Research on Submersible Safety Operations Based on the Basic Information 3D Motion Trajectory and Entropy Weight Method Models

Guanlin Li [1], Derong Deng [2, *]

[1] College of Information, North China University of Technology, Beijing, China, 100144

[2] Department of Engineering, Shenzhen MSU-BIT University, Shenzhen, Guangdong, China, 518172

* Corresponding Author Email: derongdeng.dero@gmail.com

Abstract. This research outlines a strategy to enhance ocean exploration safety, using the Ionian Sea as a case study, by integrating theoretical and practical methods. It develops a comprehensive model that predicts submersible locations, optimizes search operations, and manages multiple units simultaneously, ensuring efficient and safe underwater tours in various challenging marine environments. The model simulates the submersible's trajectory using a three-dimensional Cartesian coordinate system, accounting for propulsion power, communication status, and environmental factors such as time-variant ocean currents, position, velocity, temperature, and density. Outputs include a 3D motion trajectory char, providing visual representations of the submersible's path. Additionally, the model addresses the selection and evaluation of search and rescue equipment using the Entropy Weight Method, which calculates the weights of evaluation indicators leading to a Continuous Variable Model that enhances decision-making. This aids in choosing the optimal combination of equipment based on cost, reliability, efficiency, and service life, maximizing search efficiency within a budget.

Keywords: 3D Motion Trajectory, Entropy Weight Method, Continuous Variable Model.

1. Introduction

The exploration and utilization of underwater environments pose significant challenges, particularly regarding the safety and efficiency of submersible operations. Advances in technology have led to the development of sophisticated models that enhance the predictability and management of submersibles in complex marine settings. The utilization of 3D motion trajectory analysis, coupled with the Entropy Weight Method (EWM) and Continuous Variable Model (CVM), represents a critical evolution in this domain. These models leverage computational techniques to simulate submersible behaviors, which are crucial for operational planning and crisis management in the challenging underwater conditions of the Ionian Sea.

Historically, the safety of submersibles has been a focal point of ocean engineering research. Previous studies have predominantly focused on improving the mechanical strength and operational protocols of submersibles to withstand harsh underwater environments [1]. The incorporation of 3D modeling techniques has allowed for a more detailed understanding of submersible dynamics under various conditions [2]. Moreover, the application of the Entropy Weight Method with a further model--Continuous Variable Model has been explored in various decision-making frameworks to optimize equipment selection and operational strategies [3]. These methodologies have significantly contributed to enhancing the reliability and effectiveness of submersible operations.

Despite advancements, current models often neglect the integration of continuous variable modeling and detailed environmental interaction, crucial for real-time operational adjustments and safety improvements. To enhance underwater safety and streamline rescue operations post-accident, the research emphasizes the need for additional equipment, acknowledging uncertainties in both existing and predictive models, as well as real-world variability. The research recommends selecting this equipment based on the inherent uncertainties of the prediction model and real-world conditions. Recognizing the importance of each component at ocean depths, the research advocate for precise equipment selection using the Entropy Weight Method (EWM) and Continuous Variable Model (CVM) to improve submarine safety [4].

The research presents an integrated model that merges 3D motion trajectory precision with the robust decision-making frameworks of the EWM and CVM. This model accurately predicts potential paths and operational challenges, adapting to dynamic environmental conditions and operational states to enhance safety and efficiency in submersible operations in the Ionian Sea. This comprehensive approach optimizes safety and operational efficiency across various marine environments.

2. The basic fundamental of 3D motion trajectory based on the basic information and Entropy Weight Method (EWM) Models

2.1. The Structure of the 3D Motion Trajectory Model

3D motion trajectory models are pivotal in fields like computer vision, virtual reality, robotics, and more, enabling the tracking of position, velocity, and acceleration for target tracking, motion analysis, and environment sensing [5]. These models are integral to robot navigation, medical image processing, and understanding submerged object movements. By combining geometric parameters and motion equations, they clarify the spatial dynamics in three-dimensional space [6]. Utilizing real-time observations and sensor data, these models provide accurate tracking and analysis of trajectories, crucial for the safety and efficiency of submersible operations.

The following's a detailed explanation of the 3D motion trajectory model [5-6]:

(1) Input/output interface: Receives initial object parameters and external controls.

(2) Mathematical model: Core component with equations for object motion calculation.

(3) Output data: Provides object position, velocity, and acceleration over time.

To effectively model three-dimensional marine environments and predict submarine trajectories, it's essential to gather key environmental variables. These include current direction, seawater density, temperature, ocean wind direction, and submarine-specific parameters such as weight, acceleration, and angular velocity [7-8]. This data is crucial for building accurate models of both the marine environment and submarine dynamics [8].

However, collecting and integrating these variables is challenging due to the complexity of the equations and computational demands, making accurate fittings difficult. This study proposes a novel approach using three-dimensional modeling and positioning models that work with sparse datasets [9]. This method allows for effective modeling of the marine environment and submarine dynamics, facilitating precise simulation of submarine positional changes over time. Next, the research detail the methods used to gather and integrate the environmental variables and model the dynamics of submarine movement in the virtual marine environment.

