

Comparison of Cournot Duopoly Model and Stackelberg Model

Leqing Tan *

Guangdong Guangya High School, Guangzhou, China

* Corresponding author: leqing@ldy.edu.rs

Abstract. As a matter of fact, the game theory is a kind of mathematical study of economic interactions. The Cournot Duopoly Model and the Stackelberg Model are the two widely used concepts in the oligopoly market. Both are similar but different in firms' order of making decisions. Firms in the Cournot Duopoly Model announce the quantity of production simultaneously, which is different from the Stackelberg Model where the leader makes decisions first then the follower responds to the leader's plan. Therefore, the profits of firms are unequal in the two models. Firms in the Cournot Duopoly Model earn the same profits while the leader gains more advantage than the follower in the Stackelberg Model. This study aims to compare the two models and explain the difference between the two models is due to the order of announcing plans. It points out that the two models should consider the external factors that affect the outcome of models in future development.

Keywords: Game theory, Cournot duopoly model, Stackelberg model, oligopoly.

1. Introduction

The mathematical study of interactions between autonomous, self-interested agents is known as game theory. When John von Neumann and Oscar Morgenstern originally developed game theory in 1944, they created the first strategic games, in which several players analyze each other's best response of action and the outcome of the equilibrium [1]. Individuals make decisions according to calculation and logical reasoning.

With microeconomics serving as its primary application field, mathematicians and economists are the ones who study it the most [2]. Game theory has been used in practical military decision-making, teaching materials, and finding solutions to complex situations [3]. A market structure known as an oligopoly is one in which there are few firms. None of the firms in an oligopoly has significant market power, and no firm can raise prices above what they would in a situation of perfect competition. The theory of oligopoly explains how businesses in a market with few competitors set their prices and produce goods. Because of the complexity of the interactions between enterprises, there exist multiple models of oligopoly. The Cournot Duopoly Model and the Stackelberg Model are two oligopoly models that describe the competition between two firms. The oligopoly model can be integrated in examining the interaction between airline companies, but the Cournot Duopoly Model-based version is preferred in describing data [4].

Besides, the Cournot Duopoly Model can be applied to the structure, properties, and solution approach of a model in the international energy market [5] and consider renewable energy uncertainty costs [6]. Stackelberg Model is used in the telecommunication industry and computational systems, and it's also useful in systems where security issues are important decision-making criteria nowadays [7].

This study aims to compare two typical oligopoly models, the Cournot Duopoly Model and the Stackelberg Model to find their features and differences in outcomes. It includes the introductions of the Cournot Duopoly Model and the Stackelberg Model, a comparison of data, and the current limitations in the field of the oligopoly model.

2. Cournot Duopoly Model

Augustin Cournot gave the classical proof of game equilibrium between two companies in *Researches into the Mathematical Principles of the Theory of Wealth* in 1838. The Cournot Duopoly Model is an economic theory in which firms compete on the quantity of products while making

independent decisions. It assumes that companies make identical products and make decisions simultaneously. In the Cournot Duopoly model, it assumes that there are only 2 firms in the oligopoly market. Because the capacity of this market is limited, the market can only sell a limited quantity of products at a certain price. As a result, it causes competition in the quantity of products between two firms [8]. If one considers the market from the perspective of Firm 1, it will estimate the quantity of products that Firm 2 will produce and then decide its quantity. The price in the market is the function related to the sum of two firm's quantities.

$$p = p(q_1 + q_2^e) \quad (1)$$

Here, q_1 is the quantity of products of Firm 1, and q_2 is the quantity of products of Firm 2. Meanwhile, the two firms aim to maximize their own profits, so Firm 1's profit maximization can be represented as

$$\max_{q_1} \{p(q_1 + q_2^e) - C(q_1)\} \quad (2)$$

For any given belief q_2^e , there is a corresponding quantity of products q_1 of Firm 1. Therefore, the reaction function of Firm 1 is

$$q_1 = f(q_2^e) \quad (3)$$

It's the same as Firm 2. Firm 2 will consider its quantity according to the belief of Firm 1's quantity of products.

