

Optimization Study of Momentum Strategy for Tennis Match Based on FINX Evaluation Model and AHP

You Fu ^{1,*}, Wufan Zhou ² and Youzhan Wang ¹

¹ School of International Trade and Economics, Shanghai International Studies University, Shanghai, China

² School of Business and Management, Shanghai International Studies University, Shanghai, China

* Corresponding author: 0221138005@shisu.edu.cn

Abstract. This paper focuses on a comprehensive study of the evaluation of tennis players' match performance, aiming to explore the complex dimensions of momentum in sport by analyzing the comprehensive data set of Wimbledon 2023 and introducing FINX models to analyze and predict match flow and key transitions. Firstly, this paper divides 19 determinants of match results into four groups: off-field factors, internal situations, internal factors and external factors. Secondly, a momentum scoring equation is developed by using entropy weight, hierarchical analysis and TOPSIS methods in the FINX model. Finally, we compare the prediction results of the FINX model with the actual matching results. The results show that the correlation between the prediction accuracy of the model and the actual competition result is 0.826, which shows a strong prediction ability.

Keywords: momentum, AHP, EWM.

1. Introduction

In the fierce competition of competitive tennis, the momentum change of players has a crucial impact on the result of the match. Momentum is a key index to evaluate athletes' performance and competition situation. Its formation and evolution are influenced by many factors. To deeply analyze the complex mechanism behind the momentum of tennis players, this paper constructs a FINX combination model, which aims to reveal the influencing factors of momentum and their interactions from multiple dimensions and levels.

Through the in-depth analysis of many tennis match data, we identify four categories of first-level indicators: "off-court factors", "on-court conditions", "internal factors" and "external factors", and further refine them into 19 specific second-level indicators. These indicators cover many aspects such as players' personal skills, field effect, wonderful shots, running scores, etc., providing a rich dimension for a comprehensive assessment of athletes' momentum. On this basis, a combination weighting method is proposed, combining the analytic hierarchy process (AHP) [1] in the subjective weighting method, the entropy weight method (EWM) [2] in the objective weighting method, the standard model of entropy method (ENT) [3] in the evaluation model and TOPSIS [4] to establish a scoring system. The establishment of this system not only helps us to understand more deeply the internal law of the momentum change of tennis players, but also provides a scientific basis for the formulation of training strategies for coaches and the self-improvement of players.

2. Establishment of the FINX Model

2.1. Establishment of the Evaluation System

After referencing a substantial amount of relevant literature and reviews, this paper conducts a comprehensive analysis of the momentum indicator for tennis players. Through the Analytic Hierarchy Process (AHP) [5-6], four primary indicators analyzing how "momentum" changes are derived from extensive data. These indicators are categorized as "Off-court Factors," "On-court

Situation," "Internal Factors," and "External Factors," serving as the first-level indicators for evaluating momentum. Following a further analysis and summarization of numerous matches, we have identified 19 specific aspects as second-level indicators. These aspects are detailed in the Table 1:

Table 1. FINX Analysis

Primary index	Off-site factors	Internal situations	Endogenous	Exogenous
Secondary index	Personal skill	Set differential	Winning ratio of service game	Opponent's winning ratio of service game
	Venue effect	Point differential	Physical output	Opponent's double fault
	Popularity	Running score	Double fault	Opponent's unforced error
		Break point	Unforced error	
		Service game		
		Wonderball		

To investigate the momentum of tennis players, we draw inspiration from the definition of potential energy in physics. In physics, the momentum of particles is the vector sum of their momenta. Similarly, the generation of "momentum" in tennis involves the vector addition of multiple factors. Following Euler's first law, we propose a quantification method for calculating momentum and apply it to our research on the momentum in tennis player matches. This method is grounded in academic rigor, aligning with the principles of Euler's first law and providing a systematic approach to analyze the momentum in tennis competitions.

$$P = \sum_i M_i V_i \quad (1)$$

2.1.1. Off-site factors (OF)

Refer to external elements that are predetermined before the match, independent of the day's playing conditions but exert a certain psychological influence on the players. External factors encompass the following three aspects:

F1 Personal skill:

The player's individual technical proficiency is a critical source of confidence and performance. By consulting the ATP official website, we gathered the ATP point values for each player in Wimbledon singles matches as of June 19, 2023. These points are utilized to reflect the comparison of a player's strength against opponents. Additionally, we investigated each player's return depth (RD) and serve depth (SD). Generally, deeper returns and serves are associated with higher-quality play, leading us to consider a positive correlation between serve and return depth and player strength.