2.1.1. Building Submarine's Motion Dynamics

The research first establishes a coordinate system, given that the research considers relative positions and take the submarine as the origin, the research set it as a random value, hence defining the position vector as Formula 1:

$$\overrightarrow{P(t)}=\left(P_x(t),\ P_y(t),\ P_z(t)\right) \tag{1}$$

To meet the requirement of clear observation of the ocean and to make the submarine simulation realistic and reasonable, the research set the initial linear velocity to any value within the 0–10 m/s range; set the linear acceleration per second to any value within the -0.1–0.1 m/s^2 range; set the initial angular velocity to any value within the 0–2π rad/s range; and set the angular acceleration per second to any value within the -0.1–0.1 rad/s^2 range. Specifically, the velocity-related formulas are set as Formula 2-5 as follows:

$$\overrightarrow{v(t)}=\left(v_x(t),\ v_y(t),\ v_z(t)\right) \tag{2}$$

$$\overrightarrow{Ac(t)}=\left(Ac_x(t),\ Ac_y(t),\ Ac_z(t)\right) \tag{3}$$

$$A_{V(t)}\in(0,2\Pi) \tag{4}$$

$$A_{V_{Ac(t)}}\in(-0.1,0.1) \tag{5}$$

Subsequently, the research design and establish a submarine propulsion model, primarily involving velocity update and position update formulas, as shown in the Formula 6-7:

$$\vec{v}_{new}=\vec{v}+\overrightarrow{ac}*\Delta t \tag{6}$$

$$\overrightarrow{position_{new}}=\overrightarrow{position}+\vec{v}*\Delta t+0.5*\overrightarrow{ac}*(\Delta t)^2 \tag{7}$$

Considering the Ionian Sea's maximum depth of 5,267 meters and the need for safety and economic considerations, the research set the research submarine's maximum operational depth at 4,500 meters. After reviewing various models, the research selected China's "Jiao long" submarine, which also has a maximum diving depth of 4,500 meters. The research is using its weight and volume specifications to establish and initialize the research model parameters, as detailed in Table 1.

Table 1. Data from "Jiao long" submarine.

Quality	Volume
200000kg (20t)	$9.3\times3\times4\ m^3$

After obtaining the parameters of the submarine itself, the research set the gravitational acceleration g=9.81m/s2; in the formula, the research considers the seawater density to be 1025 kg/m31025kg/m3; at this point, the research introduces Formula 8-11:

$$\vec{G}=m*\vec{g} \tag{8}$$

$$|\overrightarrow{F_b}|=\rho*g*V \tag{9}$$

$$\overrightarrow{F_{n_v}}=\overrightarrow{F_b}-\vec{G} \tag{10}$$

From Newton's second law, the net effect of gravity and buoyancy is converted into acceleration (since F=ma):

$$\overrightarrow{a_{n_v}}=\frac{\overrightarrow{F_{n_v}}}{m} \tag{11}$$

2.1.2. Building the Virtual Marine Environment Model

Following the integration of acquired Ionian Sea data and environmental impact considerations, an Environmental Impact Function, denoted E, was formulated. To simulate the marine environment, especially accounting for the damping effect of water currents' resistance on submarine movement from various directions, the implementation of the

Navier-Stokes equations for ocean current simulation were initially contemplated, as described in Formula 12

$$\rho\left(\frac{\partial v}{\partial t}+v*\nabla v\right)=-\nabla p+\mu\nabla^{2}v+\rho*g \tag{12}$$

However, due to the high computational load and complexity associated with the Navier-Stokes equations, a simplified model for ocean currents was ultimately adopted. This model introduced a continuous dynamic system akin to an air resistance model for ocean currents, with parameters defined according to Ionian Sea data as outlined in Formula 13.

$$\vec{F}_{o_c}=-\alpha*\overrightarrow{v(t)}+\beta*\vec{F}+\sigma*rand(n\sqrt{t}) \tag{13}$$

In this model, several variables are critical: $v(t)$ represents the velocity of ocean currents at time t; α is the friction coefficient, reflecting the decelerative impact of friction on current velocity; F is the external driving force, influenced by factors like ocean swells, eddies, gyres, wind, and thermal differentials; β quantifies the impact of F on current velocity; σ is a proportionality coefficient that modulates the strength of perturbations from standard normally distributed random numbers; and t adjusts the effects of random disturbances over time, signifying increasing perturbations. These elements are crucial for simulating the dynamics of ocean currents.

A random velocity factor, was also incorporated to enhance the simulation of ocean current randomness, thereby augmenting the model's realism.

Additionally, following comprehensive analytical validation, the introduction of several factors was contemplated to more accurately model the genuine Ionian Sea marine environment.

