Dynamic Network Modeling of Optimal Water Level Control Based on RMOBM and DMOWN

Yuke Li *, Keyu Chen and Zhuojun Zhao

College of Electrical Engineering, Southwest Minzu University, Chengdu, China

* Corresponding author: cathylyk@163.com

Abstract. In this paper, the optimal water level of the Great Lakes and their connected waterways in North America is studied by using the multi-objective bargaining model based on water level risk (RMOBM model) and the dynamic multi-objective basin network (DMOWN model). First of all, we established the RMOBM model, taking into account the influence of rainfall, snowfall, evaporation of lake water and surface leakage on water level, and carried out multiple rounds of bargaining with the objective function of maximizing economic benefits and minimizing the probability of flood occurrence, and obtained the optimal water level results by using MATLAB software. Secondly, based on the RMOBM model and the maximum flow target value, we considered the flow regulation effect of Moses-Sanders Dam project and Cornwall County in the DMOWN model, obtained the maximum flow value through the Ford-Fulkerson algorithm, and then output the optimal lateral flow results of five major water level nodes. These studies provide a scientific basis for the management of North America's Great Lakes and their associated waterways and help develop effective planning and management strategies.

Keywords: Water level dynamic management, multi-objective bargaining model, watershed network dynamic model, Ford-Fulkerson algorithm.

1. Introduction

The Great Lakes of the United States and Canada are known as the largest group of freshwater lakes in the world, and this magnificent system of water not only constitutes the geographical symbol of North America, but also carries enormous ecological and economic functions. However, managing water levels in the Great Lakes is a challenging issue. The level of water level is directly related to the security, economic vitality and ecological balance of the surrounding cities, so how to effectively manage this resource has become an important issue faced by all stakeholders. In order to better manage the water level of the Great Lakes, two models are designed to meet the needs of various stakeholders and maintain the stability of the optimal water level.

First, with the help of historical data and literature review, we propose a risk model of water level in the Great Lakes based on multi-objective bargaining [1] to balance the relationship between economic development and ecological protection. Divide stakeholders into economic and environmental actors, and reach long-term stable agreements through a fair and equitable negotiation process. Secondly, considering the complexity of water level changes in the Great Lakes, we built a dynamic network flow model [2] to simplify the problem. To maintain the optimal water level of the Great Lakes, we established a multi-objective network dynamic model of the basin, and adopted the Ford-Fulkerson algorithm [3] to solve the maximum flow problem. By adopting the multi-objective optimization method, we solved the optimal water level of the Great Lakes basin to ensure the coordinated development of its ecological, economic and social functions. In summary, our research provides new ideas and methods for the management of water level in the Great Lakes, and is expected to provide scientific and effective decision support for relevant stakeholders in practice.

2. Multi-objective Bargaining Model Based on Great Lakes Water Level Risk

2.1. Multi-objective Bargaining Model Construction

The How the bargaining model is applied to a multi-objective problem requires that the two sets of stakeholders be considered as two participants, an environmental participant (Participant 1) and an economic participant (Participant 2). Participant 1's objective is to minimize the likelihood of flooding, and Participant 2's objective is to maximize the economic benefits from existing water levels.

At the very beginning it was necessary to determine the theoretical ranges of Participant 1 (environmental objectives) and Participant 2 (economic objectives) with the aim of determining the negotiation intervals for both participants. Boundary value calculations for the negotiation intervals are carried out by maximizing or minimizing the individual objective functions of the participants.

In round 0, ER_{min} is set as the target value for Participant 1, denoted as ER_{goal}, and EL_{max} is set as the target value for Participant 2, denoted as EL_{goal}.

For Participant 1: Since if too little water is drained or evaporated from the lake, flooding will result and littoral homes and businesses will be affected, this paper will consider the amount of water stored in the lake at the beginning of the month, the flow into the lake, and the number of anthropogenic releases per month. (1) is the model for Participant 1 with the optimization objective of minimizing the amount of water stored at the end of each month.

$$Minf_1(x) = ER = \sum_i \sum_j \sum_m \left(M_{i,j}^m + I_{i,j}^m - V_{i,j}^m - O_{i,j}^m - E_j\right)$$
$$s.t.\begin{cases} V_{i,j}^m \le V_{max}^m \\ E_j \le E_{max} \\ EL \ge EL_{goal} \end{cases} \tag{1}$$

Where ER - The likelihood of flooding, $M_{i,j}^m$ - The amount of water stored in the Great Lakes at the beginning of month j of year i.

