Analysis of Optimal Water Levels in the Great Lakes Based on A Comprehensive Satisfaction Function

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Abstract. The purpose of this paper is to analyze the optimal water levels in the Great Lakes to meet the needs of multiple stakeholders and thus achieve sustainable development in the region. The Weight-Function-Synthesis-Optimization (WFSO) model was used in this study. By classifying the water level needs of different interest groups, their respective satisfaction functions are established, and the overall satisfaction function is formed by weighting. In addition, the quadratic polynomial function is used to fit the optimal monthly water level time series of Lake Ontario, and the fitting effect is good, fully reflecting the periodic and seasonal changes in the water level of Lake Ontario. By considering the priorities of various stakeholders, this paper defines an overall satisfaction metric, "allsa," and identifies a time series of monthly optimal water levels for Lake Ontario and beyond. The ideal water level was determined to be 74.9 meters. The study results suggest that maintaining this water level can maximize the satisfaction of ecosystems, transportation, real estate, terminals, and hydropower companies, and promote sustainable development of the region.

Keywords: Satisfaction Function, Optimization, Lake Ontario, Polynomial function.

1. Introduction

The Great Lakes are important to the ecosystems and environment within the basin and are an important source of transportation, fisheries, tourism, and water supply. The water level of the Great Lakes not only affects the stability of its ecosystems but also directly affects the vital interests of residents, businesses, and industries in the basin [1]. Therefore, the management and optimization of water level changes are essential for the social, economic, and environmental sustainability of the region. However, due to the interconnectedness of the Great Lakes and the fact that their water levels are influenced by a combination of factors such as climate change, seasonal rainfall, evaporation, human activities, and river inflow, determining the optimal water levels in the Great Lakes is a complex task [2]. Different stakeholders have different expectations, which often makes it difficult to align management goals. Therefore, balancing these differing needs to determine the optimal lake water level is extremely challenging.

At present, there is a lot of research literature on the prediction and optimization methods of lake and river water level. Pan Liu et al. [3] used the Copula function to establish the joint distribution of seasonal floods, and clarified the frequency relationship between the seasonal flood quantile and the annual maximum, aiming to develop water systems in different seasons without reducing the original flood control standards, so as to obtain greater economic benefits. Prékopa, A et al.[4] introduced a stochastic programming model system for optimal control of storage levels. Shuo Ouyang et al. [5] studied the compensation relationship between reservoirs and proposed an improved Adaptive Electromagnetic Mechanism-like Mechanism (SEM) algorithm to deal with the ODFLL problem to maximize water resource utilization. However, the time frame is composed of many future periods, and each model in the existing studies is only used to determine the optimal strategy for the future period, and does not comprehensively consider the needs of each group to maximize water level satisfaction.

In this study, a Weight-Function-Synthesis-Optimization (WFSO) model was established to solve the optimal water level. Firstly, the water level was classified according to the needs of different groups, and the satisfaction function was obtained, and the satisfaction function was established by weighting and integrating all the satisfaction functions, finally, the seasonal characteristics of the water level were determined and the time series of the optimal water level was established.

2. Data Description and Preparation

The data source for this study is https://www.comap.com/contests/mcm-icm, using monthly water level data that has been available in recent years, to calculate the monthly average water level of each lake or river. A time series plot was generated from the monthly average water level. Figure 1, from top to bottom, illustrates the water level variations of the St. Lawrence River, Ottawa River, Niagara River, Lake Erie, and Detroit River.

