Changes in the Sex Ratio of Species Based on the Lotka-Volterra Equation

Junjie Tong *

College of Information Science and Technology, Nanjing Forestry University, Nanjing, China

* Corresponding author: tjj123456202107@163.com

Abstract. Changes in sex ratio have an important impact on the species as well as the surrounding ecological environment. In this paper, a predator-prey model and the Lotka-Volterra equation were developed to study the population dynamics of the species and the relationship between the changes in sex ratio, taking lampreys as an example. A sex-ratio adjustment function $g(P)$ was introduced to modify the model to simulate the effects of populations of species with different sex ratios on prey abundance compared to populations with a standard 1:1 sex ratio. At the same time, the evolutionary trend of the ecosystem under different scenarios was investigated by modelling the system dynamics of species populations with other species based on the Lotka-Volterra equation. After incorporating the quantitative relationship between food availability and the sex ratio of species populations to refine the model, the effects of changes in sex ratio on the ecosystem were again analysed.

Keywords: Predator-prey models, Lotka-Volterra equations, system dynamics models.

1. Introduction

The sex ratio of males and females in natural populations is usually 1:1, but varies with water, nutrients, carbon dioxide concentration, temperature, light and disturbance levels [1]. For some species, sex transition is even an obligatory part of their life cycle, for example, some hermaphroditic species start life as one sex and change to the other after a period of time, and the sex of many fish is highly plastic [2]. This adaptive sex ratio change is a key factor in biological evolution and ecological strategy. During evolution, this reproductive strategy has appeared independently in different lineages, showing the plurality of its origins [3].

In this study, changes in the sex ratio of individuals in an ecosystem and how these changes reflect and affect ecosystem functioning were simulated and analysed by developing a predator-prey model and a Lotka-Volterra equation model. After refining this model by incorporating a quantitative relationship between food availability and the sex ratio of species' populations, the aim is to provide insights into how species respond to changes in resource availability through adaptive adjustments in sex ratios, as well as the long-term ecological benefits and potential value of this strategy.

2. Modelling and solving

2.1. Modelling population dynamics through the Lotka-Volterra equation

In ecosystems, there is exchange of materials and transfer of energy among individuals, groups, communities and between them, as well as between organisms and the environment, and food chains and food webs dominate the process of energy and material flow in the entire ecosystem. Food web is a complex network of food relationships between organisms in an ecosystem, indicating the interdependence and balance of the ecosystem. Food webs play a key role in ecosystem material cycling, energy transfer, and so on [4-6].

The lampreys play a complex role in the ecosystem and has few natural enemies, except in a few areas where it serves as an ingredient for humans. We wanted to study the impact of changes in the sex ratio of the species on the ecosystem, which is a large system, so we quantified it as the number of predators to be studied. Therefore, we need to build a model that describes food web relationships so that we can analyse and predict the effects of changes in the sex ratio of a species on the number of its prey, which in turn has an impact on the larger ecosystem.

A Lotka-Volterra model was used to describe the interactions of this species with other species in the ecosystem. In addition, the dynamics of male and female lampreys should be modelled separately in order to better describe the effects under different sex ratios.

Firstly, population dynamics equations for male lampreys were developed.

$$\frac{dM}{dt} = r_M \cdot M - d_M \cdot M + b_M \cdot a \cdot P \cdot M \tag{1}$$

Where M is the population size of male lampreys, r_M is the reproductive rate of male lampreys, d_M is the mortality rate of male lampreys, and b_M is the predation efficiency of male lampreys. a is the probability of prey predation, and P is the population size of the prey.

Similarly, population dynamics equations for female lampreys:

$$\frac{dF}{dt} = r_F \cdot F - d_F \cdot F + b_F \cdot a \cdot P \cdot F \tag{2}$$

In the equation, F is the population size of female lampreys, r_F is the reproductive rate of female lampreys, d_F is the mortality rate of female lampreys, b_F is the predation efficiency of female lampreys, a is the probability of prey predation, and P is the population size of prey.

Finally, predator population dynamics equations were developed based on Eqs. (1) (2).

$$\frac{dP}{dt} = r_P \cdot P - a \cdot P \cdot (M + F) \tag{3}$$

In the equation, P is the population size of the prey, r_P is the natural growth rate of the prey, a is the probability that the prey will be taken, M is the population size of male lampreys, and F is the population size of female lampreys.

