Analysis of Combination of Four, Five Six Crease Vertices
Waterbomb

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Abstract. Origami combination is reliable because of great stabilization and controllable deployable. Potential and extensive applied in medical field, as a soft robot gripper. To investigate a possible stabilization structure, this design contained a combination of three four-crease vertexes, one five-crease vertex and one six-crease vertex. The design theory is based on waterbomb pattern design and obey the two design principles. The degree of freedom in this combination would be proof is zero, which demonstrates an over-constrained structure, and also the poor control performance and reliability. The reason might be the compelling in five-crease vertex and multiple Dof in the four-crease vertexes (which might be one Dof), which cause the overall outputs dihedral angles could not be controlled by overall inputs. Additionally, this design has faster equilibrium state because of smaller dihedral angle (smaller than 90 degrees) in each vertex, as an overstrained structure. The future improvement might be done by reduce the Dof in the four-crease vertex and promotion in five-crease vertex.

Keywords: 3D deployable structure; Four and five and six-crease vertexes; Kinematic analysis; Mechanical.

1. Introduction

Soft rigid origami as an increasingly prosperous field, which is not only benefit the aerospace area, mechanical area or even civil area because of the origami stabilize and great force-taken ability structure. Particularly, origami structure as a gripper to grasp or hold object is widely used in field of medicine and biology. The simple and interweaving structure made the origami structure maintain longer time and operate in a small error which could be taken. There are multiple designs aim at showing the feasibility and sustainability of origami gripper.

The most common type would be origami based on 3D printer [1]. Using different materials to design 3D gripper without using motors, which has more Dof and easy to apply into gripper and actuators market.

Then, the commercial type, origami paper based on spring. [2] This design also demonstrates the feasibility and great force-undertake of the origami structure even using a couples of springs to provide the stiffness force. The investigation also prove that the origami is not only suitable in commercial and high precision field, but this structure is also potential in alternative mechanical device.

Additional, to investigate the potential performance in force undertake ability and structure stability. Origami structure also used in design vacuum gripper. [3] The investigation shows that the force could be taken is surprised and big. Despite this, the structure could also be maintained in a flat and stability equilibrium state when deployed.

Origami gripper is also suitable in pneumatic gripper without external compressor. [4] Compared with above vacuum gripper design, this design demonstrated that the origami structure has great stability and force under-take when in great pressure operation. Noticed the force transmit is also efficiency in this structure.

In this paper, one special gripper design based on origami structure is waterbomb. [5] Waterbomb as a single vertex crease structure, which has flat equilibrium state when before deployable. During deployable process, the waterbomb structure would generate great grasp force and achieve maximum
force in one of the dihedral angles. Waterbomb structure is not only simple structure, but also the control system of this structure could be easy to monitor.

This paper focuses on a new design of part of waterbomb pattern, as well as mobility and kinematic analysis from separation difference vertexes to combination of these vertexes. The model design principle and method would be introduced in section 2, contained 2D planar design. The mobility and kinematic would be demonstrated in section 3. The final structure quantity and effect of special four-crease vertex would be concluded. In additional, the compelling in some of the separation vertex would be analysis.

2. Methods

A gripper could be based on rigid origami structure. [6] The degree of freedom could be gained by input one of the dihedral angles by different sensor and then output too demanded kinematic position. the rigid body gripper also shows great stability and great force undertaken ability. This paper aims to design the similarity pattern. So first using the waterbomb pattern design as Liu demonstrated,

![Figure 1. Waterbomb pattern [6]](image)

The basic components in similar origami design would be estimated, including two parts: multiple C2 control and central base which is fixed during deployable process. In specific here, the design of origami gripper would only focus on the backbone of a gripper and grasping an object by wrapping around it. At the same time, notified here, the combination of the design is constructed by four separated patterns. Then, to achieve the construction of two main features in gripper, four crease patterns as shown below [7]

![Figure 2. Stereoscopic square [7]](image)

Scratching the vertex and creases combination as the separation patterns

![Figure 3. Vertex and creases combination](image)

Then, assembling the Figure.3 combination and construct as follow
Figure 4. 2D planar design