Following are the detailed formulas definitions for each influence:

(1) Incorporating the Effect of Time Variation (time_effect):

The research utilizes sine and cosine functions to model the cyclical variation of environmental factors over time. sin(t/10) and cos(t/10) represent the cyclical changes in environmental impacts in the X and Y directions. Meanwhile, −0.02sin(t/5) introduces another type of cyclical change in the Z direction, possibly representing vertical environmental factors, such as rising or sinking ocean currents, as in Formula 14:

$$\overrightarrow{\mathrm{F}_{\mathrm{time}_{\mathrm{eff}}}}=\left[\sin\left(\frac{\mathrm{t}}{10}\right), \cos\left(\frac{\mathrm{t}}{10}\right), \text{-}0.02\sin\left(\frac{\mathrm{t}}{5}\right)\right] \tag{14}$$

(2) Introducing the Effect Independent of Position (position_effect):

By inserting the current position coordinates into trigonometric functions, the research model how environmental factors change based on the submarine's geographical location. This could represent different environmental characteristics in various regions, as in the Formula 15:

$$\overrightarrow{\mathrm{F}_{\mathrm{po}_{\mathrm{eff}}}}=\left[0.1\sin\left(\frac{\mathrm{P(t_0)_x}}{10}\right), 0.1\cos\left(\frac{\mathrm{P(t_0)_y}}{10}\right), \text{-}0.02\right] \tag{15}$$

(3) Introducing the Effect Independent of Velocity (velocity_effect):

Considering the impact of the submarine's velocity on environmental influences. For example, higher speeds may lead to greater drag. Parameters are set according to data obtained from the Ionian Sea, generating a vector perpendicular to the current velocity, which might represent lateral hydrodynamic effects caused by the submarine's movement, as in the Formula 16:

$$\overrightarrow{\mathrm{F}_{\mathrm{v}_{\mathrm{eff}}}}=0.01\left[\text{-}\mathrm{v_y(t)}, \mathrm{v_x(t)}, 0\right] \tag{16}$$

(4) Considering the Impact of Ocean Temperature and Density
(temperature_effect, density_effect):

Deeper ocean means lower water temperature, slowing submarine speed as depth increases. Simultaneously, increased depth raises water density due to thermohaline effect, boosting resistance and further slowing submarine motion. Data from the Ionian Sea informs Formulas 17 and 18.

$$\overrightarrow{\mathrm{F}_{\mathrm{tem}_{\mathrm{eff}}}}=[0, 0, \text{-}0.05\exp(\mathrm{P(t_0)_z})] \tag{17}$$

$$\overrightarrow{\mathrm{F}_{\mathrm{den}_{\mathrm{eff}}}}=[0, 0 ,0.1*\mathrm{P(t_0)_z}] \tag{18}$$

(5) Considering the Impact of Terrain (errain_effect):

With parameters set according to data obtained from the Ionian Sea, as the submersible moves horizontally, the value of the cos function changes, thereby producing the effect of terrain. Specifically, when the submersible is near the peak of the terrain, the value of the cos function is larger, indicating that the terrain exerts a downward force on the submersible; as in the Formula 19:

$$\overrightarrow{\mathrm{F}_{\mathrm{ter}_{\mathrm{eff}}}}=[0, 0, \text{-}0.02\cos(\mathrm{P(t_0)_x})] \tag{19}$$

(6) Considering the Effect of Environmental Randomness (random_effect):

Randomness is introduced to model uncertain factors in the natural environment, such as sudden storms, ocean current vortices, etc. With parameters set according to data obtained from the Ionian Sea, the research use rand (1, 3) ×0.2−0.1 to generate a random vector within the range of -0.1 to 0.1, adding unpredictability to the environmental influence, as in the Formula 20:

$$\overrightarrow{F_{r_{eff}}}=\overrightarrow{rand(1,3)*0.2\text{-}0.1} \tag{20}$$

Finally, the research synthesizes all environmental influencing factors(environment_effcet), summing up all the above factors to obtain the final environmental impact vector, which will directly influence the submarine's linear velocity movement, making it more closely mimic the behavior in a real environment, as in the environmental factors function Formula 21:

$$\overrightarrow{F_{env_{eff}}}=\overrightarrow{F_{o_c}}+\overrightarrow{F_{time_{eff}}}+\overrightarrow{F_{po_{eff}}}+\overrightarrow{F_{v_{eff}}}+\overrightarrow{F_{r_{eff}}}+\overrightarrow{F_{ter_{eff}}}+\overrightarrow{F_{tem_{eff}}}+\overrightarrow{F_{den_{eff}}}+\overrightarrow{F_{n_v}} \tag{21}$$

Similarly, akin to the impact on linear velocity, the research also introduced the environmental impact on angular velocity, considering both time and randomness. $0.05\cos(\frac{t}{5})$Represents the part of angular velocity that varies cyclically with time, while using (rand×0.1−0.05) adds randomness, simulating the environmental impact on rotational or steering actions, as in Formula 22.

$$\overrightarrow{F_{a_{v_{eff}}}}=0.05\cos\left(\frac{t}{5}\right)+(rand*0.1\text{-}0.05) \tag{22}$$

2.2. The basic fundamental of Entropy Weight Method (EWM) Models and Continuous Variable Model (CVM)

The EWM, rooted in information entropy, systematically evaluates the influence of factors on decision outcomes. Widely adopted across various domains, it aids in selecting optimal solutions, mitigating risks, and enhancing decision reliability [10].