$$F_1 = ATP' + RD + SD \quad (2)$$

F2 Venue effects:

Different court surfaces lead to distinct court surface effects, such as higher average ball speeds on clay courts and lower speeds on grass courts. We collected data on the average ball speed and rebound height during matches on three types of courts: hard courts, clay courts, and grass courts. Analyzing the impact of court surfaces, we considered the on-court performance of various players. Under objective conditions, players of different heights and playing styles adapt to different surfaces. Under subjective conditions, different court surfaces reflect the venue of the event, potentially introducing a home advantage effect.

F3 Popularity:

Considering that players are influenced by the live audience, we use the Google search popularity of players up to June 16, 2023, as a measure of player attention. This reflects the proportion of the live audience during the matches.

2.1.2. Internal situations (IS)

The Internal situation indicator reflects the impact of the ongoing match dynamics on a player's momentum. It comprises six elements:

- I1 Service Game
- I2 Wonderball
- I3 Running score
- I4 Break point
- I5 Point differential
- I6 Set differential

$$IS_2 = UW + ACE + UN \quad (3)$$

In tennis matches, a player's control of the game's rhythm and direction is more achievable during service games due to the advantage of serving first. This advantage introduces both psychological and objective factors, making it an essential consideration. Spectacular shots encompass displays of a player's exceptional technical proficiency and decision-making ability, including aces, winners, impressive net play, and front-court points. Spectacular shots not only earn applause from the audience but also boost a player's confidence, particularly when occurring in quick succession. Consecutive points represent the continuous accumulation of momentum, imposing significant psychological pressure on the opponent. Breakpoint conversions signify the successful winning of a game with a higher opponent serving probability, enhancing one's fault tolerance and putting pressure on the opponent. Game score difference refers to the difference in the number of games won, while set score difference pertains to the difference in set scores. These factors collectively constitute the on-court situation indicator.

2.1.3. Endogenous (EN)

Endogenous refers to the indicators from the players themselves that reflect the level of technical play, these indicators will affect the momentum of the players, after further analysis, we have deeply discovered the endogenous issues of the field players' performance excellence or errors, and believe that it contains the following elements:

- N1 Winning rate of serving game:

Serve as a key moment in the player's game, has a significant impact on the player's winning and losing trend, its win rate reflects the player's game state, is one of the important considerations of the player's game momentum.

$$Wr = \frac{S_{sw}}{S_s} \quad (4)$$

- N2 Physical output:

Tennis matches are long and strenuous, and as the match progresses the player's physical fitness decreases, which we consider quantitatively by combining the player's height, age, running distance, and court type with an airplane performance model.

$$P_o = \frac{1-d'}{lgY} \frac{1}{1+\frac{vdv}{gdh}} \quad (5)$$

- N3 Double fault:

Double serve errors can reflect a player's mental and technical instability when points are lost.

- N4 Unforced error:

An unforced error in a high-intensity match reflects the player's ability to play on the court and his or her calmness of mind.

2.1.4. Exogenous (EX)

Exogenous causes refer to the impact on a player's own game momentum because of changes in the opponent's "momentum".

X1 Opponent's double fault

X2 Opponent's unforced error

When the opponent has consecutive service errors, the opponent's unforced errors represent a serious decline in his physical strength, and the opponent's control of the situation in the service game will bring psychological pressure on his side, which is a positive indicator of his side's "momentum".

X3 Opponent's winning rate of serving game:

$$OWr = \frac{OS_{sw}}{OS_s} \quad (6)$$

2.2. Weight Modeling and Results

The In order to synthesize the importance of each indicator, we quantitatively weighted different indicators to reflect their relative importance in the final evaluation or decision-making. Considering the subjective nature of the weighting percentages, we decided to use a combined weighting method to calculate the weights of all indicators to improve the accuracy of the model. The combined weighting method proposed in this paper combines the hierarchical analysis method (AHP) in the subjective weighting method, the entropy weighting method (EWM) in the objective weighting method, and the criterion model with the entropy value method (ENT) in the evaluation model, and the TOPSIS respectively, to establish a scoring system, and select the scoring system for each of them, to establish a scoring system and score his performance in one of the matches selected respectively.

