

Systmatic Analysis of Magnetic Monopoles History, Current Research, and Advances

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Abstract. Magnetic monopoles, first posited by Paul Dirac in 1931, are hypothetical particles that carry a single magnetic charge, unlike ordinary dipoles which have both a north and south pole. These particles would introduce a striking symmetry to Maxwell's equations of electromagnetism and could provide a compelling explanation for the quantization of electric charge, a key aspect of our understanding of particle physics. Dirac's theoretical framework laid the groundwork for subsequent exploration of monopoles, inspiring decades of experimental efforts to detect them. These efforts have included searches for remnants of magnetic monopoles in ancient deep-sea ferromanganese crusts and attempts to produce them in advanced particle accelerator experiments, such as those conducted at CERN. Recent breakthroughs in condensed matter physics, particularly in the study of spin ice materials, as well as the creation of synthetic Dirac monopoles in Bose-Einstein condensates, have generated new avenues for understanding their properties and potential behavior. Although magnetic monopoles have not yet been observed directly, ongoing research continues to refine theoretical models and experimental techniques, keeping the search at the frontier of modern physics. The pursuit of monopoles remains significant, especially in the context of advancing toward a unified theory of fundamental forces that underpins the universe.

Keywords: Monopoles, Ferromagnetic, Maxwell's equation.

1. Introduction

Magnetic monopoles have fascinated physicists for nearly a century because they can incorporate symmetry into physical laws and unite fundamental forces. Paul Dirac initially hypothesized the existence of these theoretical particles in 1931 [1]. Magnetic monopoles possess only one magnetic charge, either north or south, in contrast to magnetic dipoles which have both charges. Dirac's proposition was not simple hypothesis; it was grounded in the exquisite symmetry that monopoles may potentially bring to Maxwell's equations, which serve as the fundamental principles regulating classical electromagnetism.

The identification of magnetic monopoles could have profound implications, which will elucidating the process of quantization of electric charge, a natural occurrence that current theories have not yet fully accounted for.

Dirac demonstrated that the presence of just one magnetic monopole in the cosmos necessitates the quantization of all electric charges in integer multiples of a fundamental charge [2]. Over time, advancements in particle detection and accelerator technology have significantly enhanced our capacity to investigate and locate monopoles. The Large Hadron Collider at CERN has played a crucial role in enhancing our comprehension of these particles.

This study seeks to offer a comprehensive summary of the progression from the initial theoretical propositions to the present state of investigation on magnetic monopoles.

2. Early Theoretical Foundations

The theoretical concept of magnetic monopoles was first introduced by Paul Dirac in his 1931 paper, "Quantized Singularities in the Electromagnetic Field." He's work sought to reconcile the asymmetry in Maxwell's equation regarding electric and magnetic charge and to provide an explanation for the quantization of electric charge observed in nature [3].

2.1. Dirac's Modification of Maxwell's Equations

In classical electromagnetism, the Maxwell equation is given by

$$\nabla \cdot E = \frac{\rho_e}{\epsilon_0} \quad (1)$$

$$\nabla \cdot B = 0 \quad (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

$$\nabla \times B = \mu_0 J_e + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad (4)$$

Where: $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, ρ_e is the electric charge density, $\mathbf{J}_e$ is the electric current density, ϵ_0 is the vacuum permittivity, and μ_0 is the vacuum permeability. The equation

$$\nabla \cdot B = 0 \quad (5)$$

Implies that there are no magnetic monopoles since it indicates that magnetic field lines are closed loops. Dirac proposed modifying Maxwell's equations by introducing a magnetic charge density, ρ_m , and a magnetic current density, $\mathbf{J}_m$.

$$\nabla \cdot B = \mu_0 \rho_m \quad (6)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} - \mu_0 \mathbf{J}_m \quad (7)$$

2.2. Dirac's Quantization Condition

Dirac demonstrated that if magnetic monopoles exist, it would lead to quantization of electric charge. Thus, the Dirac quantization condition is derived, which relates the fundamental magnetic charge g to the elementary electric charge e :

$$eg = \frac{n\hbar}{2} \quad (8)$$

Where n is an integer and $\hbar$ is the reduced Planck's constant. This condition implies that the product of the electric and magnetic charges is quantized.

