

Research on Sex-Ratio-Variable Sea Lamprey Populations Based on Logistic and Lotka-Volterra Models

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Abstract. The increasing prevalence of invasive species like the sea lamprey poses significant challenges to aquatic ecosystems. This study explores the potential ecological impacts of the sea lamprey's unique ability to alter its sex ratio. This paper developed mathematical models based on Logistic and Lotka-Volterra equations to simulate the impact of sex ratio changes on population dynamics, genetic diversity, and ecosystem stability. In this model, the carrying capacity of the sea lamprey's environment is linked to its food supply. According to the model developed in this paper, sea lampreys are more likely to develop into males when food is scarce, which is consistent with observed behavior. Additionally, the findings suggest that the sea lamprey's mechanism for adjusting its sex ratio provides it with a significant advantage over other species, both in terms of competition and predation. The findings highlight the dual role of sex ratio variability in enhancing population resilience while potentially reducing genetic diversity, providing insights into managing invasive species and preserving biodiversity.

Keywords: Lampreys, Sex Ratio, Logistic Model, Lotka-Volterra Model.

1. Introduction

Lampreys, with an evolutionary history that spans over 125 million years, are one of the most ancient extant vertebrates. Unlike most vertebrates, sea lampreys have the rare ability to change sex based on environmental conditions and growth rates, making them a subject of significant scientific interest. The ability of these creatures to alter their sex has profound implications for their population dynamics and ecological interactions, yet this phenomenon is not fully understood within the broader context of ecosystem management and stability. Previous studies on lampreys have predominantly focused on their role as invasive species in new habitats, where they often cause significant ecological and economic damage. However, the inherent biological capabilities of lampreys, such as their sex reversal, have not been thoroughly investigated in terms of their impact on native ecosystems and population regulation. Traditional research has largely overlooked how these unique traits can influence ecological stability and adaptability in their natural habitats [1-4].

In the realm of biology, the sex ratio of most organisms is maintained around 1:1. Only a few species deviate from this rule, such as the red centipede and the lamprey, which are special cases. Mark Ian Cooper employed statistical methods to gather data and utilized regression analysis, incorporating the Pearson correlation coefficient, to substantiate a significant relationship between the sex ratio of the red millipede and its population richness [5]. Similarly, research indicates that the sex ratio of the lamprey is also susceptible to the influence of external environmental factors, such as temperature and food availability [6]. York, Joshua R., and colleagues have conducted research on the sea lamprey from a molecular genetics perspective, discovering that it is possible to control the population density of the sea lamprey through genome editing techniques [7]. Johnson N.S. and associates utilized a Bayesian hierarchical logistic regression model, and through field research, discovered that the sex determination of the sea lamprey is directly influenced by the larval growth rate [8].

However, in previous studies, few researchers have utilized modeling techniques to analyze the unique growth and reproduction patterns of the sea lamprey population. This article analyzes the sea lamprey population by integrating traditional ecological dynamics with the ecological niche occupied

by the sea lamprey within its ecosystem [9-10]. This paper seek to explore the ecological consequences of the sex reversal ability in lampreys, particularly focusing on how changes in sex ratios could influence interspecific relationships, resource competition, and ecosystem resilience. The research is motivated by the need to understand the adaptive strategies of lampreys that enable them to thrive in diverse environmental conditions, and how these strategies affect other species within the same ecosystem. A combination of mathematical modeling and empirical data analysis is utilized in this paper to examine the effects of sex ratio variations on lamprey populations and their ecological impacts. This includes developing predictive models to simulate population dynamics under different environmental scenarios, and assessing the influence of sex ratio changes on the competitive interactions among species, particularly in regions where lampreys are a dominant part of the food web.

2. The basic fundamentals of the models used in the article

2.1. Logistic Model Based on Sex Ratio

The Logistic model, also known as the Logistic growth model, was initially proposed by Pierre François Verhulst in the 19th century to describe the growth of biological populations. This model assumes that the growth rate of a population slows down as its size increases, eventually reaching a stable level, known as the carrying capacity of the environment.