2.2.1. Off-site factors via AHP

Off-site factors have a significant impact on momentum, but as invariants are not suitable to be put into the objective weight allocation method and evaluation model, so the subjective hierarchical analysis method is used, and then the following weights are obtained after parameter adjustment and validation through subsequent fitting between the results of the match and the game. Among them, individual skill level has a higher weight in influencing players' performance; court type has the second highest weight in influencing players' serve, movement and rally. Although player attention influences players' psychological state and stress, it is not significant. After learning the weight ∂_i , multiply F_i by the assignment yields the standardized F'_i , derive the value of F after summation.

$$F = \sum_1^4 \partial_i F_i \quad (7)$$

The result is shown in Fig.1:

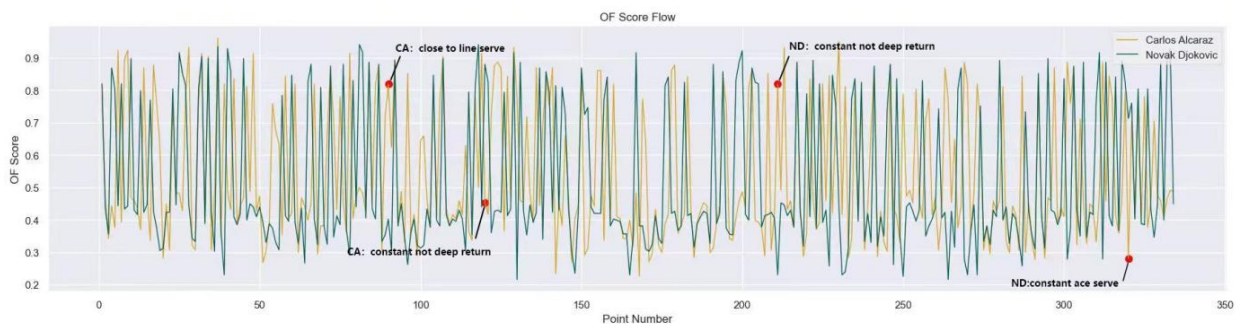


Figure 1. AHP off-site factor analysis

As can be seen from the Fig.1, the correlation between personal skill and off-site factors is extremely high. The high point of the line graph is often caused by the deeper depth of the player's stroke, while the location is usually related to the player's baseline return and the shallow depth of the return, it is not difficult to speculate that the player's baseline stroke is pressurized due to the movement that leads to the incoherence of the force, the shallow depth of the return, and the opponents are left with the opportunity of attacking the ball, which leads to the reduction of the score.

2.2.2. Internal situation via TOPSIS

The internal situation has variability, the data sample is large, and the same situation has different effects for different players, which is difficult to be judged by strict positive and negative term indicators, so we choose TOPSIS analysis method which has no strict limitation on data distribution and sample content. Calculate the evaluation object and the positive ideal solution distance D+ or negative ideal solution distance D- respectively. Combined with the distance value to calculate the comprehensive degree score C value and ranked.

Fig.2 shows that the big score difference has the biggest impact on the players' mentality of the game, followed by the innings score difference and consecutive scores.

2.2.3. Endogenous via EWM

Firstly, we analyze the weights of the indicators according to the original data and form the original matrix X. Because of the different types of indicators, we need to normalize the original matrix. Next, to eliminate the influence of dimensionality, we need to standardize the normalized matrix, and then get the matrix y. We combine the normalization and standardization process of the indicators in this paper, and the specific formula is as follows.

$$\begin{aligned} x_{ij}' &= \frac{x_{ij} - \min\{x_{1j}, \dots, x_{nj}\}}{\max\{x_{1j}, \dots, x_{nj}\} - \min\{x_{1j}, \dots, x_{nj}\}} \\ x_{ij}' &= \frac{\max\{x_{1j}, \dots, x_{nj}\} - x_{ij}}{\max\{x_{1j}, \dots, x_{nj}\} - \min\{x_{1j}, \dots, x_{nj}\}} \end{aligned} \quad (8)$$

Then, we use the entropy weight method (EWM) to calculate the weights of the indicators and calculate the probability matrix [7]. According to the concept of self-information and entropy in information theory, we can calculate the information entropy of each evaluation index, from which we can finally get the result as shown in Table 2:

Table 2. Information entropy calculation results

En	Entropy	Utility Value	Weight (%)
N1	0.996	0.004	11.768
N2	0.988	0.012	33.866
N3	0.997	0.003	8.554
N4	0.984	0.016	45.812

The above table shows that the level of physical exertion has a significant effect on the momentum of a player's game, with double serve fault having the least effect.