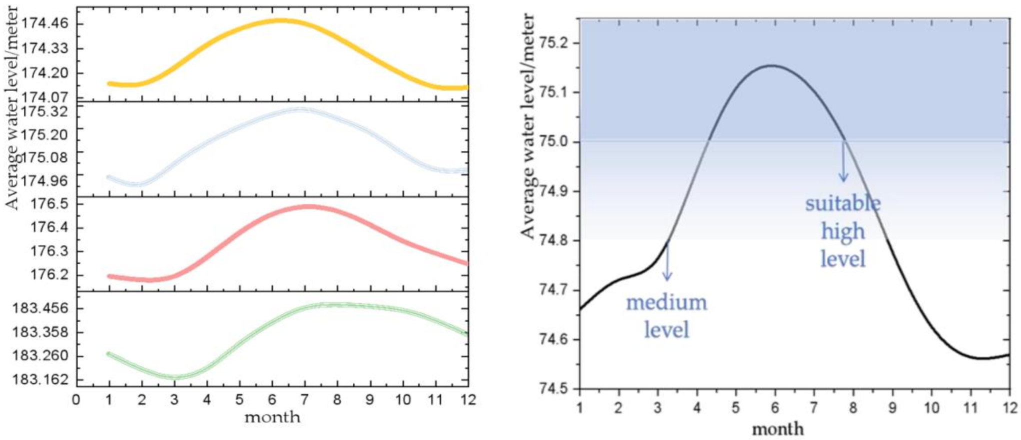

Figure 1. Changes in water levels

The water level of lakes exhibits seasonal cyclic variations. The water level changes of Lake Ontario are more pronounced compared to the other four lakes.

The water levels of lakes exhibit seasonal periodic changes[6], with Lake Ontario showing more pronounced fluctuations compared to the other four lakes.

Extensive research in the field[7-9] reveals that water levels have significant impacts on the ecosystems surrounding lakes and waterborne transportation. When the water level is maintained above 75.4 meters, grassy marshes, and shrub species along the shoreline tend to flood persistently. Around 74.8 meters, there is a noticeable increase in vegetation coverage, leading to a more diverse ecosystem. As the water level drops below 74.5 meters, biodiversity significantly decreases, and within this range, there may be severe impacts on the lake's ecosystem, including a reduction or migration of plant and animal populations[10].

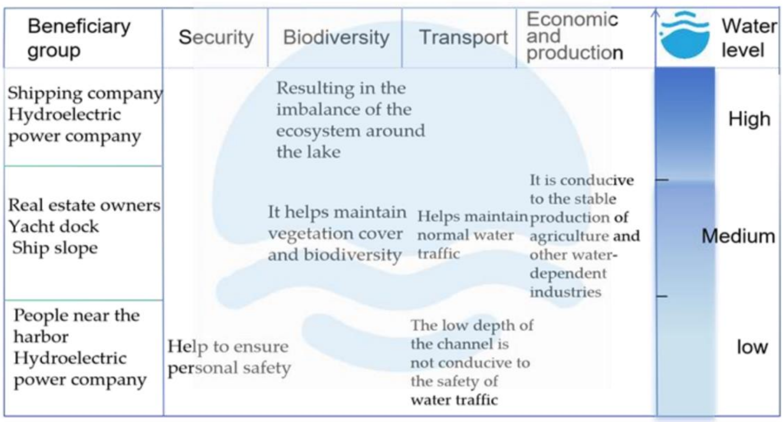

Figure 2. Water level analysis table

Combining existing data, the statistical chart as shown in figure 2, and the aforementioned time series graphs, this paper categorizes water levels into three stages based on the following criteria:

(T) High water level: 75.4 (risk of flooding above this level)

(I) Intermediate water level: 74.8

(L) Low water level: 74.1

Further subdivision of high and low water levels based on this criterion:

(ST) Suitable high water level: 75

(TL) Suitable low water level: 74.3

3. Comprehensive Satisfaction Model and Time Series of Optimal Water Levels

3.1. Determine the satisfaction function for each stakeholder

Define Stakeholder Groups: Shipping companies (S), People near shipping ports (P), Environmentalists (E), Real estate owners (R), Marinas (M), and Hydroelectric companies (H).Based on the preferences of various groups regarding water levels,

corresponding satisfaction expressions can be formulated: Airlines desire a moderate and stable water level in the Saint Lawrence River, residents near the Montreal port prefer a low and stable water level, environmentalists wish for seasonal variations in Lake Ontario's water level, property owners and yacht docks hope for a moderate and stable water level in Lake Ontario, while hydroelectric companies seek greater control over the water level. They aim to store water during high water levels and release it during peak electricity demand, aligning with low water levels during periods of high electricity demand[11-13]. The above information is summarized in table 1.

Table 1. Benefits

	Group	Water level	Water flow/surface
St.Marys River	S	High	Static
	P	Stable	Low
	H		Release in summer
Lake Ontario	E	Seasonal	
	R	Middle	Stable
	M	Middle	Stable

Regarding the water level of rivers, this paper consider the entire system connecting lakes and rivers as a connected system, where the balance of water levels between lakes and rivers requires a relatively short time compared to the time intervals the paper is considering. The water level of upstream lakes has an absolute control effect on the water level of rivers. Considering the complexity of the model and our main focus on Lake Ontario, this paper map residents' satisfaction with river water levels to the water level requirements for Lake Ontario.