The results obtained from the simulation are shown in Fig. 1. In this ecosystem, the number of females increased while the number of males remained almost constant. As a result, the proportion of females in lampreys' population increased. At the same time, the figure shows that as the proportion of females in lampreys increased, the number of preys decreased.

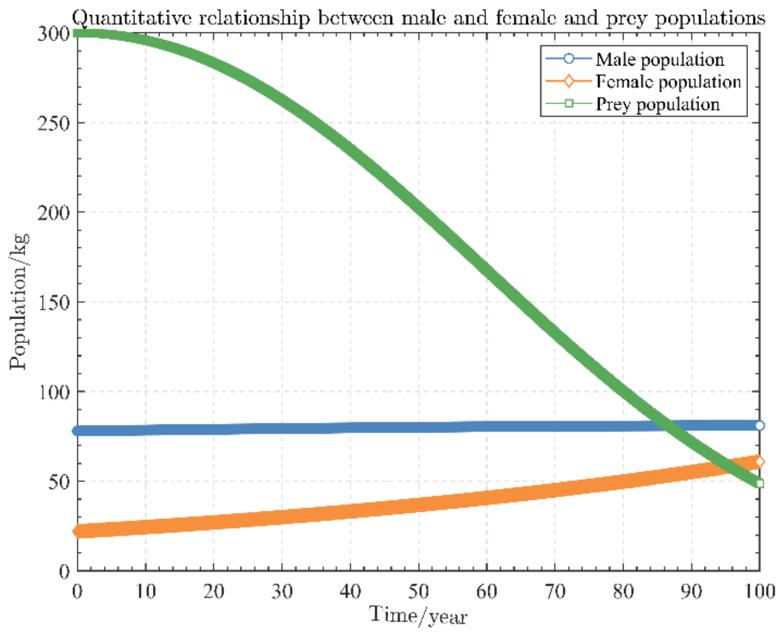


Figure 1. Quantitative relationship between male and female and prey population

Ecological networks characterize the interactions between biological components of an ecosystem. Food webs indicate a high degree of complexity in the trophic structure and processes of ecosystems, and this complexity is the basis for the cycling of matter and energy flows in ecosystems [7]. Energy is transformed into biological productivity at all trophic levels through the food web. It forms the output of ecosystem biological resources and influences ecosystem services and outputs and their dynamics.

Therefore, an increase in the female ratio causes population fluctuations, which disrupts the food web structure and affects the stability of the ecosystem. At the same time, excessive fluctuations in the sex ratio may cause instability at the ecosystem level and have a knock-on effect on the interactions of other species, ultimately affecting the stability of the whole ecosystem.

From the results, we can see that the ecosystem structure becomes unstable when the proportion of malelampreyss increases exponentially. So, we pondered how the population relationship between lampreys and predators would change if the ecosystem structure became unbalanced.

To test this conjecture, we re-simulated the model with male mortality fluctuating in the range of 0.020 ~ 0.040, as shown in Fig. 2:

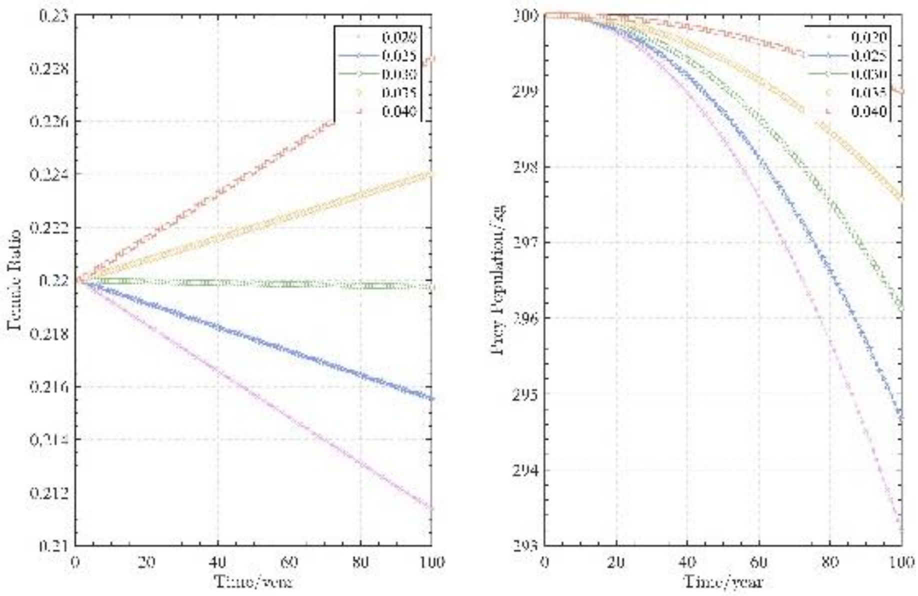


Figure 2. The male mortality rate fluctuates between 0.02 and 0.04

It can be seen that when male lampreys' mortality increased from 0.020 to 0.030, the proportion of females increased by about 0.01 after 100 years, but still tended to decrease. In contrast, when male mortality exceeded 0.030, the proportion of females tended to increase at a rate roughly proportional to male mortality. As male mortality continued to increase, the decline in predator numbers also continued to slow down, suggesting that the ecosystem will return to dynamic equilibrium following a reduction in the number of male lampreys.

Thus, not only do changes in the sex ratio of lampreys affect the ecosystem, but changes in the ecosystem also affect the sex ratio of lampreys. This also indirectly suggests thatlampreyss can select a sex ratio more suitable for the sub-ecosystem through changes in the ecosystem. Darwin's idea of evolution explains why some lives flourish and grow while others tend to die in the long course of history. Life has been able to evolve in a brutal competition for the survival of the fittest. "Survival of the fittest" is the best explanation for the survival of the lampreys to this day.

2.2. Modelling population dynamics through the modified Lotka-Volterra equation

Research has shown that in ecosystems there is constant sexual competition between species, whereby both females and males want to pass on more of their genetic material to their offspring, and when one side has a certain advantage it leads to a change in the sex ratio, and the other side evolves coping strategies to ensure a competitive advantage for the species as a whole in response to the competitive pressure.

In order to analyse the strengths and weaknesses of lamprey's population, we compared a population of lampreys with an adaptive sex ratio with a population of lampreys with a 1:1 sex ratio. It is known from searching for information that the proportion of male lampreys is greatly increased in the case of limited environmental resources [8-10]. In addition, during the development of lampreys, the rate at which it feeds will greatly influence the final sex of the individual. Therefore, we added a sex ratio adjustment function $g(P)$ to the Lotka-Volterra model developed in Section 2.1 to better study the competitive advantage of this species. Where P is the population size of the prey and the function is outputting different sex ratio adjustment values depending on the number of praises. The sex-ratio adjustment values range from 0 to 1, where 1 indicates a complete bias in favour of males and 0 indicates a complete bias in favour of females. The modified Lotka-Volterra model can be expressed as.

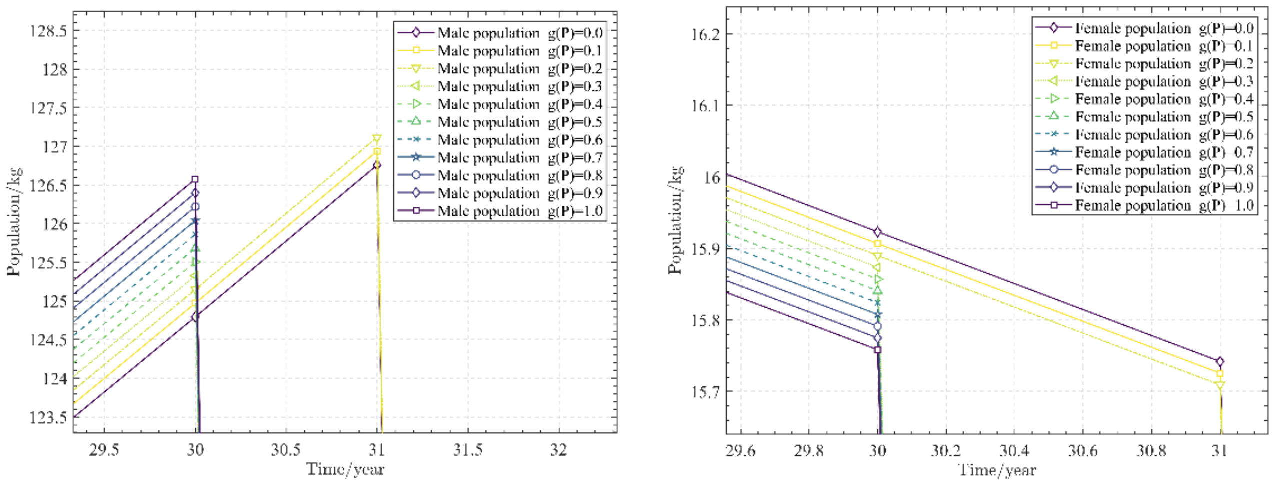
$$\begin{cases} \dfrac{dM}{dt} = r_M M g(P) - d_M M + b_M aPM \\ \dfrac{dF}{dt} = r_F(1-(P)) - d_F F + b_F aPF \\ \dfrac{dP}{dt} = r_P P - aP(M+F) \end{cases} \tag{4}$$