2.2.4. Exogenous via ENT

The entropy value method considers the uncertainty factors in decision-making, and can synthesize various possibilities when evaluating decisions, which is suitable for complex and changing decision-making environments [8]. Exogenous factors are the mistakes of opponents, strong uncertainty, suitable for the selection of entropy value method of analysis. Through the entropy value method of analysis, we got X1-X5 five kinds of secondary title weights such as shown in the Fig.3:

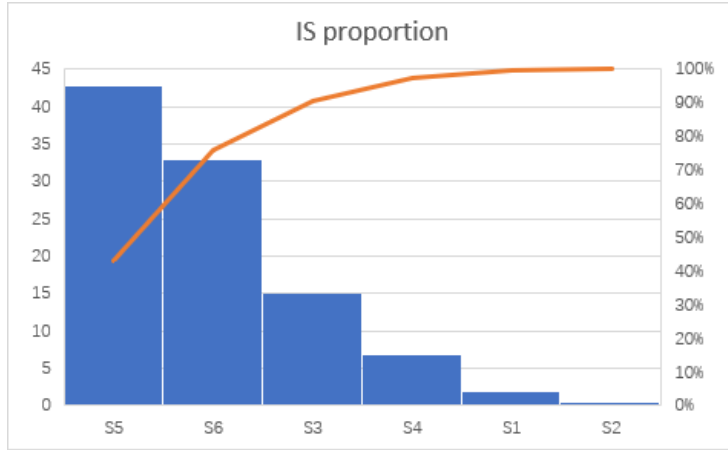


Figure 2. TOPSIS analysis results

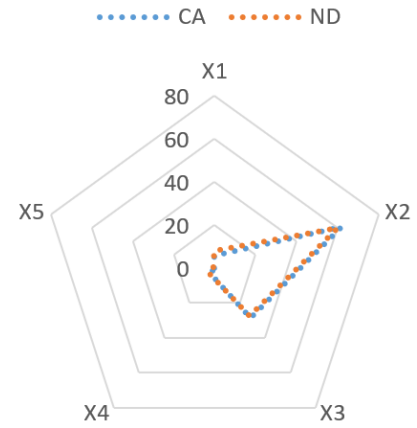


Figure 3. Entropy method weight solution results

Through the weight share we can score the two players Carlos Alcaraz and Novak Djokovic respectively, as shown in Fig.4.

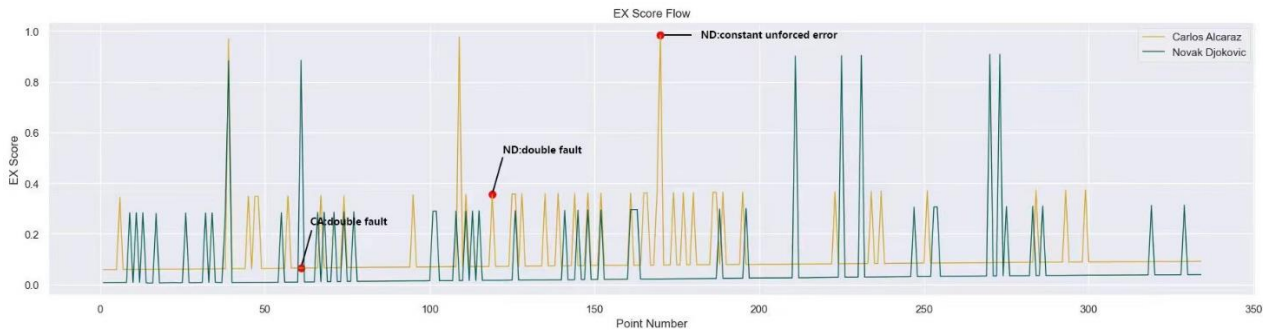


Figure 4. EX score

2.3. The Result of FINX Evaluation Model

The CRITIC method is based on evaluating the comparative strength of indicators and the conflict between indicators to comprehensively measure the objective weights of indicators [9]. Considering the size of indicator variability while considering the correlation between indicators, it completely utilizes the objective attributes of the data itself for scientific evaluation. Tennis game data has the characteristics of large data size, strong comparability, high conflict, and objective visualization, so we chose to analyze the off-site factors, internal situation, exogenous factors, and endogenous factors through the CRITIC method of four level indicators.