The mathematical expression for the Logistic model is:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right) \quad (1)$$

Where N represents the current population size; t denotes time; r is the intrinsic growth rate (the maximum rate of growth in the absence of resource constraints); and K stands for the carrying capacity of the environment (the maximum population size that the environment can support).

This paper consider the relationship between environmental capacity and food abundance, assuming that their relationship satisfies the following equation:

$$K = K_0 \varepsilon^a \quad (2)$$

Where K_0 is the maximum capacity of the environment; ε indicates the abundance of food, ranging from 0 to 1; assuming $\varepsilon = 0.8$ for high food abundance and $\varepsilon = 0.3$ for low food abundance. a Is the sensitivity parameter.

When considering the different growth rates of female and male lampreys under different food conditions, this paper will present two differential equations as follows:

$$\begin{cases} \frac{dF(t)}{dt} = r_1 \cdot F(t) \cdot [1 - (F(t) + M(t))/K] \\ \frac{dM(t)}{dt} = r_2 \cdot M(t) \cdot [1 - (F(t) + M(t))/K] \end{cases} \quad (3)$$

Under different food conditions, lamprey populations grew at different rates, r_1 for males, and r_2 for females, and they are related to K in the following equation:

$$\begin{cases} r_1 = \alpha K \\ r_2 = \beta K \end{cases} \quad (4)$$

Assume r_1 and r_2 are proportional to K with scale factors α and β . Normalize r_1 of the above equation as follows:

$$\begin{cases} r_1 = 1 \\ r_2 = \varphi \end{cases} \quad (5)$$

Since the sex ratio example has an impact on the population natality, which in turn affects the population size, this paper optimize the model with the following equation:

$$\mu = \frac{M(t)}{F(t) + M(t)} \quad (6)$$

This paper set the initial value $\mu_0 = 0.5$, which indicates that the initial ratio of male to female sea lampreys is 1 to 1. To determine the degree of deviation in the sex ratio of sea lampreys over time, the sex ratio at different time points would be divided by the initial sex ratio. This calculation would give us a measure of how much the sex ratio has deviated from the initial ratio as time progresses. This paper use m to represent the degree of deviation in the sex ratio of sea lampreys.

$$m = \frac{\mu_0}{\mu(t)} \quad (7)$$

Incorporating m modifies the established logistic differential equation, making it more reflective of real-world conditions. Because lampreys can only switch from female to male, which indicates $\mu > 0.5$, $m < 1.0$ the modified differential equation is as follows:

$$\begin{cases} \frac{dF(t)}{dt} = m \cdot r_1 \cdot F(t) \cdot [1 - (F(t) + M(t))/K] \\ \frac{dM(t)}{dt} = m \cdot r_2 \cdot M(t) \cdot [1 - (F(t) + M(t))/K] \end{cases} \quad (8)$$

The modeling process is presented as shown in Figure 1.

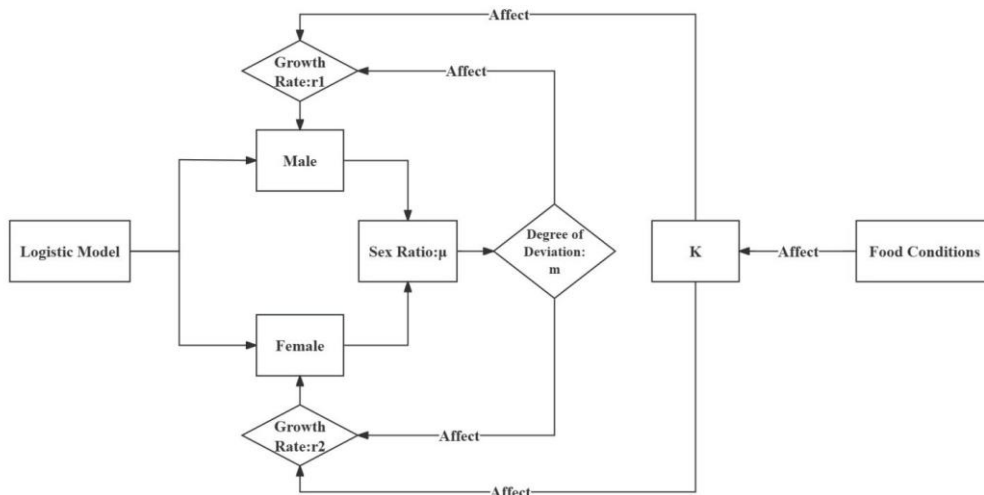


Figure 1. Logistic Model Based on Sex Ratio.