The reproduction rate of a population of lampreys with a fixed sex ratio of 1:1 is uniformly distributed between the two sexes, so that the Lotka-Volterra model can be expressed as follows.

$$\begin{cases} \dfrac{dM}{dt} = \dfrac{r}{2} M - d_M M + b_M aPM \\ \dfrac{dF}{dt} = r_F(1-g(P)) - d_F F + b_F aPF \\ \dfrac{dP}{dt} = r_P P - aP(M+F) \end{cases} \tag{5}$$

In the equation above, M is the population size of males, F is the population size of females, r_M is the reproductive rate of males, r_F is the reproductive rate of females, r is the reproductive rate of males to females, d_M is the mortality rate of males, d_F is the mortality rate of females, and b_M is the predation efficiency of males, b_F is the predation efficiency of female lampreys, r_P is the natural growth rate of prey, a is the probability of prey prey, P is the number of prey populations, and $g(P)$ is the sex ratio moderating function.

After obtaining these two sets of dynamic equations, the results obtained by setting the parameters reasonably are shown in Fig. 3:



(a) male (b) female

Figure 3. Under different $g(p)$ values, the population number changes with time.

Changes in the male population over time for different values of $g(P)$ are shown in Fig.3(a). The male population was found to have a higher proportion of males in the same year than in the previous year. In the same year, a larger value of g indicates a larger proportion of males. Changes in female population size over time for different values of $g(P)$ are shown in Fig.3 (b).

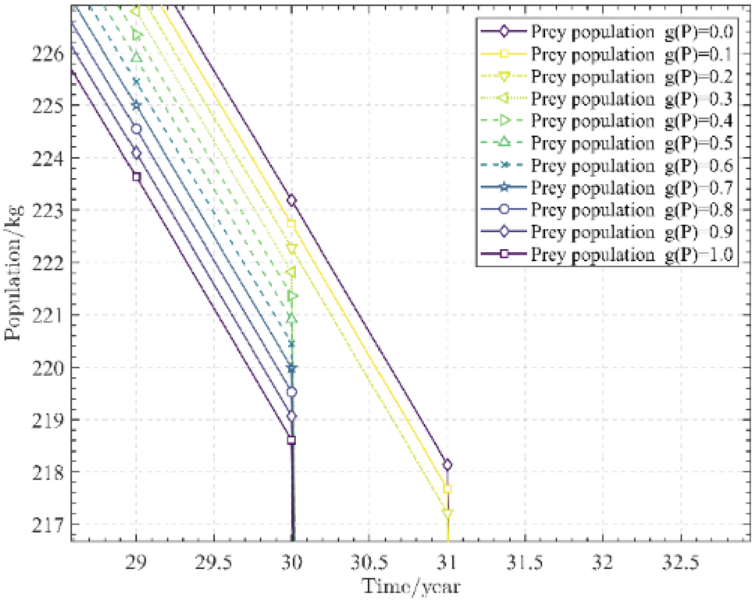


Figure 4. Number of preys under different $g(P)$ values

Fig. 4 shows the change in prey numbers over time for different values of $g(P)$. The number of preys is also shown in the figure. In the same year, it can be seen that when the g value increases, males increase and natural prey decrease. This indicates that environmental degradation leads to a decrease in the number of natural preys.