The following is the calculation process, C_j is the amount of information contained in the j -th indicator ($1 < j < 4$):

$$C_j = \delta_j \sum_{i=1}^n (1 - r_{ij}) \quad (9)$$

$$\omega_j = \frac{C_j}{\sum_{i=1}^m C_j} \quad (10)$$

The standard deviation is used to express the strength of comparison of evaluation programs, with a larger standard deviation representing a larger gap in the values taken by each program. The conflict between evaluation indicators is based on the correlation between the indicators, and if two indicators have a strong positive correlation, it means that they have a low conflict.

$$r_{ij} = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}} \quad (11)$$

We get a graph of the percentage of momentum composition for each of the two players, as shown in Fig.5 and Fig.6.

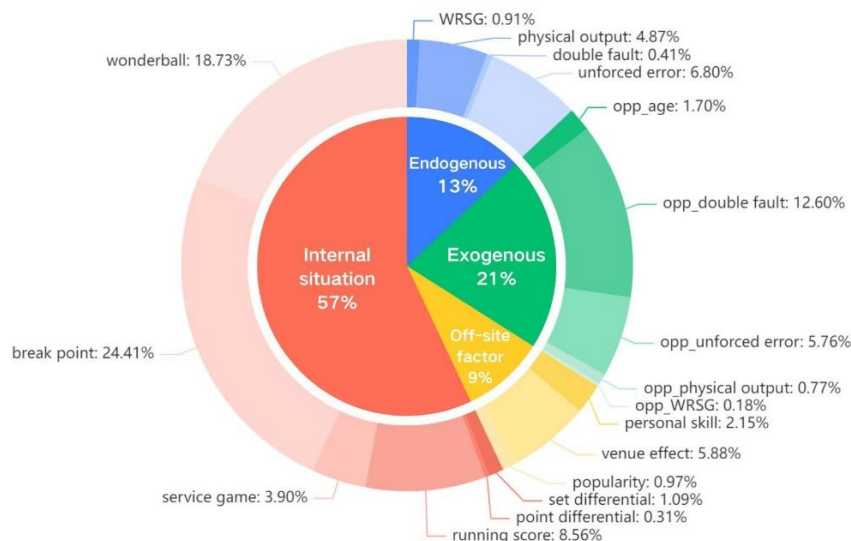


Figure 5. Player 1 momentum constitutes percentage

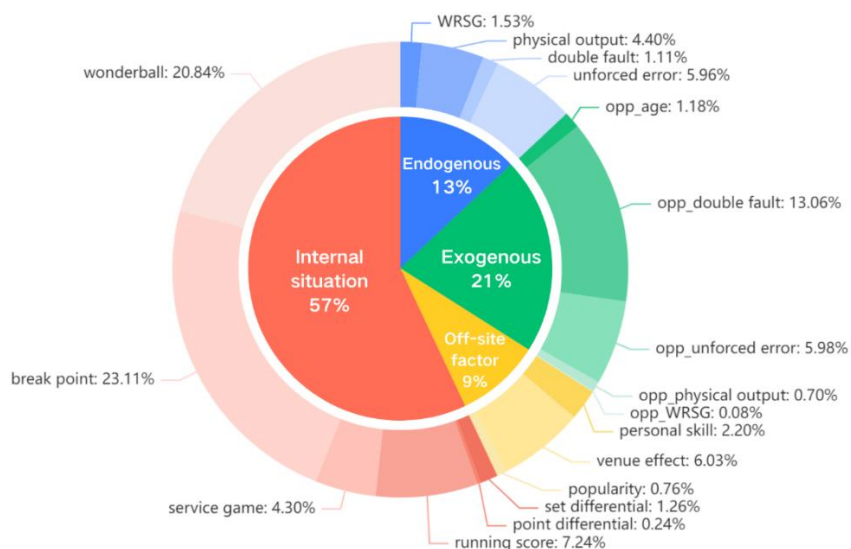


Figure 6. Player 2 momentum constitutes percentage

As can be seen from the graph, the effect of the highlights points on Djokovic's momentum is significantly smaller than that of Alcarres, which may be due to the fact that Djokovic, as a veteran with a lot of experience, is more accustomed to it. And break points for Djokovic's momentum effect is greater, and by the Wimbledon data, the second set of Alcares's break points become one of the key factors to reverse the momentum of the game.