$$q_2 = f(q_1^e) \quad (4)$$

Nash equilibrium is a strategy that each player's strategy, and each strategy plays the best response to other's action in a noncooperative n-person game [9]. In Cournot Model, Cournot equilibrium refers to the existence of such a combination of output (q_1^*, q_2^*) that satisfies:

$$\begin{cases} q_1^* = f_1(q_2^*) \\ q_2^* = f_2(q_1^*) \end{cases} \quad (5)$$

The demand curve is

$$p = a - bp \quad (a > 0, b > 0) \quad (6)$$

Since $p = q_1 + q_2$, the profit function of Firm 1 is

$$\pi_1(q_1, q_2) = (a - bp)q_1 - c(q_1) \quad (7)$$

As a result,

$$\pi_1(q_1, q_2) = aq_1 - bq_1^2 - bq_1q_2 - cq_1 \quad (8)$$

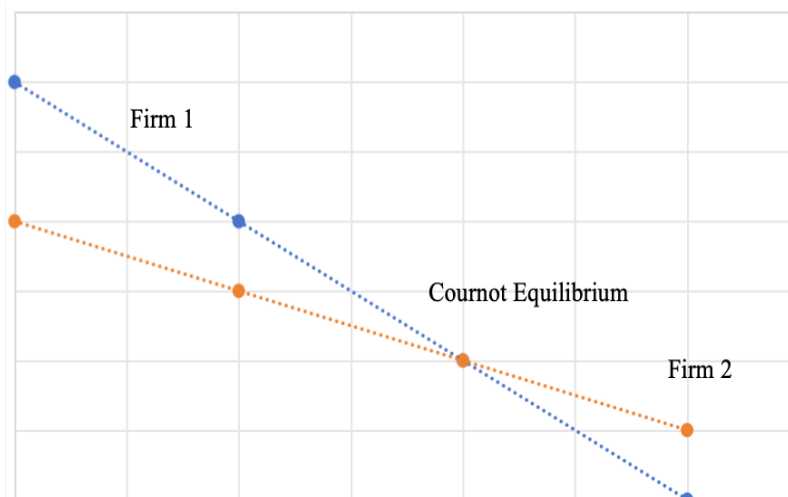


Figure 1. A sketch of the Cournot Model.

Actually, Firm 1 corresponds to every quantity of products q_2 of Firm 2, finding the maximum profits from different profit functions. The maximum profit point must be a point on a profit line where the slope of tangent line is zero:

$$a - bq_2 - 2q_1 - c = 0 \quad (9)$$

so,

$$q_1^* = \frac{a - bq_2^e - c}{2b} \quad (10)$$

Therefore, one can get Firm 2's reaction function

$$q_2^* = \frac{a - bq_1^e - c}{2b} \quad (11)$$

When it reaches equilibrium, two reaction function must intersect with each other, so $q_1 = q_1^*$ and $q_2 = q_2^*$. Then one can get that:

$$q_1^* = \frac{a - c}{3b} = q_2^* \quad (12)$$

$$\pi_1(q_1, q_2) = \frac{(a - c)^2}{9b} \quad (13)$$

$$\pi_2(q_1, q_2) = \frac{(a - c)^2}{9b} \quad (14)$$

A typical sketch for the results is shown in Fig. 1.

3. Stackelberg Model

The Stackelberg Model is named after the German economist Heinrich Freiherr von Stackelberg who described the model in Market Structure and Equilibrium in 1934. It is a two-player interactive strategy game on two different levels. One of the players makes the decision first, called the leader. The other player chooses its response, called the follower [10]. The price is decided by the sum of quantity $(q_1 + q_2)$ of production of the leader's quantity q_1 and the follower's quantity q_2 . As a result, the price is a function of $(q_1 + q_2)$. Assuming Firm 1 is the leader, it has the first-mover advantage. Since it has the right to move first, it will consider the follower's reaction function that is triggered by its decision of production of quantity. As a result, one needs to use backward induction to analyze the reaction function of the follower Firm 2, then figure out the best strategy for production of Firm 1. If Firm 1 announces its quantity of production, Firm 2 should consider:

$$\pi_2(q_1, q_2) = p(q_1 + q_2)q_2 - c_2(q_2) = (a - b(q_1 + q_2))q_2 - cq_2 \quad (15)$$