The design satisfied the theory when building an origami zero thickness pattern, using Figure 5. The red point is each vertex in the half of gripper. Because the gripper is symmetric, half analysis could be used to represent the overall performance [8]

In each vertex, the difference between valley fold and mountain fold is 2
Total creases are equal to even value 32
Then in next section, the mobility and kinematic relationship would be calculated

3. Results

3.1 The mobility and kinematic of the separated vertexes

First named each part in half origami

![Pattern vertex in rigid paper model](image)

Figure 5. Pattern vertex in rigid paper model

Then, separated the patterned to 4-creases, 5-creases and 6 creases

3.1.1 Four-crease vertexes.

There are three four-crease vertexes (A, B, E). Results demonstration as followed

![4-creases in rigid paper model](image)

Figure 6. 4-creases in rigid paper model

Using the mobility equation, the mobility for 4-creases is

\[
Dof_i = 3(N - J - 1) + \sum_{i=1}^{N} f_i = 3(4 - 4 - 1) + 4 = 1
\]  

(1)

Where N is amount of linkage (part), J is amount of joint, \( f_i \) is total degree of freedom in joint (equals to 4 because each joint is rotation joint and the Dof is 1)
The mobility of 4-creases shows that the need one input to drive the other three outputs.
However, in the particular Figure 6a and c situation, the Dof is actually more than one, the vector equation would be introduced to proof.

Assumed a zero-thickness origami paper with a four-crease vertex. Set up the coordinate with original at the vertex, x axis along $a_1$ and facet set at the ground.

Using dihedral angle $\theta$ as input, $\delta$ & $\varepsilon$ as output, respectively.

The unit vector could be written in coordinate form

\[
\begin{align*}
  a_1 & = (1,0,0) \\
  a_2 & = (\cos \alpha, \sin \beta, \sin \alpha \sin \theta) \\
  a_3 & = (-\sin \beta^2 \cos \delta - \cos \beta^2, -\sin \beta \cos \beta + \sin \beta \cos \beta \cos \delta, \sin \beta \sin \delta) \\
  a_4 & = (\cos \beta, -\sin \beta, 0)
\end{align*}
\]

The vector multiplication of $a_3$ and $a_4$ is

\[
a_3 \ast a_4 = -\cos \beta
\]

Gives

\[
\cos \delta = 1
\]

Also, the multiplication of normal vector of plane A3-4 with A2-3 would be

\[
n_{23} \ast n_{34} = \cos \varepsilon
\]

Gives

\[
\cos \varepsilon = \sin \beta \sin \alpha \sin \theta
\]

Similarity

\[
n_{12} \ast n_{23} = \cos \mu
\]

Gives

\[
\cos \mu = \sin \alpha^2
\]

For Figure 6. (a), $z^A_1$ is along $a_1$ axis (Figure 7), $z^A_2$ is along $a_4$ axis, respectively.

\[
\begin{align*}
  \cos \varphi^A_2 & = 1 \\
  \cos \varphi^A_1 & = \sin 135^\circ \sin 135^\circ = 0.5
\end{align*}
\]

Where $\alpha = 135^\circ$

\[
\cos \varphi^A_3 = \sin 135^\circ \sin 135^\circ \sin \varphi^A_1 = 0.5 \sin \varphi^A_1
\]

For Figure 6. (c), $z^B_2$ is along $a_2$ axis (Figure 7), $z^B_2$ is along $a_2$ axis, respectively.

\[
\begin{align*}
  \cos \varphi^B_2 & = 1 \\
  \cos \varphi^B_1 & = \sin 90^\circ = 1
\end{align*}
\]

Where $\alpha = 90^\circ$

\[
\cos \varphi^B_3 = \sin 90^\circ \sin 90^\circ \sin \varphi^B_1 = \sin \varphi^B_1
\]

For Figure 6. (b), using matrix form [9],

\[
\begin{align*}
  \varphi^B_1 & = \varphi^B_3 \\
  \varphi^B_2 & = \varphi^B_4 \\
  \tan \frac{\varphi^B_2}{2} & = -\frac{\sin \alpha_{12} - \alpha_{23}}{2} = -\frac{\sin 90^\circ - 45^\circ}{2} = -1
\end{align*}
\]