Here's a detailed explanation of the EWM method:

(1) Data Collection: Gather data related to decision factors.

(2) Entropy Value Computation: Compute entropy values for each factor to quantify uncertainty.

(3) Weight Calculation: Calculate weights based on entropy values; lower entropy indicates higher importance.

(4) Decision Evaluation: Use weights to evaluate alternatives; select the option with the highest overall score as the optimal solution.

EWM finds application across various domains and decision-making scenarios, including engineering, environmental assessment, financial investment, and more. It provides decision-makers with a structured framework to analyze complex decision landscapes, prioritize decision factors, and select optimal solutions [11].

The Continuous Variable Model (CVM) is a decision-making model that integrates multiple evaluation indicators to provide a comprehensive assessment of decision alternatives. It is commonly used in conjunction with methods like the Entropy Weight Method (EWM) to evaluate the performance of different alternatives across various criteria [12].

The Continuous Variable Model (CVM) is constructed through several steps:

(1) Integration of Evaluation Indicators: Identify and integrate relevant evaluation indicators representing different aspects of decision alternatives.

(2) Normalization of Indicators: Normalize indicators to ensure comparability across different scales, typically ranging from 0 to 1.

(3) Weighting of Indicators: Assign weights to indicators based on their importance, often determined using methods like the Entropy Weight Method or expert judgment.

(4) Aggregation of Scores: Aggregate normalized scores of each indicator by multiplying them by corresponding weights and summing up across all indicators.

(5) Selection of Optimal Alternative: Choose the alternative with the highest overall score as the optimal solution, indicating its suitability based on aggregated performance across all evaluation criteria.

The Continuous Variable Model provides decision-makers with a structured framework to evaluate decision alternatives comprehensively and objectively [13]. By integrating multiple evaluation indicators and weighting them based on their importance, the CVM facilitates informed decision-making and helps identify the most suitable solution among competing alternatives.

In the research submersible safety operations, the EWM offers a structured framework to address complexities like cost and reliability. By objectively quantifying indicator weights, it optimizes resource allocation, maximizing search efficiency within budget constraints [14]. The research merged the EWM with CVM so that to enable decision-makers to strike a balance between cost considerations and operational requirements through a Continuous Variable Model [14-16].

2.2.1. Data Collection: Selecting this additional equipment

Environmental factors, such as biological activities interfering with sonar and the impact of waves, contribute to uncertainties affecting submarine stability and safety. Within the submarine, uncertainties include variations in propulsion due to fuel consumption and wear, alongside sensor inaccuracies. Also, model uncertainties stem from simplifications in mathematical representations and parameter estimation errors due to measurement inaccuracies or incomplete knowledge.

From the above uncertainties, the research can infer the following information that the submarine can send to the mother ship:

(1) Propulsion status: Including battery level, fuel level, operation condition of the propulsion system, etc.

(2) Sensor data: Including water temperature, salinity, seabed terrain scanning results, nearby object detection (such as sonar data), etc.

(3) Communication system status: Including the working status of the communication system, signal strength, etc.

(4) Biological activity detection: Oceanic biological activity conditions that the sonar system may detect.

(5) Meteorological data: If the submarine can surface near the sea surface, it may include wind speed, atmospheric pressure, etc.

(6) Forecast data: Data information on the next movement plan of the submarine.

Therefore, the research analyzes the equipment needed for the submarine to include the following Tabel 2.

Table 2. Underwater Surveillance and Navigation Equipment Specifications.

Device Name	Purpose	Unit	Typical Normal Range
Underwater Acoustic Communication System	Data Transmission Speed	Bits per second	Varies significantly based on distance and conditions
GPS Receiver	Positional Accuracy	Meters (m)	5 - 10 m
Ultra-Short Baseline System	Positional Accuracy	Meters (m)	<1m (Short Range)
Long Baseline Acoustic Positioning System	Positional Accuracy	Meters (m)	1 - 3 m(Long Range)
Side Scan Sonar	Detection Range	Meters (m)	100 - 500 m
Multibeam Sonar	Detection Range	Meters (m)	200 - 8000 m
Biological Activity Detection Sonar	Detection Frequency	Hertz(Hz)	20,000 - 40,000 Hz
Drifting Buoy	Surface Current Speed	Meters/second (m/s)	0 - 2 m/s
	Surface Temperature	Celsius(°C)	-2 - 35 °C
Pressure Sensors and Depth Gauges	Depth	Meters (m)	0 - Measurement Limit

The selection and evaluation of search and rescue equipment involve various types, including:

(1) Multibeam Sonar: A sonar system that forms multiple beams simultaneously within a certain spatial range, used for mapping the seabed.