When one differentiates it, one derives the maximization of profit of Firm 2.

$$a - 2bq_2 - bq_1 - c = 0 \quad (16)$$

So,

$$q_2 = \frac{a - bq_1 - c}{2b} \quad (17)$$

Once the leader knows that its q_1 will cause q_2 , it will announce a q_1 to influence the function of Firm 2 in order to maximize its own profit. As a result, the leader needs to consider that

$$\pi_1(q_1, q_2) = p(q_1 + q_2)q_1 - c_1q_1 \quad (18)$$

One can get:

$$q_1^* = \frac{a - c}{2b} \quad (19)$$

$$q_2^* = \frac{a - c}{4b} \quad (20)$$

$$\pi_1(q_1, q_2) = \frac{1}{4b}(a - c)^2 \quad (21)$$

$$\pi_2(q_1, q_2) = \frac{1}{16b}(a - c)^2 \quad (22)$$

4. Comparison of Cournot Duopoly Model and Stackelberg Model

The Cournot equilibrium and the Stackelberg equilibrium are the two most widely used in the oligopoly concept. In the Cournot model, two firms decide the quantity of production simultaneously. However, the leader moves first then the follower responds to the leader in the Stackelberg model [11]. Comparing the firms' profits under Cournot equilibrium and Stackelberg equilibrium, the leader's profit in the Stackelberg Model is greater than firms' profit in the Cournot Model. Follower's profit in Stackelberg Model is less than the firms' profit in the Cournot Model [12].

The difference between the Cournot Duopoly Model and the Stackelberg Model is due to the order of making the decisions. For the Cournot Duopoly Model, two firms decide their quantity of production at the same time. Instead of simultaneously announcing production plans, firms make decisions sequentially in the Stackelberg Model. The follower must make a response after the leader announces the plan. The leader has the first-mover advantage, so its profit is greater than the firms' profits in the Cournot Duopoly Model. The follower has to respond to the leader, so its profit is less than the firms' profits in the Cournot Duopoly Model.

5. Limitations and Future Outlooks

There are still limitations of the Cournot Duopoly Model and the Stackelberg Model in practice. Firstly, they ignore the actual environment of competition. The competition in oligopoly is more competitive and complicated, so there are more factors that influence the quantity of production of firms and profits. Because the number of firms is relatively small, the needed bargaining cost and opportunity cost are relatively less. As a result, many oligarchs choose to collude with others in real life. So, the supply of products in the market decreases, prices then increase, and firms earn more profits [13]. This causes the actual profits and quantity of production to not match with the one in the Cournot Duopoly Model and the Stackelberg Model. Besides, technology plays an important role in the market. Technology revolution may cause a change in competitive relations, so it will affect the plans and decisions of firms in an oligopoly market to make greater profits. Because of the unstable relation and variational environment, the Cournot Duopoly Model and the Stackelberg Model are not suitable.

In the future, there is room for oligarchic models to grow. Facing the multiple environments, the Cournot Duopoly Model and the Stackelberg Model can consider more factors that influence the decisions of firms in an oligopoly market. Both of the two models can be more practical instead of only calculating the profits of firms. In addition, the change in competitive relations should be included in the models. Using a suitable constant or factor to describe the consequences is necessary.

6. Conclusion

This paper introduces the Cournot Duopoly Model and the Stackelberg Model and discusses the difference between the two models. In the Cournot Duopoly Model, firms make decisions at the same time and the profits of firms are equal. By contrast, firms announce production plans sequentially and the profit of the leader is greater than the one of the followers. The reason why the profits of firms are different in the two models is that firms move in different order, so the follower gains the first-mover advantage which increases its profit. However, the two models ignore other reality factors, like collision and technological revolution. Models can consider the change in the market environment and the relationships in competition can be involved in the future. The paper aims to summarize the current achievements of the Cournot Duopoly Model and the Stackelberg Model and

contrasts the two models through calculation. Then figuring out the place needs to improve and future direction.

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