1. Calculation of Water Storage Capacity

Based on the Great Lakes profile in the title annex, the lake storage volume can be approximated as a cone volume, with a constant cone slope and a diameter proportional to depth: $D = kH$, and H can be considered as the water level height. By querying the relevant water level data of the Great Lakes Coordinating Council, the historical water level of the Great Lakes at the beginning of the month is obtained, from which the storage volume of the Great Lakes at the beginning of the month is calculated. Reference (2):

$$V = \frac{\pi}{3} H \left(\frac{D}{2}\right)^2 \tag{2}$$

2. Calculation of Evaporation

Evapotranspiration (E) was calculated using the Hargreaves-Samani equation [4] calculation, which is a commonly used method under data scarcity conditions and is known for its simplicity and effectiveness. The method requires minimum data input while producing satisfactory results. In this paper, evapotranspiration can be calculated with the help of daily maximum and minimum temperature, daily average temperature and extraterrestrial solar radiation. The Hargreaves-Samani equation is given below:

$$E = \frac{\left(0.0023 \times R_a \times (T_{max} - T_{min})^{0.5} \times (T + 17.8)\right)}{2.45} \tag{3}$$

Where R_a is the extraterrestrial solar radiation (MJ/m^2), T_{max} and T_{min} are the daily maximum and minimum temperatures (℃), respectively, and T is the daily average temperature (℃).

For Participant 2: Since large ships will not be able to deliver supplies through the waterways if the discharge is too high, this will affect the local economy. The main issue is to regulate the water level so that all stakeholders can benefit. (4) is the model for Participant 2, whose optimization objective is to maximize the total economic benefit from each lake, which can be achieved at the current water level where multiple stakeholders benefit and can be maximized.

$$Maxf_2(x) = EL = \sum_{m=1}^{5}\sum_{i=1}^{20}\sum_{j=1}^{12} x_{i,j}^{m} \cdot VC$$
$$s.t.\begin{cases} \beta Q_m \le x_{i,j}^{m} \le Q_m \\ P_l \le x_{i,j}^{m} \le P_u \\ ER \le ER_{goal} \end{cases} \quad (4)$$

Where EL - The likelihood of flooding

VC - Revenue generated per average cubic meter of water per year

β - Coefficient for determining the minimum level

Q_m- Flood occurrence

1. Average Annual Revenue Per Cubic Meter of Water

The average annual revenue generated per cubic meter of water is closely related to the product generated per cubic meter, the unit price of the product, and taxes. Establish the formula:

$$VC = (CPD \cdot NB) - ST \quad (5)$$

Where VC is the average annual revenue generated per cubic meter of water, CPD is the number of products or services generated per cubic meter, NB is the unit price of products and services, and ST is taxes.

2. Calculation of $x_{i,j}^{m}$

The water level of a lake in the j-th month of year i is related to local precipitation, lake evaporation, inflow, outflow, and the previous month's lake level. Establish the formula:

$$x_{i,j}^{m} = \left(P + I_{i,j}^{m}\right) - \left(E + O_{i,j}^{m}\right) + x_{i,j-1}^{m}$$
$$x_{i,0}^{m} = x_{i-1,12}^{m} \quad (6)$$

In the (6), where P is the local precipitation and E is the evaporation from the lake.

3. Parameter Determination

Determine the parameters Q_m, P_l, P_u in this objective function, and determine the range of water level values by consulting the literature to obtain the number of floods occurring each year in the Great Lakes basin. The water level change is to be maintained between 2-3 feet: the lower limit of water level change P_l is 0.6090m and the upper limit of water level change P_u is 0.9144m.

2.2. Determination of Optimal Water Levels in the Great Lakes

In the objective functions established above for the two participants, Participant 1's objective is to minimize the likelihood of flooding, and Participant 2's objective is to maximize the economic return from existing water levels. These two objectives are in conflict with each other, but Participant 1's strategy is to minimize the likelihood of flooding without compromising economic returns, and Participant 2's strategy is to maximize economic returns while reducing the likelihood of flooding. This leads to the fact that the two participants will be dissatisfied with each other's results, and they will then have to engage in round after round of bargaining. With each additional round of bargaining, both participants make concessions: EL_{goal} gradually decreases and ER_{goal} gradually increases.