3. Application and Testing of the FINX Model

3.1. Application of the FINX Model to the Wimbledon Finals

We analyzed the 2023 Wimbledon final data using the FINX composite rating model, and the fluctuations in the performance ratings of the two players during the match are shown in Fig.7.

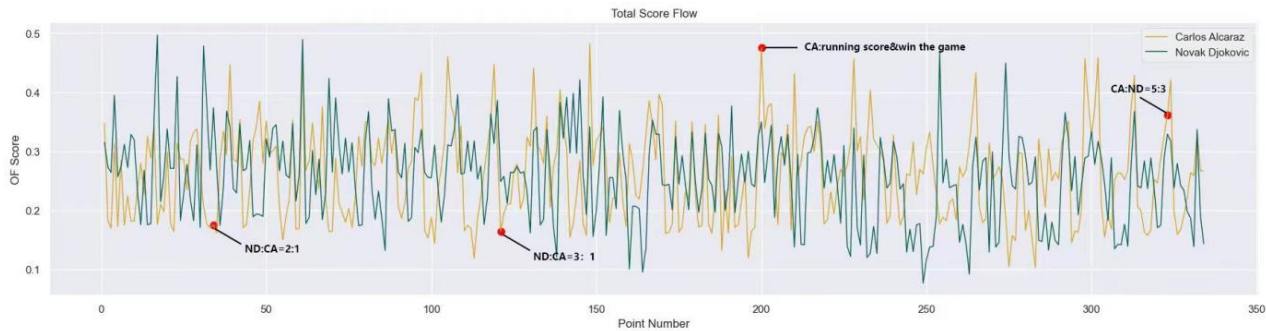


Figure 7. Performance score fluctuation chart

The fluctuations in the "momentum" of the two players in the match showed a competitive and volatile situation. As can be seen from the chart, the first 25 sets of Djokovic's momentum dominated the upper hand, the score of Djokovic to 6-1 control of the first set. In the 33-61th game Djokovic's lead was gradually recovered by Carlos Alcaraz, from the match, the second set was very tense, although Djokovic initially had the lead, but the momentum of Alcaraz had surpassed Djokovic when the score was 3-1 in both, and finally Carlos Alcaraz won 7-6. 270-333 games in which Carlos Alcaraz was always in the lead, and we can infer that in the late stages of the game in the later stages of the match, Alcarres mastered the "momentum" of the match and finally realized the reversal, winning 6-4.

The FINX comprehensive scoring model analyzed this final match and compared it with the real data of the match and concluded that the model has a high degree of fit, reflecting the objectivity and fairness of the scoring system.

3.2. Application of the FINX Model to Table Tennis Matches

To test the fit of the FINX model on table tennis, we extend the application of the FINX model to the sport of table tennis. We can evaluate the model's adaptability to different sports with similar sport characteristics. According to the FINX prediction model, we will count one match of table tennis men's singles world No. 1 Ma Long and world No. 2 Wang Chuqin to score their on-court performances separately. Despite the differences in rules and courts between table tennis and tennis, they still share similarities in many aspects of the sport, such as interactions between opponents, technical requirements, and tactical choices. By testing the FINX model match on table tennis, we can assess the model's adaptability to different sports with similar motor characteristics.

$$Y = B_0 + B_1X + \epsilon \quad (12)$$

B_0 is the dependent variable, B_1 is the slope, ϵ is the inaccuracies.

Table 3. FINX model ping-pong regression analysis

	Standardized coefficient Beta	t	P	VIF	R^2	F
En_wrsg	0.542	8.098	0.000**	1.926	0.826	32.292
En_po	-0.122	-1.619	0.000***	1.392		
Is_set differential	-0.443	-7.387	0.000***	1.94		
Is_wonderball	0.192	1.804	0.023	1.546		
Ex_opprsg	-0.811	-14.188	0.000***	1.406		

$R^2 = 0.826$, From the Table 3 it is shown that the FINX model explains the variability of the dependent variable better. This implies that the independent variables considered by the model (e.g., points scored, points conceded, highlights, errors, etc., in tennis sets) play a greater role in explaining the observed table tennis sports performance. The results where the feedback of the correlation coefficients such as serve win percentage, set point difference, etc. were highly significant. This indicates that the FINX model is more effective in explaining the variability of table tennis sports performance, but further analysis and validation are needed to confirm the robustness and usefulness of the model.