2.3. Implications for Quantum Field Theory

Dirac illustrated that monopoles could exist at the endpoints of nodal lines with non-zero phase winding in a wavefunction. This theory describes how magnetic fields can be generated by the vector potential associated with these nodal lines, leading to the concept of the 'Dirac string'—a nonphysical line singularity terminating at the monopole. The incorporation of monopoles into electromagnetic has significant contribution in quantum field theory and unification of force. It introduces a duality between electric field and magnetic field, indicating a deeper understanding of fundamentals, which is a cornerstone of several grand unified theories and string theories [4, 5].

3. Historic Searches

Despite the Dirac theoretical significance, no direct evidence has been found, promoting a series of experimental searches over the decades. The following are summary of some notable historic attempts to detect monopoles.

3.1. Search for Magnetic Monopoles in Deep Ocean Deposits

One of the significant experimental efforts was by R. L. Fleischer in 1969. It was aimed to detect monopoles that are trapped in ferromanganese crusts from the deep ocean floor. This is due to the

hypothesis of that cosmic ray might produce monopole pairs, which might be trapped for billions of years [6].

The researches identify monopoles by using high-field magnets and solid-state track detectors. The devices used were pulsed-field magnet for small sample processing up to 265kg, and a steady-field magnet for larger samples with fields up to 104kg. Detectors contains two 250- μ -thick polycarbonate and cellulose nitrate, are designed to capture the tracks left by any monopoles extracted from the samples.

Despite the presence of over 8 kilograms of marine crust, no monopoles were observed. Nevertheless, the experiment has established stricter constraints on the prevalence of magnetic monopoles. It was specifically found that the rate at which monopoles reach the ocean floor is less than $4 \times 10^{-18} \text{cm}^{-2} \text{sec}^{-1}$ with 90% confidence. The findings indicate that cosmic rays with energies of up to 1020 eV are not primarily made up of magnetic monopoles. Thus, setting the upper bound on the monopole flux that can be expected from the cosmic ray under Earth's atmosphere.

3.2. Search for Magnetic Monopoles at Fermilab (NAL)

Another notable experiment was proposed by L.WAlvarez, P. H. Eberhard, R. R. Ross, and R. D. Watt at Fermilab which was called National Accelerator Laboratory(NAL) back then. Utilizing the property of the magnetic monopoles' electromagnetic to detect them over a macroscopic distance [7].

By placing a ferromagnetic trap in a beam dump, where proton-proton collisions happen, it might potentially produce monopoles. The existences of the trap are too well trap any monopoles that are produced, and existing detector would then be used to search for them.[8]

However, we have to assume that the monopoles would generate an electromotive force when moving through a coil for it to be trapped and thus detected. The researcher's approach was to grind the ferromagnetic material into powder and analyze each sample separately to prevent any possible annihilation between north and south monopoles [9].

In order to capture the monopoles without them escaping, the binding energy must withstand thermal agitation. In which the binding energy is given by

$$W = 3 \times 10^{-4} \mu \left(\frac{1}{r} \right) \quad (9)$$

Where W is the binding energy in electron volts, is the permeability of the ferromagnetic and r is the distance [10].

Despite its forward-thinking nature, this approach encountered substantial experimental difficulties. It addressed the necessity of extended periods of exposure and the management of radioactive substances, which presented practical challenges. However, it provided a basis for future studies by indicating that the absence of monopole detection would create stricter boundaries on their production cross section and mass.