The paper normalizes the water level values L, LH, I, TH, and T of upstream lakes to eliminate the dimensional impact, serving as the threshold values for river water levels. The standardized values for moderate, high, and low water levels are presented in the table 2.

Table 2. Nnormalizes the water level

L	LH	I	TH	T
0	0.153846153846155	0.538461538461536	0.692307692307691	1

Creating satisfaction functions for each group based on their specific water level preferences. The specific function images and equations are shown in figure 3 below.

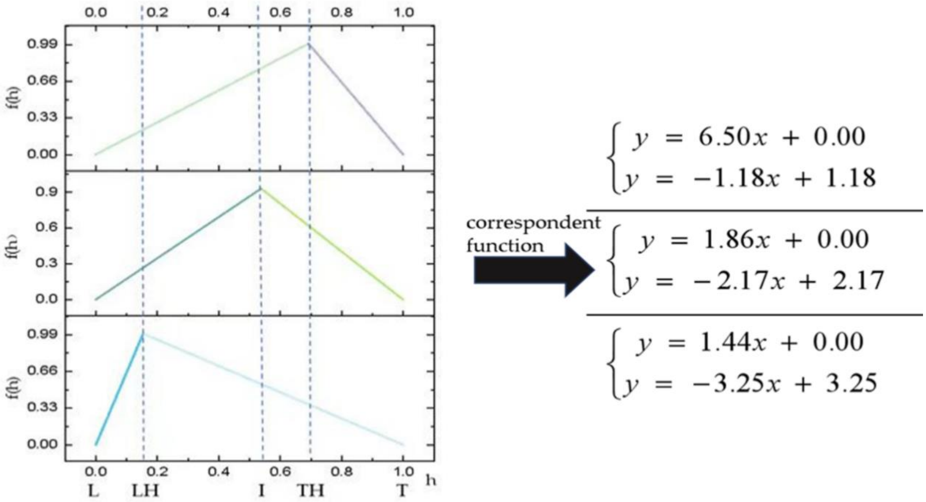

Figure 3. Create the satisfaction functions

3.2. Determining the priority factor λ

Considering various needs and balancing the interests of different groups is essential to ensure the sustainability and equilibrium of the overall system (all priorities have been normalized). The article will show the priorities of each stakeholder in the form of a radar chart, as shown in figure 4.

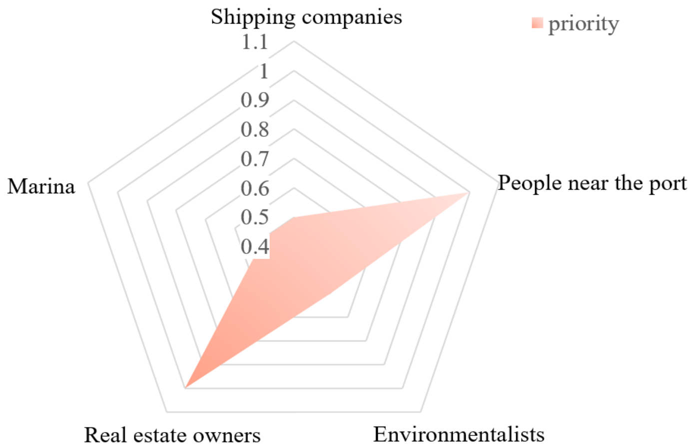

Figure 4. Priority factor analysis

People constitute the fundamental components of society, and their needs are directly related to the overall well-being and quality of life in society. Safeguarding the living environment, safety, and health of individuals is crucial for societal stability and development, making their priority the highest with a weight assigned of 1. The needs of real estate owners, involving property value and economic interests, play a role in the stability and preservation of property values within the real estate market, influencing the economic health of communities and the financial well-being of residents. The priority for real estate owners is similarly assigned as 1.

Environmentalists focus on ecological balance and sustainability, recognizing the importance of preserving the natural environment for long-term ecological health and human survival. While environmental concerns are important, there may be a need to balance them with the prioritization of basic needs and socio-economic development, assigned by a priority factor of 0.6. The demands of shipping companies and ports are directly linked to trade and economic development. However, compared to the needs of individuals and the real estate market, their impact on the overall society may be more localized and flexible, given a priority factor of 0.5. Considering the fulfillment of basic needs for people and communities as a prerequisite, the demands of shipping companies and ports can be considered. This prioritization reflects a values-oriented approach centered around people and the well-being of society[14].