(2) Side-scan Sonar: A device that utilizes the principle of echo sounding to detect seabed topography and underwater objects.

(3) Sonobuoy: A buoy-type sonar equipment for detecting underwater targets, a type of hydroacoustic remote sensing detector, forming part of a sonobuoy sonar system for fixed sonar surveillance of submarines.

(4) Satellite Communication Equipment: Facilitates communication between the mother ship and the submarine (if possible), the rescue team, and the command center.

(5) Life Rafts: Emergency rescue equipment for the sea, used for transferring stranded individuals and also for surface search.

(6) Communication Buoys: Used for communication with underwater devices, such as

(7) Underwater robots and trapped submarines.

To quantify and facilitate the selection process, a device attribute table is established with X=1, 2, 3, 4 representing the four different search devices mentioned above. Let C_x represent the total cost of the device, C_B the purchase cost, C_K the maintenance cost, C_U the usage cost, and T the usage period. The cost function is defined as follows Formula 23:

$$C_x=C_B+C_KT+C_U \tag{23}$$

To ensure the selected equipment achieves maximum search and rescue efficiency within a certain budget, let *F* represent the funding limit, establishing the following constraint Formula 24:

$$\sum C_x \leq F \tag{24}$$

2.2.2. Establishment of Scoring Indicators and Acquisition of Standardized Weights

For the four evaluation indicators considered, the entropy weight method is used to calculate weights. Given the varying sizes and ranges of each indicator, the research first normalize the data using the following formulas. For positive indicators such as reliability, efficiency (range), and service life, the research use Formula 25:

$$\chi_{ij}=0.998*\frac{\chi_{ij}-\min\left\{\chi_{1j},\dots\chi_{nj}\right\}}{\max\left\{\chi_{1j},\dots\chi_{nj}\right\}-\min\left\{\chi_{1j},\dots\chi_{nj}\right\}}+0.002 \tag{25}$$

For negative indicators such as total cost, the research use Formula 26:

$$\chi_{ij}=0.998*\frac{\max\left\{\chi_{1j},\dots\chi_{nj}\right\}-\chi_{ij}}{\max\left\{\chi_{1j},\dots\chi_{nj}\right\}-\min\left\{\chi_{1j},\dots\chi_{nj}\right\}}+0.002 \tag{26}$$

Then, the weight P_{ij} of the i device under the j indicator is calculated as Formula 27:

$$P_{ij}=\frac{\chi_{ij}}{\sum_{i=1}^{n}\chi_{ij}}\ (j=1,2,3,4) \tag{27}$$

$$e_j=-\sum_{i=1}^{n}P_{ij}*\ln(P_{ij}) \tag{28}$$

Subsequently, using the weights and evaluation indicators with the Formula 29.

$$S_x=e_1S_1+e_2S_2+e_3S_3+e_4S_4 \tag{29}$$

2.2.3. Continuous Variable Optimization Model Establishment

In practice, submarine agencies or administrators have the flexibility to select search and rescue equipment models and manage expenditures within budgetary constraints. To identify the most effective combination of equipment models and devise contingency plans for cases where precise equipment purchases aren't feasible, the research introduce the following simplifications:

(1) For a certain type of search and rescue equipment, the market cost of such products ranges within [70%, 130%] of the costs listed in the table above.

(2) For a certain type of equipment, its reliability, search range, service life, and cost are unified into a total score.

(3) Assumed that the agency can purchase up to two types of products at the same time.

(4) The research introduces an efficacy function as Formula 30.

$$S_X=\gamma_x^{\beta}\times R_x \tag{30}$$

Taking β=0.5 and establishing constraints, the research derives the following Formula 31.

$$\begin{cases} S_x=0.5\times\gamma_x^{0.5}\times R_x \\ \sum\gamma_x\times C_x\leq F \end{cases} \quad \gamma\in[0.7,1.3] \tag{31}$$

For ease of model solving, the research assumes F=62000 (USD) and select multibeam sonar and underwater drones, resulting in the equation as in Formula 32.

$$\begin{cases} S=426\times\gamma_1^{0.5}+66.9\times\gamma_2^{0.5} \\ \gamma_1\times 53800+\gamma_2\times 9800=62000 \end{cases} \tag{32}$$

3. Results

3.1. Data acquisition

The research acquire data from the websites of follow Table 3.

Table 3. Data source collation.

Database Names	Database Websites	Data Type
GEBCO	https://download.gebco.net/	Digital Elevation Model
CMEMS	https://marine.copernicus.eu/	Marine physical parameters
Earth weather	https://earth.nullschool.net/	Marine physical parameters
Google Scholar	https://scholar.google.com/	Academic paper
Wikipedia	https://en.wikipedia.org/	Initial Data

3.2. 3D Visualize and Analyses

In the final phase of study, the research utilized the established theoretical models and computational codes to generate visual representations of the submarine's trajectory. This resulted in two key visual outputs: the "Submarine 3D Trajectory Graph" and its magnified version, as displayed in Figure 1.