4. Monopoles In Spin Ice

Spin ice materials are a class of magnetic system that have are able to host quasi particles similar to a magnetic monopole. They are characterized by a pyrochlore lattice structure in which magnetic moments (spin) are arranged in a way that mimics disorder in water ice, thus the name spin ice [11]. An example can be seen in figure 1.

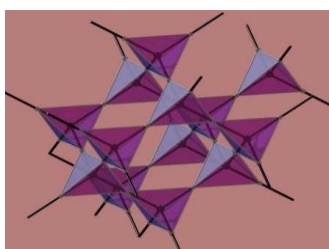


Figure 1. The pyrochlore and diamond lattices

4.1. Emergent Magnetic Monopoles

The magnet moment in spin ice is restricted by “ice rules”, where two spin point into and two spins point out of each tetrahedron in the lattice. This results in a highly frustrated system, where conventional magnetic ordering is inhibited, but thermal fluctuations can create defects which would lead to an isolated north or south magnetic charges that behave like monopoles [12].

These monopole-like excitations can be represented by,

$$\nabla \mathbf{B}_{\text{eff}} = \rho_m^{\text{eff}} \quad (10)$$

Where the divergence of the effective field $\mathbf{B}_{\text{eff}}$ is related to the emergent magnetic charge density ρ_m^{eff} . This highlights the link between the defects in spin ice and the Dirac’s theoretical monopoles [13].

The motion of these monopoles is influenced by an applied magnetic field H , which will exert a force on monopoles due to Lorentz force,

$$\mathbf{F} = q_m (\mathbf{v} \times \mathbf{B}_{\text{eff}}) \quad (11)$$

Where q_m is the magnetic charge of the monopole, $\mathbf{v}$ is its velocity, and $\mathbf{B}_{\text{eff}}$ is the effective magnetic field. Suggesting how monopoles move through the spin ice lattice under external influences [14].

4.2. Experimental Evidence

4.2.1 Neutron Scattering

Materials such as $\text{Dy}_2\text{Ti}_2\text{O}_7$ and $\text{Ho}_2\text{Ti}_2\text{O}_7$ proved direct evidence of monopoles excitations using neutron scattering [15]. Using this method allows them to observe the spatial and temporal correlation between spins which can be interpreted as the movement of monopoles-antimonopolies pairs through the lattice [16].

These experiment reveals peaks from the structure factor $S(Q)$ measured for presence of monopoles, which is represented by:

$$S(Q) = \sum_{i,j} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle e^{i\mathbf{Q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \quad (12)$$

Where $\mathbf{Q}$ is the momentum transfer vector, $\mathbf{S}_i$ and $\mathbf{S}_j$ are spin vectors at sites i and j , and $\mathbf{r}_i$ and $\mathbf{r}_j$ are their respective positions [17].

4.2.2 Magnetic Relaxation and Specific Heat

The observation monopoles in spin ice are further supported by magnetic relaxation and specific heat. Studies revel the presence of free-moving magnetic charges that contribute to its thermodynamic properties, e.g. increased entropy and temperature-dependent heat capacities. Indication of formation and movement of monopole-antimonopoly pairs is shown in the specific heat experiments has shown peaks at low temperatures. The heat capacity can be expressed as:

$$C(T) = \frac{\partial}{\partial T} \left(\sum_i E_i n_i \right) \quad (13)$$

Where E_i is the energy level of state i , n_i is the occupation number, and T is the temperature [18]. The phase diagram of spin ice is shown in figure 2.

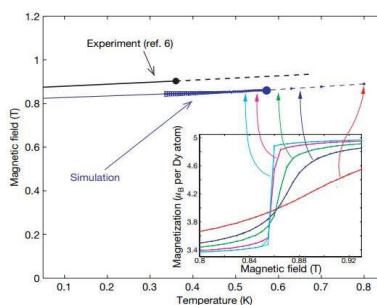


Figure 2. The Phase diagram of spin ice [18]

4.3. Theoretical Implications

Although the monopoles are emergent phenomena rather than fundamental monopoles, it still offers a unique platform for testing theories and inspired theoretical models that explore how monopoles interact each other with external magnetic fields. The motion of monopoles in response to external magnetic field can be represent by equation similar to those that is governing electric charge transport.