Get

\[
\varphi^B_2 = 2\pi - \varphi^B_1
\]
3.1.2 Five-crease vertex

![Figure 8](image)

**Figure 8.** Top view of the design

Using the mobility equation, the mobility for 4-creases is

\[
Dof_2 = 3(N - J - 1) + \sum_{i=1}^{N} f_i = 3(5 - 5 - 1) + 5 = 2
\]  

(22)

The mobility of 5-creases shows that the need two input to drive the other three output. The rotation axis satisfied [9]

\[
\alpha_{s1} = \alpha_{12}, \alpha_{23} = \alpha_{45} = \frac{\pi}{2}, \alpha_{34} = \pi - 2\alpha_{12}
\]  

(23)

When fixed the relevant axis, the rotate angle satisfied

\[
\phi_4^D = \phi_3^D, \phi_3^D = \phi_2^D
\]  

(24)

\[
\frac{\tan \frac{\phi_3^D}{2}}{\tan \frac{\phi_2^D}{2}} = \frac{1 - \sin 45^\circ}{\cos 45^\circ} = \sqrt{2} - 1
\]  

(25)

Where \( \alpha_{12} = 45^\circ \)

\[
\tan^2 \frac{\phi_2^D}{2} - \frac{2\tan \frac{\phi_2^D}{2}}{\tan \frac{\phi_2^D}{2}} * \frac{\cos 45^\circ}{1 - \sin 45^\circ} - \frac{1 + \sin 45^\circ}{1 - \sin 45^\circ} = 0
\]  

(26)

3.1.3 Six-crease vertex

![Figure 9](image)

**Figure 9.** Six-crease in rigid paper model

Using the mobility equation, the mobility for 4-creases is

\[
Dof_3 = 3(6 - 6 - 1) + 6 = 3
\]  

(27)

The mobility of 6-creases shows that the need of three inputs to drive the other three. The kinematic calculation as followed

The rotation axis in this symmetry six-crease vertex satisfied [10]

\[
\alpha_{12} = \alpha_{34} = \alpha_{45} = \alpha_{61}, \alpha_{23} = \alpha_{56} = \pi - 2\alpha_{12}
\]  

(28)

When fixed the relevant axis, the rotate angle satisfied

\[
\phi_4^C = \phi_3^C, \phi_2^C = \phi_5^C = \phi_6^C
\]  

(29)

\[
\tan \frac{\phi_1^C}{2} + \cos 45^\circ * \tan \frac{\phi_2^C}{2} = 0
\]  

(30)
3.2 Mobility and kinematic of the overall combination

According to the following vertex A. Starting with A. Take the dihedral angle of crease 1 as input (overall input +1), and get the identical dihedral angles of creases 2, 3 and 4. Then for B, take 3 as input, which is already known from vertex A, getting 5, 6 and 7. Now checking vertex C. It has six creases and thus three inputs are required. But in this situation, C is symmetry. Only two inputs needed. So that 4 (from vertex A) and 7 (from vertex B could be used as input. Thus, output 14, 15 and 16 could be determinate. For vertex D, in order to find 9, 10 and 11, using 6 and 11 from vertex B and C. Finally, for vertex E, which required only one input, but there are two known inputs: 14 from vertex C and 10 from vertex 8. Therefore, there is one additional input (overall input -1). It is likely that only zero DoF in this combination mechanism because there will be a coupling equation at vertex E. The dihedral angle relationship analysis as followed

First considering vertex A

\[ \cos \varphi_2 = 1 \]  
\[ \cos \varphi_4 = 0.5 \]  
\[ \cos \varphi_3 = 0.5 \sin \varphi_1 \]

Get 4 in Fig.10 as a constant output, and

\[ \varphi_4 = 60^\circ \]

Using 3 as input, the dihedral relationship is

\[ \varphi_3^B = \varphi_3^B \]

The vertex B is satisfied

\[ \varphi_3 = \varphi_6 \]  
\[ \varphi_5 = \varphi_7 \]  
\[ \varphi_7 = 2\pi - \varphi_3 \]

Get 6 and 7. Then, for vertex C

\[ \varphi_4 = \varphi_{14}, \varphi_7 = \varphi_{11} = \varphi_{15} = \varphi_{16} \]

\[ \tan \frac{\varphi_4}{2} + \cos 45^\circ \times \tan \frac{\varphi_{16}}{2} = 0. \]