2.2. Lotka-Volterra Model Based on Sex Ratio

The Lotka-Volterra model, also known as the predator-prey model, is a mathematical model used to describe the interactions between two species in an ecosystem. This model was independently proposed by Alfred J. Lotka and Vito Volterra in the early 20th century and is widely used to study the dynamic relationships in ecosystems, such as those between predators and prey and competing species.

Assuming a species P that is in competition with lampreys, in conjunction with the model from 2.1, also combined with the Lotka-Volterra Model, this paper give the differential equation for its competitive relationship with lampreys as follows:

$$\begin{cases} \frac{dF(t)}{dt} = r_1 \cdot F(t) \cdot \left[1 - \frac{F(t) + \varsigma_1 \cdot P(t)}{K_1} \right] \\ \frac{dM(t)}{dt} = r_2 \cdot M(t) \cdot \left[1 - \frac{M(t) + \varsigma_2 \cdot P(t)}{K_1} \right] \\ \frac{dP(t)}{dt} = r_3 \cdot P(t) \cdot \left[1 - \frac{P(t) + \xi_1 \cdot F(t) + \xi_2 \cdot M(t)}{K_2} \right] \end{cases} \quad (9)$$

Where ς denotes the effect of lampreys on P, ξ denotes the effect of P on lampreys, and subscripts 1 and 2 denote females and males, respectively. Since male lampreys are more dominant in competition, assuming that $\varsigma_1 = 0.2$ and $\varsigma_2 = 0.6$; $\xi_1 = 0.2$ and $\xi_2 = 0.5$, environmental carrying capacity of lamprey populations $K_1 = 1000 \cdot \varepsilon^{0.178}$, environmental carrying capacity of P populations $K_2 = 1000$. Assume that in the initial state, the sex ratio of lampreys is 1:1, and their total number is the same as that of species P.

Assuming that population Q is preyed upon by the sea lampreys, this paper have also established corresponding differential equations for it based on the Lotka-Volterra model as follows:

$$\begin{cases} \frac{dF(t)}{dt} = [m \cdot r_1 \cdot F(t) \cdot (1 - (F(t) + M(t))/K_1)] + \frac{\delta_1 \cdot F(t) \cdot Q(t)}{1 + \gamma_1 \cdot Q(t)} - d \cdot F(t) \\ \frac{dM(t)}{dt} = [m \cdot r_2 \cdot M(t) \cdot (1 - (F(t) + M(t))/K_1)] + \frac{\delta_2 \cdot M(t) \cdot Q(t)}{1 + \gamma_2 \cdot Q(t)} - d \cdot F(t) \\ \frac{dQ(t)}{dt} = [r_3 \cdot Q(t) \cdot (1 - Q(t)/K_2)] - \frac{\delta_1 \cdot F(t) \cdot Q(t)}{1 + \gamma_1 \cdot Q(t)} - \frac{\delta_2 \cdot M(t) \cdot Q(t)}{1 + \gamma_2 \cdot Q(t)} \end{cases} \quad (10)$$

Where δ denotes the effect of lampreys on Q, γ denotes the effect of Q on lampreys, and subscripts 1 and 2 denote females and males, respectively. Assuming that $\delta_1 = 0.02$ and $\delta_2 = 0.03$; $\gamma_1 = 0.03$ and $\gamma_2 = 0.02$, d represents the mortality of a species' environmental carrying capacity of lamprey populations $K_1 = 100 \cdot \varepsilon^{0.178}$, environmental carrying capacity of Q populations $K_2 = 200$.