5. Synthetic Dirac Monopole

The observation of Dirac monopoles was achieved by M. W. Ray, E. Ruokokoski, S. Kandel, M. Mottonen, and D. S. Hall in their study using Bose-Einstein condensate (BEC) to simulate the conditions necessary for creation of Dirac monopoles. Using ferromagnetic spin-1 ^{87}Rb BEC it will generate a synthetic magnetic field by manipulating the spin texture of the condensate. This will involve adiabatically rotating the spin of the BEC in response to a time-varying magnetic field [19]. Figure 3 shows what their approach looks like.

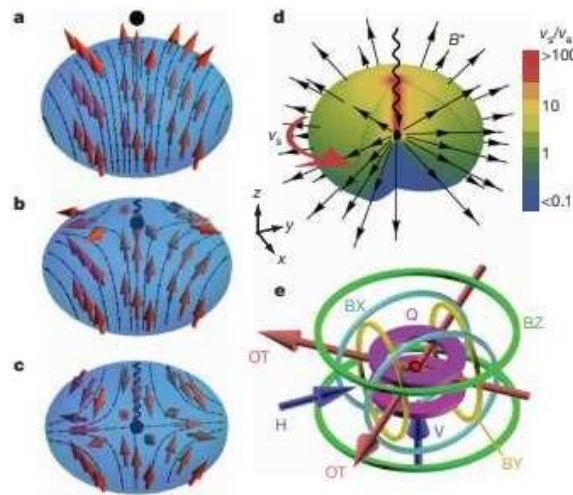


Figure 3. How monopole was created in theory and the experimental approach [14]

The experiment relies on the nonlinear Schrodinger equation that describe the quantum mechanical order parameter of the BEC. The synthetic gauge potentials were used to simulate a north pole, thereby creating a controlled environment to observe the monopole's effects. The magnetic field that was produced by the monopole is defined as:

$$B_1 = \frac{\hbar}{r_0^2} \hat{r}_0 \quad (14)$$

Which shows that the field behaves like a point source similar to how monopoles would theoretically produce a radial magnetic field. Inside the magnetic field, the flow of the BEC is given by the superfluid velocity equation:

$$v_s = \frac{\hbar}{Mr_0} \frac{1 + \cos \theta_0}{\sin \theta_0} \hat{\phi}_0 \quad (15)$$

Thus characterize the azimuthal flow around the monopole that is influenced by the condensate spins, and the rotational behavior of the superfluid is given by:

$$V = -\frac{\hbar}{Mr_0^2} \hat{r}_0 + \frac{4\pi\hbar}{M} \delta(x_0)\delta(y_0)H(z_0)\hat{r}_0 \quad (16)$$

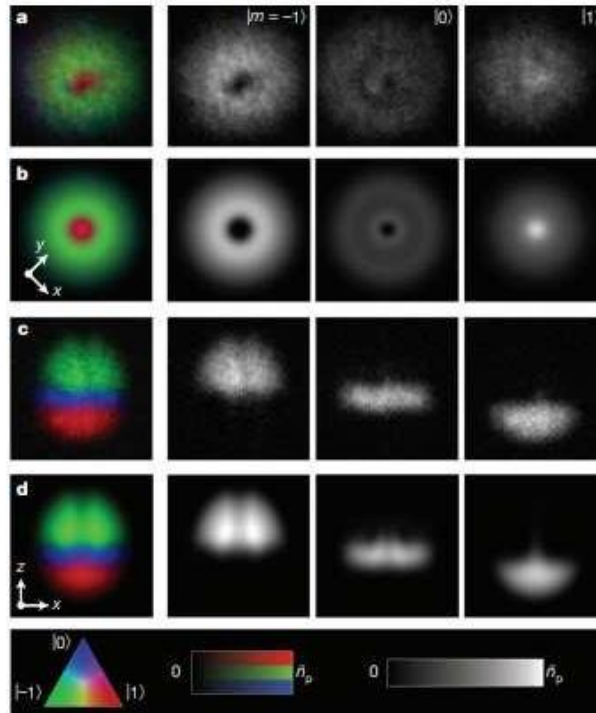


Figure 4. The Experiment data (a, c) vs the simulation data [14]