2.3. Modelling system dynamics and sex ratio and food availability

2.3.1. System Dynamics Modelling

In order to investigate the effects of changes in species sex ratios on ecosystem stability, we will consider the interactions of lampreys populations with other species, including competition and predation.

We introduced another new ecotype species and studied its interactions with lampreys' population. In order to develop a more complex system dynamics model to analyse the dynamic behaviour of lamprey's population, we combined a multi-species competition model, a logistic growth model and a Lotka-Volterra model to consider competition between lampreys' population and other species. food chain relationships and the effects of changing sex ratios on the ecosystem as a whole.

Firstly, the population dynamics equation is established:

$$\begin{cases} \dfrac{dN}{dt} = r_N \cdot N \cdot (1 - \dfrac{N}{K_N}) - \alpha_{NO} \cdot (N - O) \\ \dfrac{dF}{dt} = r_F \cdot F \cdot (1 - \dfrac{F+N}{K_F}) + \alpha_{FN} \cdot (N - F) \\ \dfrac{dM}{dt} = r_M \cdot M \cdot (1 - \dfrac{F+M}{K_M}) + \alpha_{MN} \cdot (N - M) \end{cases} \tag{6}$$

Then, other population dynamics equations were established:

$$\frac{dO}{dt} = r_O \cdot O \cdot (1 - \frac{O}{K_O}) - \beta_{ON} \cdot (O - N) \tag{7}$$

Ecosystem stability equations.

$$\frac{dS}{dt} = \gamma \cdot (\frac{dN}{dt} + \frac{dF}{dt} + \frac{dM}{dt} + \frac{dO}{dt}) \tag{8}$$

In order to visualize the complex system dynamics model of lamprey's population and to observe the changes in the number of lampreys, sex ratios and the populations of other species, the complex system dynamics model was simulated using the ODE45 function and the results were visualized.

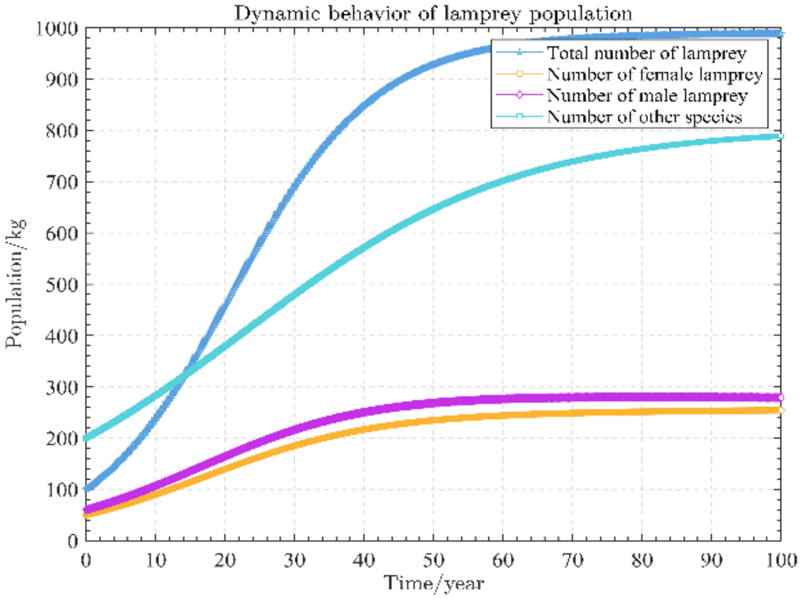


Figure 5. Dynamics of lamprey's population and other species

As can be seen in Fig. 5, when the male ratio increases, the resources available to lampreys decrease, while the populations of other species increase. The ecosystem reached stability when the sex ratio of lampreys was close to 1:1. The increase in the number of females favoured the competition of lampreys eel populations, the increase in the number of males favoured the competition of other populations, and the similarity of the sex ratio favoured the stability of the ecosystem.

In addition, we observed the equilibrium point and stability of the system by plotting phase diagrams and phase planes, and analysed time series to determine whether the system exhibited cyclic or chaotic behaviour. Finally, the stability curve of the ecosystem as a function of sex ratio was obtained based on the established stability equation.