(7) gives the calculation of the yield value, and C_n denotes the participant's yield value. The calculation of the concession value for a participant is done by dividing the difference between the maximum and minimum values of the likelihood of a flood occurring and the difference between the maximum and minimum values of the economic benefit from the existing water level into equal parts. At the same time, the target values of both participants need to be reset: the target value of Participant 1 will increase by $C_n 1$ per round, while the target value of Participant 2 will decrease by $C_n 2$ per round.

$$C_n1 = \frac{ER_{max} - ER_{min}}{k}$$
$$C_n2 = \frac{EL_{max} - EL_{min}}{k} \tag{7}$$

where k is the value of a reasonable concession coefficient that can satisfy both participants. The following equation is the condition to reach the Nash equilibrium: the bargaining is terminated if it satisfies that the final solution of participant 1 is less than the objective value set by him and the final solution of participant 2 is greater than the objective value set by him. Where the Nash equilibrium solution of the multi-objective problem is ER_{final}, EL_{final} [5].

$$\begin{cases} ER_{final} \leq ER_{goal} \\ EL_{final} \geq EL_{goal} \end{cases} \tag{8}$$

Fig.1 gives the specific solution process for solving the RMOB-I model. First, set the objective values of the two objective functions; second, determine whether the model reaches the equilibrium conditions, if yes, output the model results, reach an agreement, and terminate the bargaining; if no, it is necessary to enter the next round of bargaining, and then go on to repeat the above steps until the model reaches the equilibrium conditions, and then the bargaining is over, and output the model results [6-7].

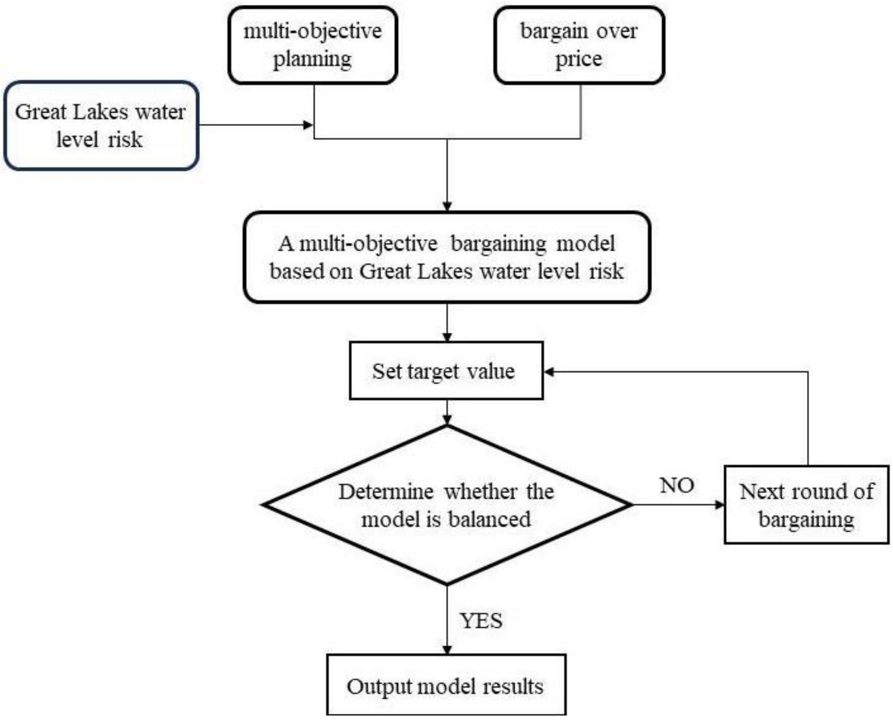

Figure 1. RMOB-I Model solution flow

In this paper, the optimization toolbox in MATLAB is used to solve the model. Through this model solving process, we obtained the optimal water level of the Great Lakes at any time of the year.

3. Dynamic Modeling of Multi-objective Watershed Networks

3.1. Dynamic Network Flow Model for the Great Lakes Basin

Modeling a network flow from Lake Superior to the Atlantic Ocean requires detailed modeling of the flow of water throughout the Great Lakes system, including rivers, canals, and other water bodies between the lakes. The model is intended to depict the direction and volume of flow: how water flows from Lake Superior to the Atlantic Ocean through different paths and the maximum flow.