Which the presence of the Dirac delta function signifies a point where the rotational field has a discontinuity, indicative of a monopole's presence [20].

There were also numerical simulations that was based on solving the Gross-Pitaevskii equation (GPE), which describes the behavior of the wave function of a BEC at zero temperature:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2M} \nabla^2 + V(r, t) + g|\psi|^2 \right) \quad (17)$$

To simulate the influence of magnetic field on neutral atom in a BEC. It revealed a nodal line where the density of the BEC drops to zero indicating a vortex line. This nodal line also has a 4π phase winding, indicating the presence of a Dirac string. The numerical result was in an agreement with the experimental data, confirming the presence of a synthetic Dirac monopole within BEC, the simulation confirmed the experimental method [21].

6. Current Method

The ATLAS collaborations using the proton-pronton (pp) collision for detection. The unique capabilities of the ATLAS detector allow for a detailed explortion of highly ionizing particles, e.g. magnetic monopoles [22].

6.1. ATLAS Detector

The ATLAS detector is a particle detector at CERN designed to capture a wide array of particle interactions with near 4π coverage. The detector includes these several components. First the inner tracking detector (UD) surrounds the beam pipe and tracks charged particles paths using silicon pixel and strip detectors which is useful for measuring the trajectories of charged particles, operating with a magnetic field of 2T provided by a superconducting solenoid [23].

The calorimeter containing electromagnetic and hadronic calorimeters that measures the energy of particles, using liquid argon as medium to identify highly ionizing particles through energy deposits. Thus, can detect the ionization patterns of magnetic monopoles [24]. Lastly, the moun spectrometer detects and measures the momentum of mouns, utilizing an air-core toroidal magnet. It provides additional layer of detection for particles that penetrate through the calorimeter mentioned above. Overall, the detector is capable to identify high ionization properties is monopoles.

6.2. Simulation and Models

The method used involves simulating the production of monopoles through Drell-Yan and photon-fusion process which is shown in figure 5, accounting for their potential spin-0 and spin-1/2 nature.

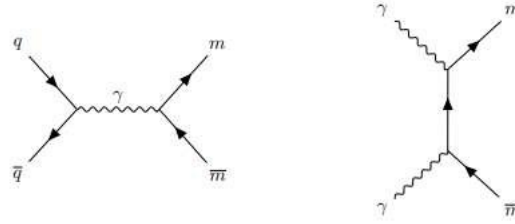


Figure 5. Feynman diagrams for Drell-Yan and photon-fusion pair-produced monopoles [19]

The Drell-Yan process involves the production of monopole pairs via quark-anti-quark annihilation, this process is highly predictable and calculate within the quantum field theory framework. The photon fusion process occurs when two photons interact to produce monopole pairs, it can have higher cross-section at LHC energies, thus providing an alternative production channel. Monte Carlo simulations are employed to generate events and assess detection [25].

The simulations for monopoles behavior are based on the GPE that was described in (17). Being a highly ionizing particle, monopoles have a distinct energy loss when traveling through matter, which is given by:

$$-\frac{dE}{dx} = \frac{4\pi N_A z^2 \alpha^2}{m_e c^2 \beta^2} \left(\ln \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - 2\beta^2 \right) \quad (18)$$

Where z is the charge of the monopole, α is the fine structure constant, N_A is Avogadro's number, m_e is the electron mass, and T_{max} is the maximum energy transfer in a single collision.