Here, only the quantitative impact of water level changes is considered, and the qualitative consideration of water surface stability will be addressed in the third step.

3.3. Determining the optimal water level using the overall satisfaction – the allsa model

Considering the established priority factors and their correlation with water level satisfaction, the study defines an overall satisfaction metric, denoted as "allsa."

$$s.t.\lambda_s f_s(h) + \lambda_p f_p(h) + \lambda_r f_r(h) + \lambda_m f_m(h) = allsa_{max} \tag{1}$$

Organize the relevant condition information in table 3:

Table 3. The allsa model

Stakeholders	Priority factor	Water level satisfaction	Expression
Residents near the port	1	$\lambda_p f_p(h)$	$\begin{cases} y = 6.50x + 0.00 \\ y = -1.18x + 1.18 \end{cases}$
Real estate owners Marina owners	1 0.5	$\lambda_r f_r(h)$ $\lambda_m f_m(h)$	$\begin{cases} y = 1.86x + 0.00 \\ y = -2.17x + 2.17 \end{cases}$
Shipping company	1	$\lambda_s f_s(h)$	$\begin{cases} y = 1.44x + 0.00 \\ y = -3.25x + 3.25 \end{cases}$

3.4. Determine the optimal monthly water levels for Lake Ontario and beyond

When fitting the time series of Lake Ontario water levels, we attempted various fitting methods such as linear and polynomial models. However, both fitting goodness and model simplicity were found to be suboptimal. After examination and comparison, we opted for a model that balances good data fitting with a certain level of simplicity: a fourth-degree polynomial function for fitting. Below are the results and corresponding visualizations obtained through the fitting process using Matlab.

$$f(x) = 0.000938x^4 - 0.0234x^3 + 0.1736x^2 - 0.3592x + 74.88 \tag{2}$$

The R-square value is 0.9853, and the sum of squared residuals (SSE) is 0.007613, indicating that the model can explain 98.53% of the variability in the data, reflecting the superiority of the fitted model. Additionally, the coefficients p1 to p5 all have 95% confidence intervals, indicating the significance and reliability of the coefficients. Therefore, this fourth-degree polynomial fitting can effectively capture the water level characteristics of Lake Ontario: the water level exhibits periodic and seasonal variations, with higher levels typically observed from May to July, which is usually a season with concentrated precipitation. During spring to early summer, increased rainfall and, simultaneously, warming temperatures result in additional snowmelt, releasing a substantial amount of water into the lake and causing the water level to rise.

Next, this paper will derive the optimal monthly water level data for Lake Ontario through the following approach:

In the derivation process mentioned above, we provided the optimal water level, de-noted as h, which represents the average water level for the twelve months of the year in Lake Ontario. Throughout the year, the water level in Lake Ontario experiences a high-water season around June, two low-water seasons in January and November (excluding significant seasonal changes in water levels caused by major climate variations and disregarding major natural disasters), and two moderate-water seasons around April and August. The water level data for these months characterize the features of Lake Ontario's water levels throughout the year (maximum, minimum, and average).

We will derive the optimal water levels for these five months, and then use a fourth-degree polynomial to determine the optimal water levels for all twelve months in Lake Ontario. Based on the earlier derivation process, the optimal average water level for Lake Ontario is denoted as h. Using this obtained optimal average water level, we determine the average water levels for April and August, aiming to maintain the overall average water level of Lake Ontario approximately equal to h throughout the year. For the two lowest months and one highest month, a qualitative discussion is conducted, considering the combined priority weights discussed earlier for various stakeholder groups regarding the rate of water level changes.

$$T = S_s \times \lambda_s + P_s \times \lambda_p + E_{ss} \times \lambda_s + R_s \times \lambda_r + M_s \times \lambda_m + H_s \times \lambda_h \tag{3}$$

Regarding water level stability, there are two aspects. First is the intensity of seasonal water level changes throughout the year, and second is the turbulence of water flow during the year. For this model, the intensity of seasonal water level changes can be described as the difference between the extremes of water levels within a year. The larger the difference between the extremes, the greater the seasonal variation, and the smaller the difference, the smaller the seasonal variation. As for the turbulence of water flow, considering its multidimensional nature, we will simplify by stating: for the water level function in this context, if the difference between extremes decreases, the rate of water level $\frac{dh}{dt}$ changes between extremes will also decrease, and vice versa.