This figure illustrates the trajectory variations of the submarine within the maritime environment. The visualization includes a yellow ellipsoid representing the submarine, a red dashed line depicting its trajectory, and a blue area simulating the oceanic region. This graphical representation allows for an intuitive understanding of the submarine's path through complex marine terrains.

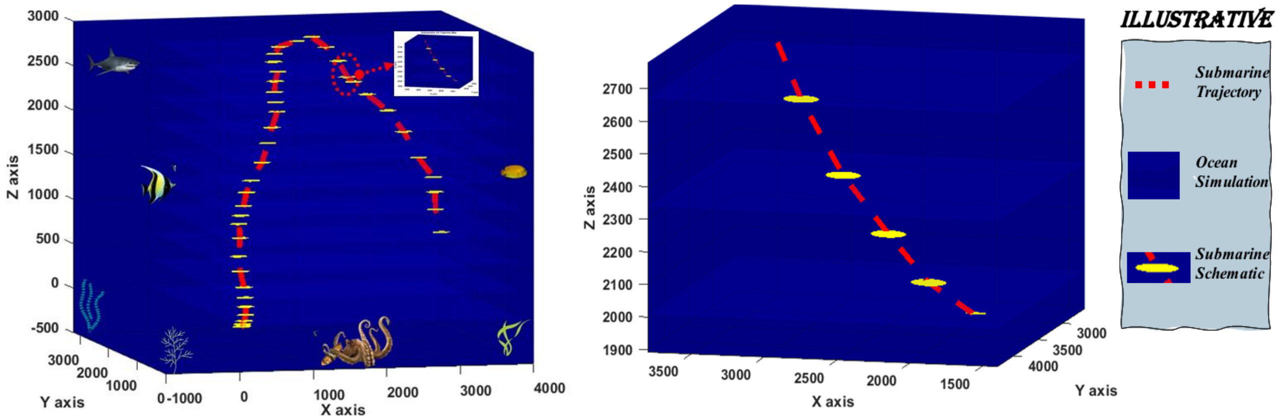

Figure 1. Submarine 3D Motion Trajectory Chart with Zoomed-in View.

3.3. Visualization of Equipment Rating Situations

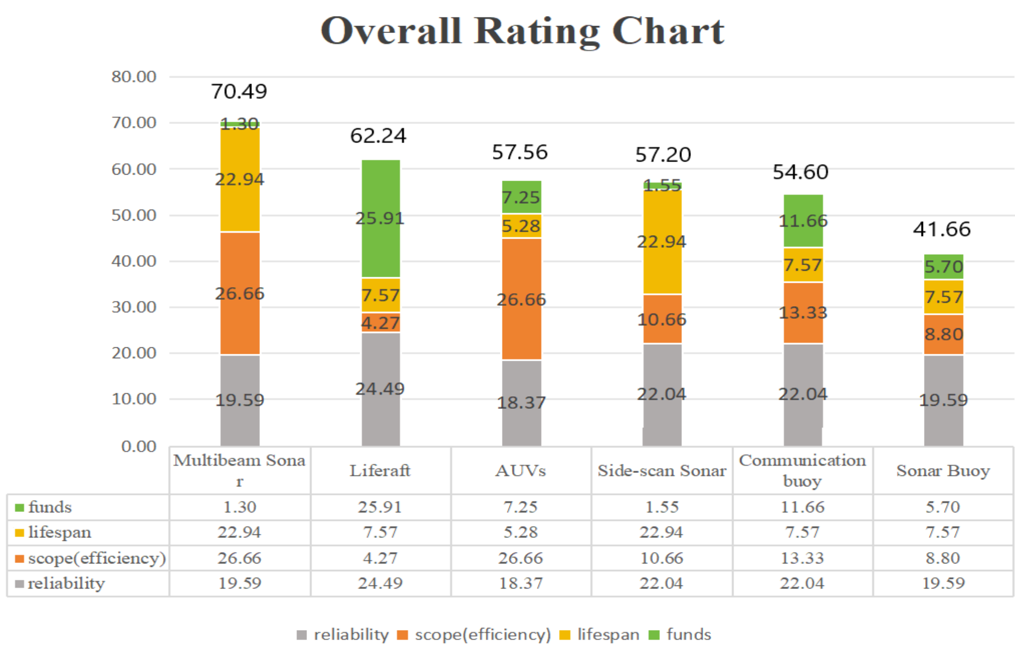

Figure 2. Overall Rating Chart.

The research also conducted evaluations of various search and rescue equipment, which are crucial for the submarine operations. This involved an analysis of equipment ratings based on criteria such as reliability, service life, and cost-effectiveness, resulting in a comprehensive rating chart (Figure 2). The chart ranks the equipment from highest to lowest score: Multibeam Sonar, Life Rafts, Underwater Drones, Side-scan Sonar, Communication Buoys, Sonobuoy. Notably, the Multibeam Sonar ranks first despite its higher cost, reflecting its superior reliability and efficiency.