The analysis hinges on distinguishing highly ionizing particles from background noise. HIP is expected to produce extensive ionization along their trajectory, identified through the transition radiation tracker and as mentioned above the calorimeter deposit. To calibrate to maximize signal detection while minimizing false positives, two function is introduced. The fraction of high-threshold hits (f_{HT}) is calculated by

$$f_{HT} = \frac{N_{HT}}{N_{total}} \quad (19)$$

Where N_{HT} is the number of high-threshold hits, and N_{total} is the total number of hits in the TRT. This represents the number of high-energy hits compares to the total number of hits in the TRT. Where we can see it in figure 6. This can be used to identify the ionization characteristics expected of monopoles.

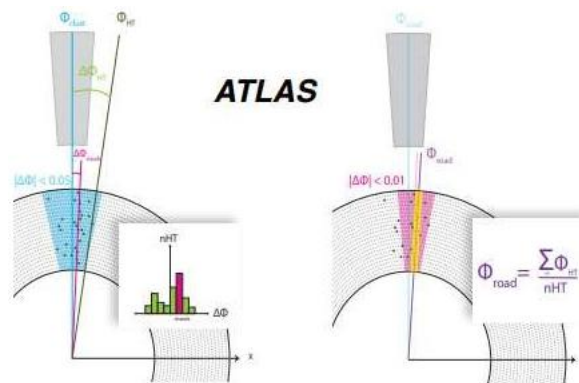


Figure 6. Depicts the signal selection criteria using TRT hits and EM cluster analysis [19]

Another component is the narrowness of the electromagnetic energy cluster, reflecting how the energy from a detected event is distributed in the calorimeter. A narrow energy cluster is characteristic of a monopole, as it does not induce an electromagnetic shower like conventional particles, and it is represented by:

$$w = \frac{\sum_i E_i^2}{(\sum_i E_i)^2} \quad (20)$$

Where E_i represents the energy deposited in each calorimeter cell.

6.3. Results and Implications

Despite extensive data collection and analysis, no magnetic monopoles nor any high-electric-charge objects were identified in the data, which encompassed an integrated luminosity of 138 fb^{-1} . The search set new limits on the production cross-section for monopoles, refining previous constraints and establishing benchmarks for future investigations. The upper limits on the production cross-section for monopoles are calculated by

$$\sigma < \frac{N_{obs}}{L \cdot \epsilon} \quad (21)$$

Where N_{obs} is the number of observed events, L is the integrated luminosity, and ϵ is the detection efficiency.

7. Conclusion

The continuing search for magnetic monopoles, although lacking conclusive proof, has greatly enhanced our comprehension of theoretical physics and experimental techniques. Every endeavor to identify monopoles, ranging from investigations conducted in the depths of the sea to high-energy collisions between particles, has enhanced our understanding of the theoretical structure related to these mysterious particles and limited the range of possible values for their existence. These efforts have not been futile; they have established crucial limitations that direct current and future study.

Advancements in the investigation of spin ice materials and the creation of synthetic simulations of Dirac monopoles using Bose-Einstein condensates have brought forth novel methods for studying monopole phenomena. These systems provide distinct chances to examine behavior similar to that of a monopole and research how they interact in controlled conditions. This allows for vital insights into their possible attributes and impacts.

The discovery of magnetic monopoles has significant ramifications, since it has the potential to enhance our comprehension of fundamental forces and provide fresh insights into the unification of electromagnetism with other natural forces. This significant discovery has the potential to result in innovative technology and revolutionary scientific progress, fundamentally altering our comprehension of the cosmos.

In the future, combining new theories with advanced experimental methods is crucial for discovering the mysteries of magnetic monopoles. As our tools and technologies advance, our capacity to explore the enigmas of the universe also expands. The pursuit of monopoles exemplifies the scientific curiosity and ongoing endeavor to push the limits of human understanding.

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