3.1.1. Network Flow Model

1. Define network nodes and edges

Nodes: each lake, river confluence, and other key water level locations serve as nodes.

Edges: the river, canal and lake interfaces connecting the nodes are used as edges to represent possible paths of water flow.

Fig.2 is a network model map of the water flow path to help model the network flow.

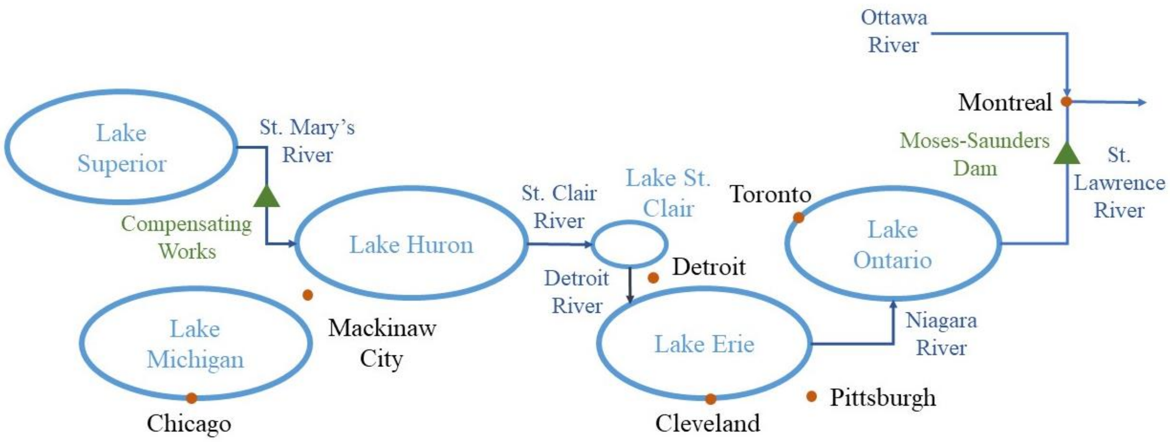

Figure 2. Schematic diagram of water flow pathway network flow model

The volume of water in the lakes is a state variable, controlled by inflow (precipitation, surface water inflow, and groundwater inflow) and outflow (evaporation, seepage, and surface water outflow). The dynamics of water levels in the Great Lakes can be schematically plotted.

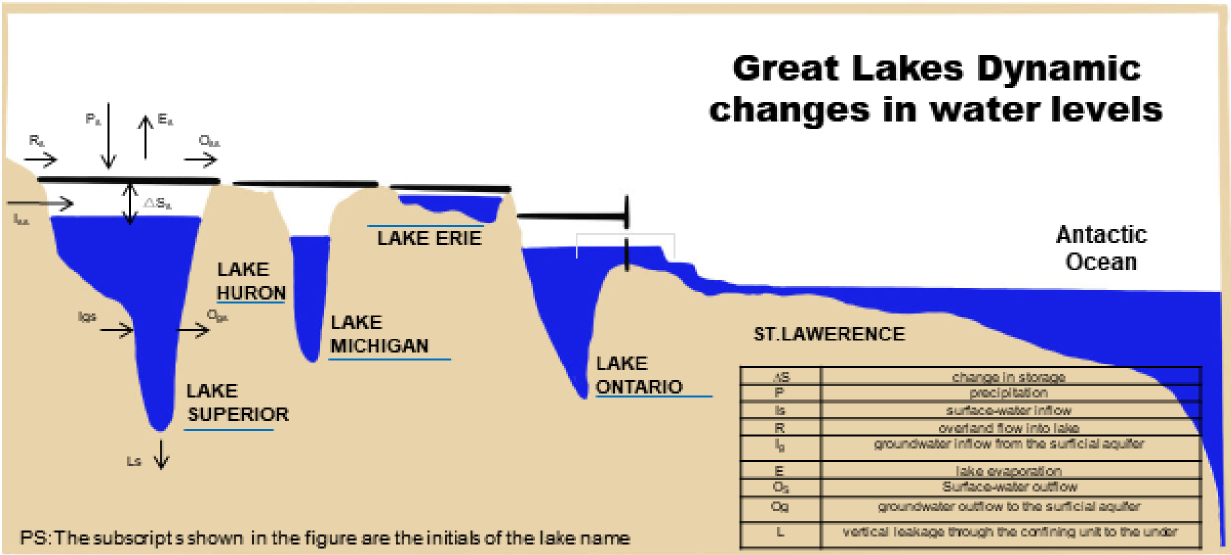

Figure 3. Schematic diagram of the dynamics of changes in the volume of water in the Great Lakes

In the Fig.3, we have used a number of symbols to describe the dynamics of water levels in the lake, listed in Table 1.