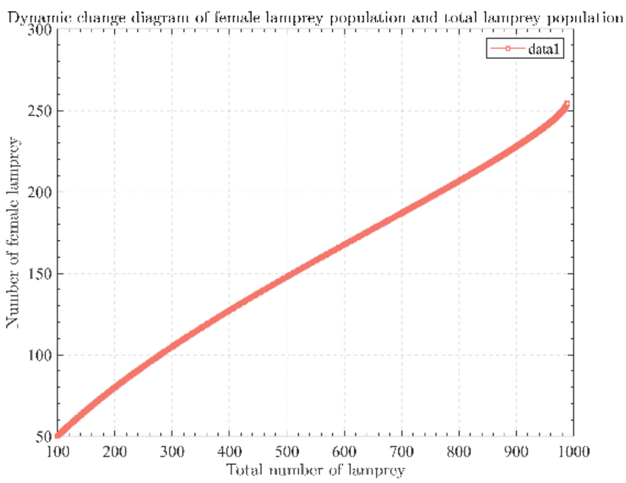


Figure 6. Phase plot of the number of female lampreys versus the total number of lampreys

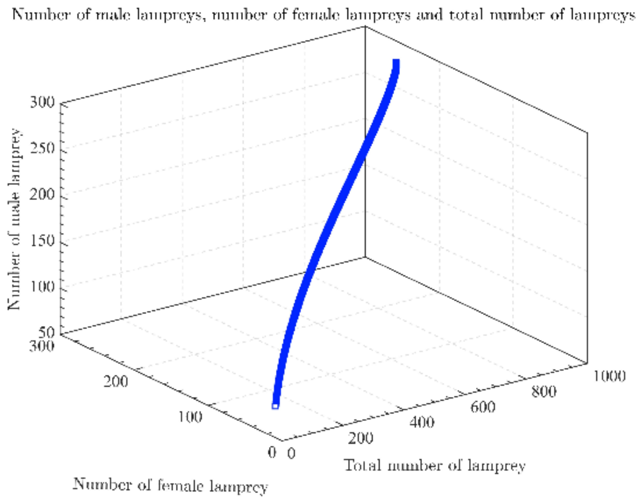


Figure 7. The phase plane of the number of male lampreys, the number of female lampreys and the total number of lampreys

Figure 8. Time series of changes in lamprey population with sex ratio

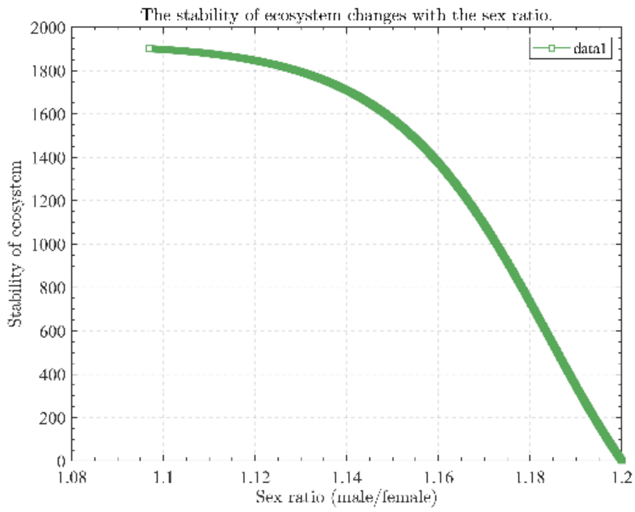


Figure 9. Relationship between ecosystem stability and gender ratio

From the Fig. 6 to 9, it can be seen that the stability of the ecosystem decreases when the proportion of female lampreys rises to a certain level. It also shows that female lampreys consume more resources, resulting in the sex ratio of lampreys population not being able to stabilise at one value all year round. Its population sex ratio will change according to the external environment.

This shows that when studying the stability of the ecosystem from the perspective of species interactions, the sex ratio of lampreys will change according to the environment, and at the same time, it will also carry out a negative feedback regulation of the stability of the ecosystem to improve the ability of the ecosystem to self-regulate and adjust.

Whether lampreyss are male or female also depends on the growth rate in the larval stage. Therefore, it is hypothesized that changes in sex ratio are influenced by food supply, which in turn influences the birth rate of male and female lampreys. This relationship between sex ratio and food supply is combined with the system dynamics equations described above.