2.3. CA model

The CA model, or Cellular Automaton model, is a mathematical model used to simulate the dynamic behavior of complex systems. Cellular automata consist of a series of simple units, called cells, which update their states according to specific rules at discrete time steps. Due to their flexibility and scalability, cellular automata are widely used in various fields such as physics, biology, computer science, and social sciences. From simulating material diffusion and ecosystem dynamics to designing parallel algorithms and modeling urban traffic flow, cellular automata provide a powerful tool for studying and understanding the emergent behavior of complex systems.

First, a two-dimensional grid is initialized, and each grid point can be a male, a female lamprey, or a competitor, replaced by different colors. Next, a cellular update is performed according to the cellular state update rule, and when the density of surrounding competitors exceeds a threshold, females will probabilistically invert males according to the degree of exceeding the threshold as a way to model the change in sex ratio under intensified competition for resources. The update rules of the CA model are in the form of difference equations derived from the differential equations in the established Lotka-Volterra model. Next, the model changes in competitor density. The probability of

increasing competitors when lamprey density is high. This is used to model competitor response to resources and competition for ecological niches.

Considering the discretization of the time of the tuple automaton, in the function, this paper show the evolution of the tuple state over time by iteratively applying the update rule successively. Finally, this paper visualize the whole process.

3. Results

3.1. Analysis of Logistic Model and Lotka-Volterra Model Results

Images of the changes in population size and sex ratio of lamprey over time under adequate and deficient food conditions are drawn as Figure 2 and Figure 3:

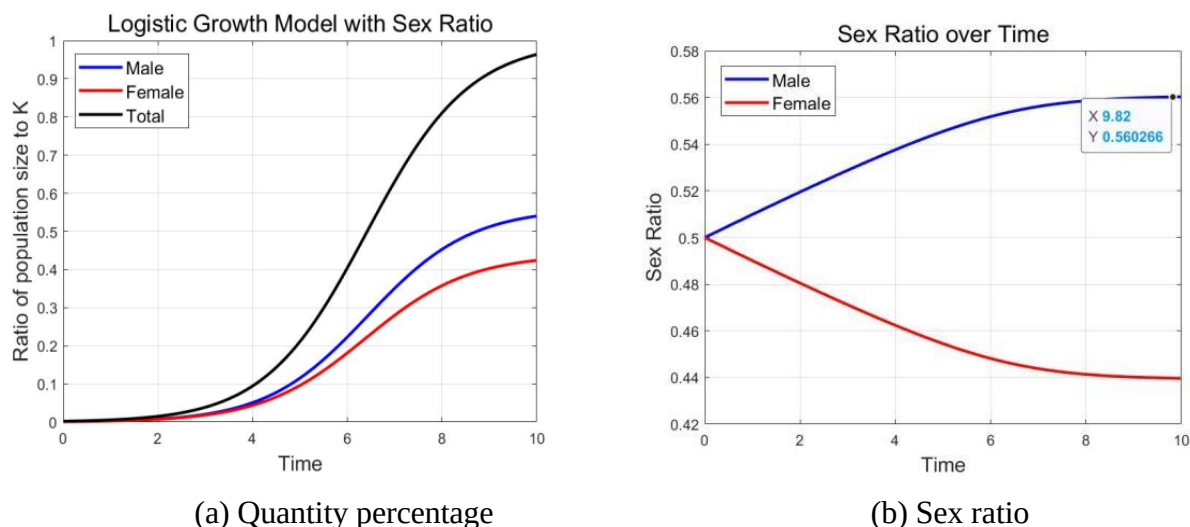


Figure 2. Ratio of population size to K and population sex ratio over time when food is sufficient ($r=0.8$).