Table 4. Reliability and cost comparison of underwater detection and communication equipment.

Device name	Reliability/%	Scope/m²	lifespan(year)	Funds(USD)
Multibeam Sonar	80	300	30	53800
Side-scan Sonar	90	120	30	49800
Sonar Buoy	80	100	10	11500
AUVs	75	300	7	9000
Liferaft	100	50	10	2500
Communication Buoy	90	150	10	5500

PS: Assuming an average usage period of 10 years for these devices

Excluding necessary communication equipment and devices unsuitable for search and rescue, the remaining six types of searching and rescue equipment are evaluated based on reliability, efficiency (range), service life, and total cost. The research obtained the following information for the six types of equipment in the following Table 4:

The information entropy e_j for the j indicator is then calculated, and normalized to obtain the weights *a* for each evaluation indicator as shown in Table 4 below.

Table 5. Evaluation Indicator Weights.

Reliability	Scope(efficiency)	lifespan(year)	Funds
0.24	0.27	0.23	0.26

Take the best condition of each indicator as a full score, divide the rest by the best condition to obtain the corresponding score value, and obtain the corresponding score value of each search and rescue equipment, as shown in the following Table 6:

Table 6. Search and Rescue Equipment Ratings.

Device name	Reliability	scope(efficiency)	lifespan(year)	Funds
Multibeam Sonar	80	100	100	5
Side-scan Sonar	90	40	100	6
Sonar buoy	80	33	33	22
AUVs	75	100	23	28
Liferaft	100	16	33	100
Communication Buoy	90	50	33	45

3.4. Visualization of equipment combination selection solutions.

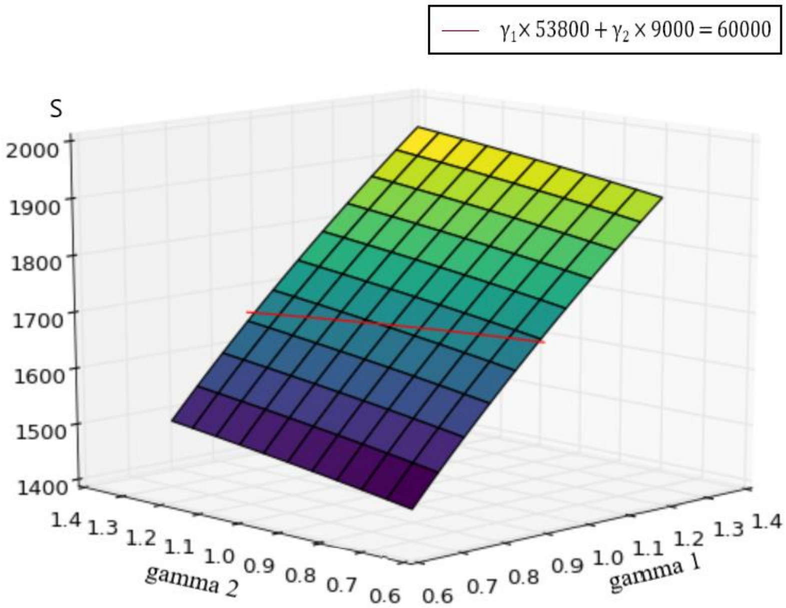

Figure 3. 3D Equation Graph.

After formulating equations to solve for the optimal combination of equipment within given budget constraints, the research visualized the solutions as shown in Figure 3. This process highlighted how different funding levels influence the selection strategy.

Then the research got the solved: γ_1=1.003 γ_2=0.67 z=1629. Thus, the research can infer that for a funding cap of 60000 USD and selecting multibeam sonar and underwater drones, the highest score is 1690 points.

For a funding cap of 62800 USD, using the above model for solving, the research find the highest score to be 1730 (γ1=1.05 γ2=0.61), while the score obtained directly from previously researched data is 1724.6. The optimization effect is not significant, but this model can still serve as an optimization algorithm when the funds are insufficient to purchase all the aforementioned search and rescue equipment simultaneously.

Due to the significant differences in cost relative efficiency score among different search and rescue equipment. Since each equipment has its advantages during the rescue process, the research adjust the scores according to the cost ratio and calculate the cost/score for each device, as shown in Table 7.

Table 7. Cost-benefit assessment of search and rescue equipment.

Device name	Funds	Score	Funds/Score
Multibeam Sonar	53800	1517.2	36
Side-scan Sonar	49800	1139.4	43.7
Sonar Buoy	11500	192	59.8
AUVS	9000	207.4	43.3
Life rafts	2500	62.2	40.1
Communication Buoy	5500	120.12	45.7

4. Conclusions

The research presented in this paper represents a significant step forward in the field of ocean exploration and submersible safety. The integrated model provides a powerful tool for predicting submersible locations, optimizing search operations, and managing multiple units simultaneously. It is a testament to the potential of combining theoretical and practical methods to address complex challenges in ocean engineering and enhance the safety and efficiency of submersible operations.