Based on the schematic diagram of the dynamics of water volume changes in the Great Lakes and the flow pathway network model diagram, a schematic diagram of the dynamically changing network flows was developed, as shown in Fig.4.

Table 1. Legend

Symbol	Specification
ΔS	change in storage
P	precipitation
Is	surface-water inflow
R	overland flow into lake
Ig	Groundwater inflow from the surficial aquifer
E	lake evaporation
Os	Surface-water outflow
Og	Groundwater outflow to the surficial aquifer
L	vertical leakage through the confining unit to the under
ΔS	change in storage

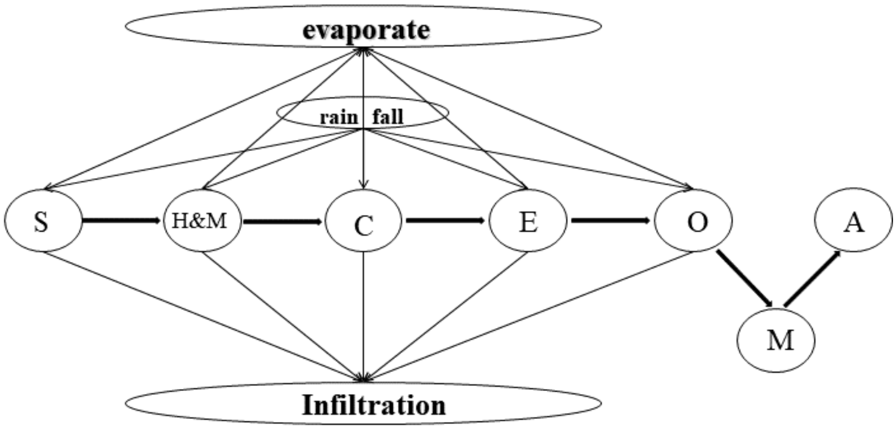

Figure 4. Schematic diagram of the network flow model

2. Flow and Capacity

Each edge has a flow rate that represents the amount of water that passes through that path in a unit of time.

Each edge also has a capacity limit that indicates the maximum amount of traffic the path can accommodate.

3. Network flow modeling

Using the network flow model from graph theory, the Ford-Fulkerson algorithm was used [8] to model the water flow.

Define a source (Lake Superior) and a sink (Atlantic Ocean)

The goal of the model is to find the maximum flow from the source to the sink, the maximum amount of water that can be delivered from Lake Superior to the Atlantic Ocean while meeting all capacity constraints.

4. Impact of environmental factors

Consider the effect of water level controls (e.g., dams and gates) on flow.

Consider environmental protection regulations, such as the minimum amount of water required to maintain a particular ecosystem during a given season.

3.1.2. Dynamic Network Flow Modeling Process for Water Level Changes

1. Defining Network Diagrams

Define a directed graph $(G = (V, E))$, where: v is the set of nodes including all inlet and outlet points of lakes, rivers other bodies of water. e is the set of directed edges denoting the paths of water flow. Each edge e points from node u to node v and indicates that water can flow from u to v.

2. Properties of nodes and edges

For each node v, define that $S(v)$ is the amount of water at the source (Lake Superior) and $T(v)$ is the amount of water received at the sink (Atlantic Ocean).

For each edge $e = (u, v)$, define two main properties:

Capacity $c\ (u, v)$: denotes the maximum flow rate of edge e: the maximum amount of water that can flow from node u to node v per unit time.

Flow rate $f\ (u, v)$: the actual water flow rate, which needs to satisfy $0 \leq f\ (u, v) \leq c\ (u, v)$.

3. Flux conservation constraint

For each node v in the graph (except for the source and sink), the flow conservation constraints require:

$$\sum_{in(u,v)} f(u, v) = \sum_{out(v,w)} f(v, w) \tag{9}$$

(9) means that the sum of flows entering any node must equal the sum of flows leaving that node.