In table tennis, the duration of each game is relatively short, and tennis usually requires more endurance and persistence, whereas table tennis focuses more on explosive power and reaction speed. In terms of technique, the fast-rotational speed of the table tennis ball in table tennis matches can greatly affect the ball's movement, and changing the speed of the ball is the most important part of table tennis, while tennis matches pay more attention to the power, speed, and precise hitting technique. After analyzing the limitations of the FINX model in predicting the performance of table tennis are mainly reflected in two aspects: firstly, the model fails to adjust the weight proportion of the secondary indicators according to the match system and venue factors, which leads to its inability to accurately reflect the characteristics of athletes' performance in different environments; secondly, the model is unable to judge the overly complex rotation factors. This may cause a large prediction error in the actual competition. Therefore, further optimization of the model to take these key factors into account will help improve its prediction accuracy and applicability in table tennis.

4. Conclusion

This paper develops the FINX model, a hybrid evaluation and prediction framework that combines methods including exponentially weighted moving average (EWM), analytic hierarchy Process (AHP), CRITIC, entropy (ENT), preference ranking techniques similar to the ideal solution (TOPSIS), etc., to analyze and predict momentum changes in players.

First, in the weight modeling part, AHP (Analytic Hierarchy Process) is used to evaluate off-site factors, TOPSIS method to look at the internal situation, EWM (entropy weight method) to deal with endogenous factors, and ENT to deal with exogenous factors. The comprehensive application of these methods makes the analysis more comprehensive and in-depth.

Secondly, the prediction results of the FINX model are compared with the actual match results, demonstrating its strong predictive power in tennis and table tennis matches. Adapted to the unique dynamics of each sport, emphasizing the model's adaptability and potential utility in the broader context of sports performance analysis.

References

- [1] Suartini, Ni Komang Yanti; Divayana, Dewa Gede Hendra; Dewi, Luh Joni Erawati. Comparison Analysis of AHP-SAW, AHP-WP, AHP-TOPSIS Methods in Private Tutor Selection [J]. International Journal of Modern Education and Computer Science, 2023, Vol.15 (1): 28 - 45.
- [2] Gu, Yingkui; Wang, Yuanjin; Shi, Changwu. Remaining useful life prediction method of rolling bearing based on EWM and SVR [J]. China Safety Science Journal, 2023, Vol. 33 (9): 49 - 55.
- [3] Yijian Lu; Aiwei Liu; Dianshi Yu; Jingjing Li; Irwin King. An Entropy-based Text Watermarking Detection Method [J]. 2024.
- [4] Zaïbi Oussama; Hachicha Ahmed; Chaabane Nabil. Comparison of Macroeconomic Performance of MENA Countries with TOPSIS Method [J]. Operations Research Forum, 2024, Vol.5 (1).
- [5] Božanić, Darko; Tešić, Duško; Komazec, Nenad; Marinković, Dragan; Puška, Adis. Interval fuzzy AHP method in risk assessment [J]. Reports in Mechanical Engineering, 2023, Vol. 4 (1): 131 - 140.
- [6] Yusuf Ersoy; Ali Tehci. Relationship marketing orientation in healthcare organisations with the AHP method [J]. Operations Research and Decisions, 2023, Vol. 33 (1): 35 - 45.
- [7] Ziyuan Luo; Jian Tian; Jian Zeng; Francesco Pilla. Flood risk evaluation of the coastal city by the EWM-TOPSIS and machine learning hybrid method [J]. International Journal of Disaster Risk Reduction, 2024, Vol. 106: 104435.
- [8] Qun Sui & Sujit K. Ghosh. Entropy-Based Subsampling Methods for Big Data[J]. Journal of Statistical Theory and Practice, 2024, Vol.18 (2): 24.
- [9] Yi He, Jingyu Liu, Yanhua Duan, Xiaofei Yuan, Lulu Ma, Ratan Dhar, Yan Zheng. A critical review of on-site inorganic arsenic screening methods [J]. Journal of Environmental Sciences, 2023, vol. 125 (3): 453 - 469.

- [10] Xu, Xiuqing; Li, Fagen; Zhao, Xuehui; Yang, Fang. Permeability Model of Liquid Microcapsule Based on Multiple Linear Regression Method [J]. COATINGS, 2023, Vol. 13 (8): 1361.