This research presents several key findings and contributions. The 3D motion trajectory model effectively simulates the spatial dynamics of submersibles by integrating propulsion power, communication status, and environmental factors, providing crucial visual tools like a 3D trajectory chart for operational planning and crisis management. The combination of the Entropy Weight Method (EWM) and Continuous Variable Model (CVM) forms a decision-making framework for selecting search and rescue equipment, optimizing the balance between cost, reliability, efficiency, and longevity. The model enhances safety and operational accuracy through real-time adjustments and environmental interactions, allowing for precise predictions of submersible position changes.

The research highlights the model's capability to optimize search and rescue equipment selection within budget constraints, improving resource allocation. Simulation and analysis validate the model's effectiveness, with visualizations of the submarine's trajectory and predictive distribution maps providing insights into operational paths and challenges, confirming its reliability and practical value. Implications for future research and practice include setting a precedent for integrating advanced modeling techniques in submersible safety operations, applying the equipment selection and optimization approach to other complex decision-making domains, refining the framework to include additional environmental variables and operational scenarios, and encouraging the development of more sophisticated models to better predict and respond to real-world uncertainties and variability.

References

[1] PRANESH S B, KUMAR D, ANANTHA SUBRAMANIAN V, et al. Numerical and Experimental Study on the Safety of Viewport Window in a Deep Sea Manned Submersible[J/OL]. Ships and Offshore Structures, 2020, 15(7): 769-779.

[2] FU J, KHAN F. Operational Failure Model for Semi-submersible Mobile Units in Harsh Environments [J/OL]. Ocean Engineering, 2019, 191: 106332.

[3] ZHU Y, TIAN D, YAN F. Effectiveness of Entropy Weight Method in Decision-Making [J/OL]. Mathematical Problems in Engineering, 2020, 2020: 1-5.

[4] SHENGJIE Q, XIAOHUI L, YI Z, et al. Effect of Manned Submersible Operation on Structural Safety[C/OL]//2019 CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS). Xiamen, China: IEEE, 2019: 375-378[2024-04-24].

[5] LI J, FU Z. Real-time 3D Object Detection and Tracking in Automated Driving: A Review [J/OL]. IEEE Transactions on Intelligent Transportation Systems, 2021: 1-12.

[6] CHEN X, et al. Dynamic 3D Motion Trajectory Prediction Using Deep Neural Networks [J/OL]. IEEE Robotics and Automation Letters, 2020, 5(4): 5315-5322.

[7] ALBARAKATI S, LIMA R M, GIRALDI L, et al. Optimal 3D Trajectory Planning for AUVs using Ocean General Circulation Models[J/OL]. Ocean Engineering, 2019, 188: 106266.

[8] WANG B, XIONG J, WANG S, et al. Steady Motion of Underwater Gliders and Stability Analysis [J/OL]. Nonlinear Dynamics, 2022, 107(1): 515-531.

[9] ADCROFT A, ANDERSON W, BALAJI V, et al. The GFDL Global Ocean and Sea Ice Model OM4.0: Model Description and Simulation Features [J/OL]. Journal of Advances in Modeling Earth Systems, 2019, 11(10): 3167-3211.

[10] CHEN C H. A Novel Multi-Criteria Decision-Making Model for Building Material Supplier Selection Based on Entropy-AHP Weighted TOPSIS [J/OL]. Entropy, 2020, 22(2): 259.

[11] KUMAR R, SINGH S, BILGA P S, et al. Revealing the Benefits of Entropy Weights Method for Multi-Objective Optimization in Machining Operations: A Critical Review [J/OL]. Journal of Materials Research and Technology, 2021, 10: 1471-1492.

[12] LI Y, et al. Entropy Weight Method-Based Integrated Evaluation of Regional Water Security: A Case Study of the Yangtze River Economic Belt [J/OL]. Water, 2021, 13(15): 2133.

[13] YANG S, et al. Entropy Weight Method-Based Evaluation of Regional Ecological Security: A Case Study of the Beijing-Tianjin-Hebei Urban Agglomeration [J/OL]. Sustainability, 2019, 11(24).

[14] WANG Z, et al. A Novel Entropy Weight Method for Determining the Weight of Attributes in Group Decision-making [J/OL]. Sustainability, 2020, 12(12): 5105.

[15] SIQUEIRA J R, PEDROSO C A, BASTOS F, et al. Atlanta Field: Operational Safety and Integrity Management[C/OL]//Day 1 Tue, October 29, 2019. Rio de Janeiro, Brazil: OTC, 2019: D011S002R004 [2024-04-24].

[16] ALMEIDA J, SILVESTRE C, PASCOAL A. Cooperative Control of Multiple Surface Vessels in the Presence of Ocean Currents and Parametric Model Uncertainty [J/OL]. International Journal of Robust and Nonlinear Control, 2010, 20(14): 1549-1565.