4. Objective function

The goal is to maximize the total flow from the source (Evaporate) to the sink (Infiltration).

$$max \sum_{(S,v)\in E} f(S, v) \tag{10}$$

Simultaneously satisfy the capacity constraints at all edges and the flow conservation constraints at all non-source and non-sink points.

3.2. Algorithms for Realizing Dynamic Models of Multi-objective Watershed Networks

It is necessary to ensure that the water level is in the optimal position and also to consider the input and output quantities to maintain the dynamic water level. A network flow algorithm is used to simulate the water flow dynamics and an objective optimization method is applied to find a strategy to maintain the optimal water level. In this paper, a multi-objective bargaining model and a dynamic network flow model are integrated to form a multi-objective watershed network dynamic model, and a multi-objective optimization method is used to solve for the optimal side flow rates of the five major water level nodes [9-10]**.** The algorithm maintains the water level better at the optimal water level so that the fluctuation of the water level is as small as possible, which better satisfies the stability of the optimal water level and the interests of the neighboring stakeholders.

3.2.1. Inverse solution of multi-objective bargaining model based on Great Lakes water level risk

The algorithm takes the optimal water level obtained by the RMOBM model as the input value, and the maximum flow from Lake Superior to the Atlantic Ocean obtained by the Ford-Fulkerson algorithm as the objective value. The objective function established in the multi-objective bargaining model based on the Great Lakes water level risk, the optimal water level is known, and only the decision variables need to be changed, so that the input flow and the output flow become unknown quantities, and the optimal flow of the six rivers is optimized, and the optimal flow of the six rivers is artificially controlled by the use of dams. The optimal flow of the six rivers can then better maintain the optimal water level of the Great Lakes.

3.2.2. Optimal Side Flow at Five Water Level Nodes

To implement the aforementioned network flow model, this paper uses Python's networkx library to construct a graph theoretic model and solve the maximum flow problem from Lake Superior to the Atlantic Ocean.

In this paper, the optimization toolbox in MATLAB is used to implement the inverse solution, and the optimal flow rate for each side of the network flow model as the six rivers, to be able to maintain the optimal water level of the lake is obtained, and the results are shown in Table 2:

Table 2. Flow of the six major rivers capable of maintaining optimal lake levels

Month	Niagara River	St. Mary's River	Detroit River	Ottawa River	St. Lawrence River
Jan	6369.64	1891.57	5569.52	1843.58	8411.88
Feb	6477.36	1837.56	5389.97	2033.17	7940.20
Mar	6485.52	1838.63	5999.77	2370	8455.25
Apr	6962.69	1815.39	6061.63	2180	8573.47
May	7011.36	1930.25	6022.62	4037	8440.69
Jun	7046.79	2155.38	6056.96	4533	8550.31
Jul	6963.63	2369.56	5993.51	3244	8551.61
Aug	6941.30	2416.62	5986.34	1393	8311.67
Sep	6762.54	2255.57	6039.35	942	7899.46
Oct	6167.52	2207.73	6058.93	992	7242.73
Nov	6417.67	2138.32	6072.52	1440	6742.30
Dec	6498.31	2001.90	5989.35	2745	6684.56

4. Summary

This paper establishes a multi-objective bargaining model and a multi-objective basin network dynamic model based on the risk of water level in the Great Lakes, analyzes the optimal water level to meet the needs of various stakeholders, and studies the control algorithm to maintain the optimal water level and its dynamic stability. Based on the multi-objective bargaining model (RMOBM model), we optimized the optimal water level data of the Great Lakes to meet the needs of various stakeholders and the optimal flow data to maintain the optimal water level. Stakeholders are divided into economic participants and environmental participants, and the final optimization result is obtained through multiple rounds of bargaining with the objective function of maximizing economic benefits and minimizing flood occurrence probability. Secondly, the multi-objective Basin Network Dynamic model (DMOWN model) was established to model the dynamic network flow from Lake Superior to the Atlantic Ocean. Based on the multi-objective bargaining model, the ecological and economic balance of the Great Lakes was maintained at the optimal water level through accurate calculation of the inflow and outflow data of the Great Lakes, so as to realize the management and sustainable utilization of this precious water resource.

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