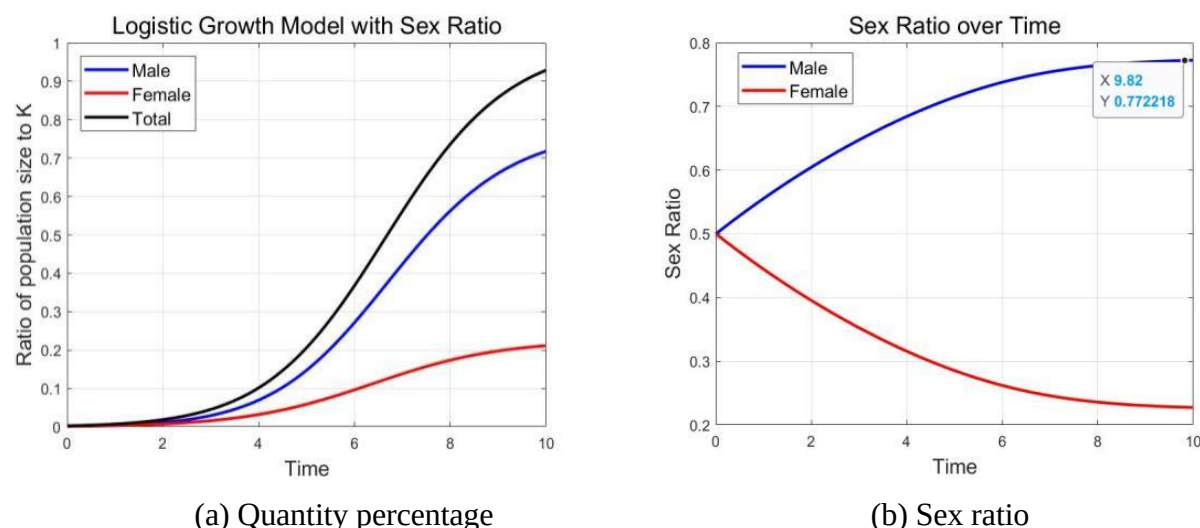


Figure 3. Ratio of population size to K and population sex ratio over time when food is shortage ($r=0.3$).

Based on the real-world data obtained: when food conditions are plentiful, the proportion of males in the population can reach up to 56%, whereas in times of food scarcity, this proportion can escalate to 78%. To accurately reflect these dynamics in the model, this paper meticulously adjusts the value of the sensitivity coefficient a . After numerous iterations and recalibrations, the optimal value for a was determined to be 0.178. This adjustment ensures that the model's predictions are in complete

harmony with the observed scenarios, thus providing a reliable tool for understanding the impacts of environmental conditions on sex ratios.

According to the Lotka-Volterra Model, the image obtained from the solution of the final differential equation is as Figure 4 and Figure 5.