2.3.2. Sex ratio and food availability modelling

Studies have shown that the proportion of male lampreys decreases as food availability increases, and based on this relationship, we can model this with a simple S-shaped function.

$$p = \frac{1}{1 + e^{-k(X - X_0)}} \tag{9}$$

In the equation, p denotes the proportion of males, X denotes the level of the food supply chain, X_0 denotes the threshold of food availability that affects the sex ratio, and k is a parameter that affects the steepness of the curve.

Based on equation (9), we can optimize the complex system dynamics model of lamprey's population by incorporating the relationship between sex ratio and food availability into the equation. It is assumed that the birth rate of female lampreys is proportional to food supply, while the birth rate of male lampreys is proportional to food supply and sex ratio.

$$\begin{cases} \dfrac{dN}{dt} = r_N \cdot N \cdot (1 - \dfrac{N}{K_N}) - \alpha_{NO} \cdot (N - O) \\ \dfrac{dF}{dt} = r_F \cdot F \cdot (1 - \dfrac{F + N}{K_F}) + \alpha_{FN} \cdot N \cdot P \cdot (N - F) \\ \dfrac{dM}{dt} = r_M \cdot M \cdot (1 - \dfrac{F + M}{K_M}) + \alpha_{MN} \cdot (N - M) \\ \dfrac{dO}{dt} = r_O \cdot O \cdot (1 - \dfrac{O}{K_O}) - \beta_{NO} \cdot (O - N) \end{cases} \quad (10)$$

We embedded the relationship between sex ratio and food supply into the calculation of birth rate of female lampreys, so that the effect of food supply on sex ratio could be considered in the system dynamics equation. By adjusting parameters k and X_0, the effect of sex ratio on the population dynamics of lampreys at different levels of food supply can be modelled.

To simulate the relationship between the total number of lampreys, the number of male lampreys, the number of female lampreys, the number of other species and the parameter when the system reaches the steady state for all possible values of parameters k and X_0. And simulate the relationship between the number of other species at steady state and the sex ratio of lampreyss at steady state.