Ultimately, based on qualitative analysis, higher satisfaction is associated with greater stability in the water level of Lake Ontario. Combining the provided statistical data in the question, we introduced a stability factor for the high-water month and the two low-water months in Lake Ontario, aiming to reduce the difference between the extremes of water levels.

The water levels for the five characteristic months are determined in table 4 below:

Table 4. Water level results

c	1	4	6	8	11
Water level/meter	74.7	74.9	75	74.9	74.6

The fitted function is

$$y = 0.0005x^4 - 0.0130x^3 + 0.0946x^2 - 0.1770x + 74.7949 \tag{4}$$

4. Conclusions

This study establishes a Weight-Function-Synthesis-Optimization (WFSO) model to analyze and optimize the optimal water levels of the Great Lakes by considering the needs of various stakeholders. The results indicate that the ideal water level is 74.9 meters, which maximizes the satisfaction of multiple stakeholders including ecosystems, transportation, real estate, marinas, and hydroelectric companies, thereby achieving sustainable development in the region.

Among other things, the study found that water levels above 75.4 meters cause flooding of shoreline grasses and shrubs, and then at around 74.8 meters, vegetation coverage increases, enhancing ecosystem diversity. Last, water levels below 74.5 meters lead to decreased biodiversity and negatively impact the lake ecosystem.

By establishing satisfaction functions for each stakeholder and determining their priority weights, the study defines an overall satisfaction metric, "allsa," and determines the optimal monthly water level time series for Lake Ontario. A fourth-degree polynomial function was chosen for fitting, explaining 98.53% of the data variability, effectively capturing the periodic and seasonal variations of Lake Ontario's water levels.

References

[1] John H. Hartig, Gail Krantzberg, Peter Alsip. Thirty-five years of restoring Great Lakes Areas of Concern: Gradual progress, hopeful future [J]. Journal of Great Lakes Research, 2020, 46(3): 429-442.

[2] Sun Jiaqi, Wang Xiaojun, Shahid Shamsuddin. Precipitation and runoff variation characteristics in typical regions of North China Plain: a case study of Hengshui City [J]. Theoretical And Applied Climatology, 2020, 142(3-4): 971-985.

[3] Pan Liu, Liping Li, Shenglian Guo, Lihua Xiong, et al. Optimal design of seasonal flood limited water levels and its application for the Three Gorges Reservoir [J]. Journal of Hydrology, 2015.

[4] A. Prékopa, T. Szántai. An optimal regulation of a storage level with application to the water level regulation of a lake [J]. European Journal of Operational Research,1979,3(3): 175-189.

[5] Ouyang, S., Zhou, J., Li, C. et al. Optimal Design for Flood Limit Water Level of Cascade Reservoirs [J]. Water Resour Manage,2015.

[6] Song Lijuan, Song Chunqiao, Zhan Pengfei, et al. Seasonal Amplitude of Water Storage Variations of the Yangtze-Huai Plain Lake Group: Implicaion for Floodwater Storage Capacity [J]. Frontiers In Environmental Science,2022.

[7] Elrod JH, Ogorman R, Schneider CP, et al. Lake trout rehabilitation in Lake Ontario [J]. Journal of Great Lakes Research,1995.

[8] Hudon C, Wilcox D, Ingram J. Modeling wetland plant community response to assess water-level regulation scenarios in the Lake Ontario-St. Lawrence River basin [J]. Environmental Monitoring and Assessment, 2006,113(1-3): 303-328.

[9] Carter E, Steinschneider S. Hydroclimatological Drivers of Extreme Floods on Lake Ontario [J]. Water Resources Research,2018,54(7): 4461-4478

[10] Liu Junyan, Wang Shijie. Monitoring of Hulun Lake water level changes based on ICESat-2 satellite altimetry data [J]. Arid Zone Research, 2023,40(9): 1438-1445.

[11] Shao Mingliang, Tan Yan, Yang Yasha. Sichuan: Accurate water accounting, water storage and people's livelihood[N]. China Water Conservancy News, 2023-08-22 (007).

[12] Wei Haoxing, Li Ao, Geng Geng, et al. Hubei: Facing the "baking" test to win the battle against drought and ensuring supply[N]. China Economic Times, 2022-08-29 (001).

[13] Kong Jiehui. Operation planning of Xili hydropower station considering electricity consumption and flood control demand[J].Water Resources Technical Supervision, 2021, (05): 134-138+159+162.

[14] YU Hao. On the change of the value orientation of environmental legislation in China[D]. Shanghai University of Finance and Economics, 2023.