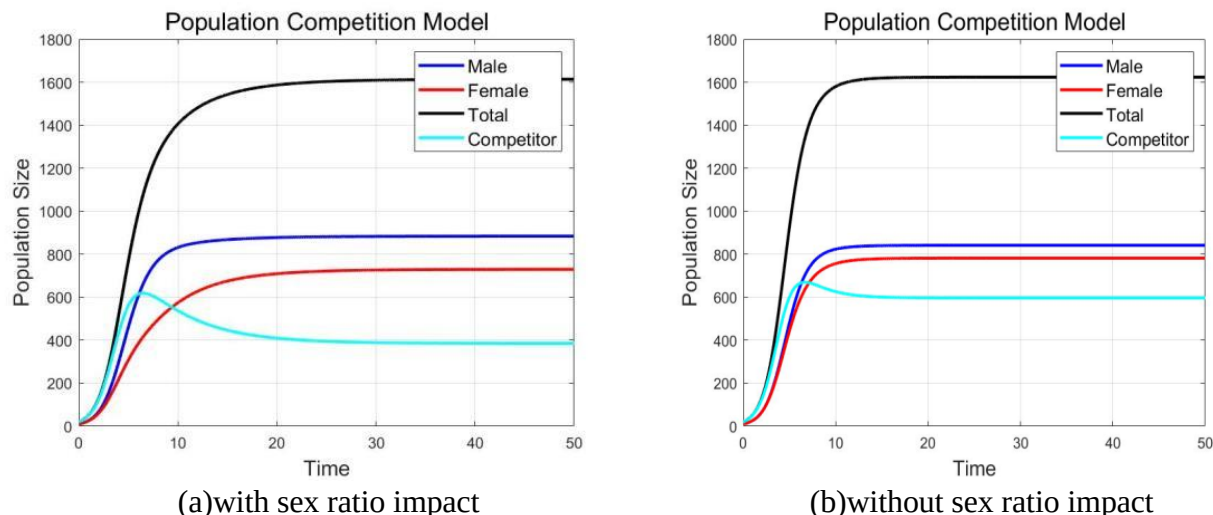


Figure 4. Effects of lampreys with and without sex ratio on P species when food conditions are adequate.

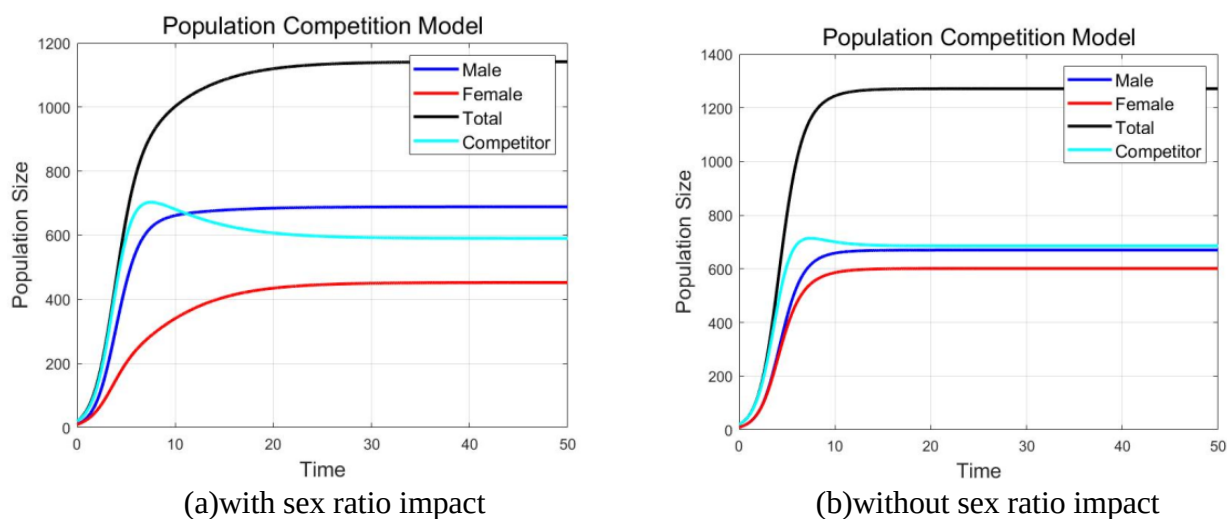


Figure 5. Effects of lampreys with and without sex ratio on P species when food conditions Are shortage.

When the population size of lampreys is correlated with sex ratio, compared to when it is not (assuming the same growth rates for females and males), lampreys are more competitive for resources. It is evidenced by the fact that there are fewer of them when the growth of P species reached a plateau. When food is plentiful, the lamprey population remains more competitive even though the proportion of males that are more competitive at this time is lower, but the environmental capacity of lampreys has a greater effect on their competitiveness.

3.2. Analysis of Cellular Automaton model results

Next, the model changes in competitor density. The probability of increasing competitors when lamprey density is high. This is used to model competitor response to resources and competition for ecological niches.

Considering the discretization of the time of the tuple automaton, in the function, the evolution of the tuple state over time is shown by iteratively applying the update rule successively. Finally, the

whole process is visualized. Competitor P is taken as an example, when the threshold is 0.4, as shown on the left, and when the threshold is 0.2, as shown in Figure 6.

