
Marine LiDAR	0.4287	0.2102	0.5028	4

Table 4 shows the optimal ordering of eight types of equipment. After comprehensive consideration, the main ship can carry three types of equipment: Teledyne BV5000 machine sonar, infrared thermal imager, and deep-sea camera in case of emergency.

4. Conclusions

Research on deep-sea operations involving submarines, we have established dynamic and Monte Carlo random walk models to predict the precise position of the submarine over some time, thereby enhancing the safety of submarine operations and improving our understanding of the surrounding environment. In the event of an accident or loss of contact with the submarine underwater, it is necessary to quickly locate its position using search and rescue equipment. Therefore, we adopted the VIKOR evaluation ranking model to comprehensively assess the existing equipment. By calculating the weights of various factors, we ultimately determined the optimal selection of search equipment, including carrying devices such as the Teledyne BV5000 sonar, infrared thermographic camera, and deep-sea camera.

The topography of the underwater world is complex, and we ignore the influencing factors of the topography in the seawater in the construction of the model, so our model needs to be further improved.

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