Using the dihedral relationship

\[ \varphi_4^A = \varphi_4^C = 60^\circ \]

Then substitute into above equation

\[ \varphi_{16} = -78.46^\circ \]

Then

\[ \varphi_7 = \varphi_{16} = 2\pi - \varphi_3 = 2\pi - \varphi_6 \]  
\[ \varphi_6 = 2\pi - \varphi_{16} = 438.46^\circ \]  
\[ \varphi_{11} = \varphi_{16} = -78.46^\circ \]

for vertex D

\[ \varphi_9 = \varphi_{10}, \varphi_8 = \varphi_{11} \]

\[ \tan \frac{\varphi_{10}}{2} = \sqrt{2} - 1 \]

Then, obtain \( \varphi_{10} \)
Using 10 as input into vertex E, then obtained
\[
\cos\varphi_{10} = \sin\varphi_{13}
\]
\[
\varphi_{13} = 52.63°
\]

4. Discussions

As Results demonstration, interesting phenomenon occurs. The design mechanism is actually zero Dof. Indeed, the mechanism is totally over-constrained structure. Meaning whatever input is, the output is fixed in each vertex. In the other word, the design is fixed in input and output dihedral angle. There are three possible reasons. First, the design is actually obeyed the design property, mentioned in Introduction. The difference between valley fold and mountain fold is not 2 in vertex D (five-crease), which mean the Dof in this vertex would be reduced to zero and cause overall Dof reduced. Secondly, the design in vertex A is defective, which cause Dof increased. The four-crease vertex has one Dof because of the input could be used to control the other three dihedral angle. [9] But in vertex A, the input could only control one dihedral angle (the opposite one). Other two adjacent dihedral angles, one would remain constant (zero), and one would depend on the crease angle in plane, which mean the Dof in this vertex is more than one, the input could control one and one would be control by plane geometry. Additionally, the Dof in vertex C is hard to estimated. The vertex C is a six-crease vertex, which would have Dof of three. However, the geometry of the vertex is symmetry, which means the Dof in this situation is reduce to one. The other dihedral angle could be obtained (equilibrium or satisfied equation) by input random one angle, which would cause conflict between vertex A, B and D. Because in imaginary, a better control mechanism would require additional input (one or more than one) to drive the system. Also, the additional input would follow the order of Dof. Said get two inputs from two four creases (one Dof), one input from five crease (two Dof), which driven by one input from one four crease and additional input from outside the system. There are three inputs in total and could be used to operate six-crease vertex. In this case, if only gaining input from A and B vertex, C could be driven, then the five creases would collapse.

In contrast, this design is not reliable to be a mechanism. However, the design might be suitable for a structure. There are two main advantages. First, for an over-constrained structure, the needed input is small only (\(\varphi_{13} = 52.63°\)). Therefore, if using this structure to a specific cover temp, the suitable input angle could be used as pillar. Because this design structure is symmetry, so that there are four inputs could be used. So maybe four normal forces could be generated. Despite this, another advantage is that all the dihedral angles are smaller than 90°. Which mean the actual volume under the structure cover would be approximately equals to cover volume. Small dihedral angle also means faster to achieve equilibrium state. Using opposite direction dihedral angle as input and pushing it to easily chieve the state.

5. Conclusion

In this paper, the special combination of four-crease, five-crease and six-crease vertexes have been discussed and demonstrate that this structure is an over-constrained structure with zero Dof. The structure also demonstrates the unstable of special four-crease vertex which has more than one Dof, which might cause compelling in the other vertexes and poor control performance. Despite this, this combination structure investigates the feasibility of four and six-crease vertexes. If adopted this special four-crease and six crease combination, the dihedral angle demanded to enter equilibrium state would faster, as the dihedral angle are all smaller than 90 degrees, which might be considered a good application structure. The design theory also be examined. The difference between two folds (valley and mountain) might influenced the actual Dof of a single vertex. In this paper, the compelling of five creases might be affected by this principle and distortion. Further investigation could be
improved by reducing the DoF of the special four-crease vertex to one DoF and change one of the crease type in five creases.

References