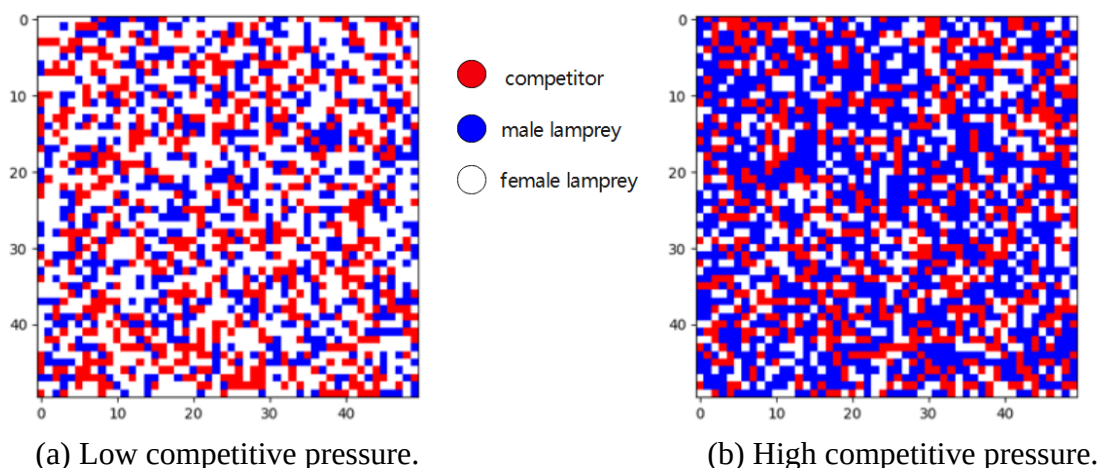


Figure 6. Population status of lampreys and competitor at different competitor density thresholds.

As the thresholds decrease, the level of competition increases, typically leading to more intense resource competition. Simultaneously, there is a significant increase in the proportion of male lampreys. It can be inferred that since males usually exhibit stronger competitiveness in resource conflicts, the increase in the male proportion may be an adaptive response to heightened competition. Based on this, this paper conclude that changes in the male sex ratio of lampreys make their populations more competitive. This finding not only validates the connection between the male proportion and competitiveness but also provides strong support for the accuracy of the conclusions.

4. Conclusions and Outlooks

In this paper, the complex issue of predicting the dynamics of sea lamprey populations based on changes in their sex ratio was addressed. By integrating traditional ecological models with advanced mathematical modeling, this paper developed a comprehensive framework to simulate how fluctuations in sex ratio affect population dynamics and ecosystem stability. The models incorporate real-world environmental variables, providing a robust tool for predicting the impacts of these factors on sea lamprey populations.

The approach involved constructing logistic and Lotka-Volterra models to capture the effects of sex ratio changes on the competitive interactions and survival strategies of sea lampreys. The logistic model highlighted the impact of environmental capacity and food abundance on population growth rates, while the Lotka-Volterra model explored interspecies interactions under different scenarios of sex ratio adjustments. Additionally, the Cellular Automaton model is utilized to simulate sea lamprey populations under varying competitive pressures to corroborate the conclusions. The results from these models demonstrated a significant correlation between sex ratio variability and population resilience, emphasizing that sea lampreys' behavior of altering their population sex ratio in response to environmental pressures is beneficial for enhancing their competitiveness in macro-systems.

The significance of the research lies in its ability to offer insights into the adaptive mechanisms of sea lampreys, which can aid in managing invasive species and conserving biodiversity. The models are not only applicable to ecological studies of lampreys but also extend to other species and ecosystems, providing a valuable framework for understanding the broader ecological implications of sex ratio dynamics. Thus, this work contributes to the field by enhancing the understanding of how intrinsic biological traits interact with environmental factors to shape population structures in both natural and invaded ecosystems.

However, it should be noted that although this model incorporates multiple parameters to ensure flexibility, some of these parameters are difficult to calculate precisely. Therefore, appropriate estimates are provided based on references. Nonetheless, the key parameters within the model are rigorously derived and analyzed. Additionally, various types of water bodies have distinct environmental conditions. Lamprey populations also vary in many other aspects, and some of the potential influences have been idealized during the study process. Moving forward, further research is needed to refine these models and validate them under more diverse environmental conditions to enhance their applicability and accuracy in real-world scenarios.

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