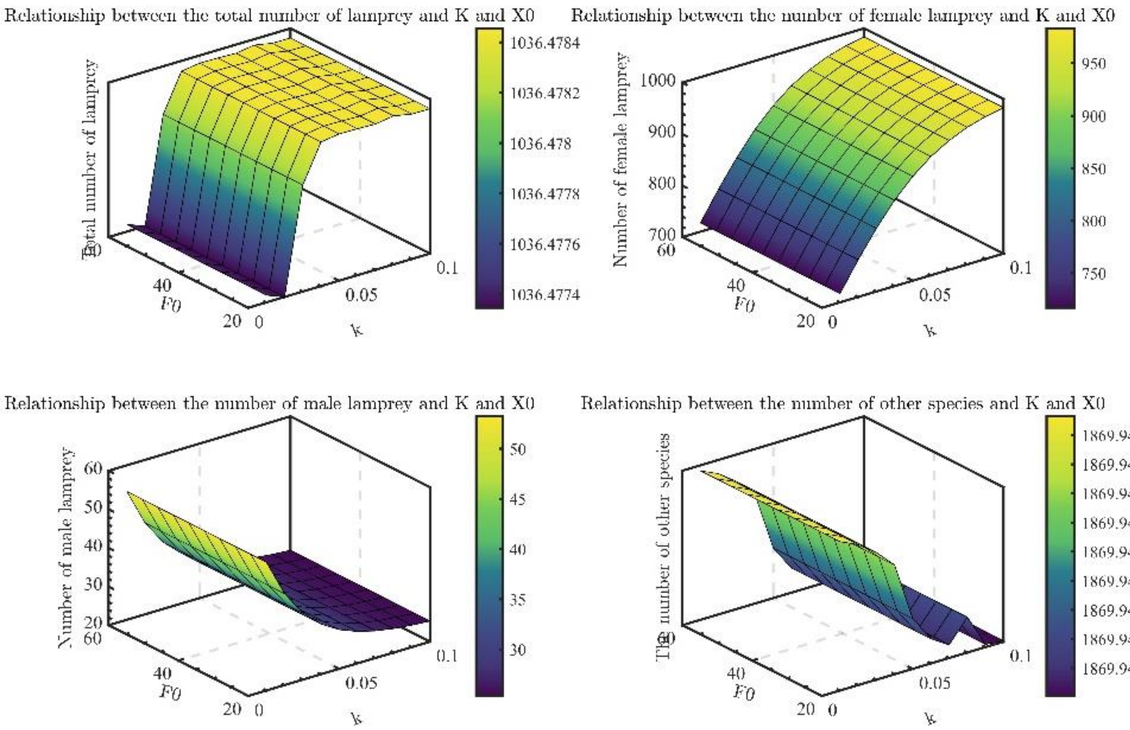


Figure 10. Relationship between k and X_0 and lamprey population, female lamprey population, male lamprey population, and other species

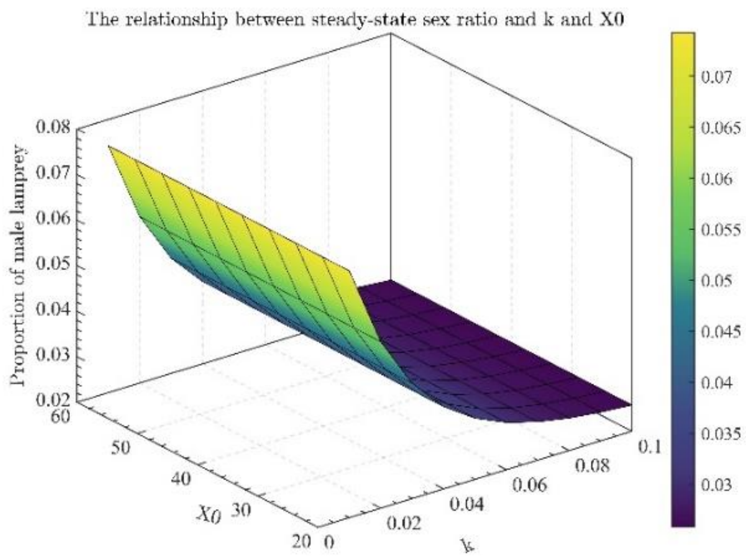


Figure 11. The relationship between steady-state sex ratios and k and X_0

Where k is the rate of change in sex ratio and X_0 is the food chain threshold that affects sex ratio.

As can be seen in Figures 10 and 11, the number of female lampreys increases and the number of male lampreys decreases in the presence of adequate food supply. This suggests that when the sex ratio of lamprey's changes, the available resources in the ecosystem will be affected.

Combined with the above conclusions, it can be concluded that when the sex ratio oflampreyss changes, it will not only affect the ecosystem food chain and cause changes in the number of other species, but also affect the change of available resources in the ecosystem.

3. Summary

In this study, we explored the effects of changes in sex ratio in the population of lampreys on the ecosystem through the Lotka-Volterra equation. We found that changes in the sex ratio of lampreys not only had an impact on its own survival and reproduction, but also had a significant effect on the stability of the entire ecosystem and the populations of other species.

First, changes in the sex ratio of lampreys affect their ability to hunt prey. When the proportion of female lampreys increases, the number of preys taken decreases, which may lead to fluctuations in prey resources in the ecosystem. This fluctuation may further affect other species that depend on this prey, thus creating a chain reaction throughout the food chain.

Second, our study also found that changes in the sex ratio of lampreys were closely related to environmental resource availability. The fact that the proportion of male lampreys increases when resources are limited may reflect an adaptive strategy that allows lampreys to better survive in environments where resources are scarce. However, this adaptive strategy may reduce the genetic diversity of lampreys, thereby affecting its long-term adaptive capacity.

Finally, our study suggests that changes in the sex ratio of lampreys have important implications for ecosystem stability. When the proportion of female lampreys is too high, the stability of the ecosystem decreases. This may be due to the fact that female lampreys consume more resources, resulting in a population with a sex ratio that cannot be stabilized at one value for a long period of time. Therefore, lampreys need to adjust their sex ratio according to changes in the external environment to maintain the stability of the ecosystem.

In summary, the change of the sex ratio of lampreys has a complex and far-reaching impact on the ecosystem. It not only affects the survival and reproduction of the lampreys itself, but may also trigger fluctuations in the populations of other species in the ecosystem and changes in resource availability. These findings are important for understanding the dynamic balance of the ecosystem and the interactions between species.

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