Study of Tennis Competition Based on Factor Analysis and Comprehensive Evaluation Methods

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Abstract. This paper introduces an innovative methodology for evaluating and forecasting the performance dynamics of tennis players during matches using a momentum-based model. We conceptualize "momentum" as a real-time indicator that reflects the competitive state of players and propose a robust analytical framework for quantifying and comparing this momentum. Our approach involves the extraction and downscaling of multidimensional match data into eight technical indicators through linear combinations. Factor analysis is employed to distill these indicators into three principal factors—negative state, break, and serve—which are further refined via factor rotation to enhance interpretability. To objectively weight these factors, we apply the CRITIC method, ensuring a comprehensive assessment of their significance. Through calculating the RSR value, we provide a comprehensive score of each player’s real-time momentum situation. In summary, our study establishes an effective methodology for real-time momentum assessment in tennis, contributing valuable insights into predictive analytics and strategic decision-making in sports analytics.

Keywords: Tennis Match, Factor analysis, CRITIC method, RSR method.

1. Introduction

Tennis has now developed into one of the most popular sports in the world in terms of spectatorship and fan favorite. Current research on tennis involves the analysis of players' technical and tactical levels, the influencing factors of tennis matches, and the prediction of tennis results, which can help coaches and players to provide better data analysis in pre-match preparation and post-match summarization [1]. Cui Yixiong (2018) showed that players with better serve-receiving ability perform better at red clay venues like the French Open. On the other hand, on hard courts such as the Australian Open and the US Open, players with better serves and fewer unforced errors performed better [2]. Meng Fanming and Huang Wenmin (2019) utilized a decision tree branching tree model and concluded that 41% serve receive percentage is an important indicator of victory or defeat in women's tennis matches [3]. Luo Weiquan and Zhang Lei (2020) used logistic regression analysis to construct a logistic regression-based judgment model of the winning factors of professional tennis players for 254 matches of the men's singles tournament at the 2019 Wimbledon Open. The results showed that the key factors affecting the winning of professional tennis players were first serve scoring rate, second serve scoring rate, first serve reception scoring rate, second serve reception scoring rate, and backhand grip type. [4]. In summary, research on tennis matches has yielded many universal laws, but there is a lack of in-match analysis based on real-time data of tennis matches to help coaches and players make adjustment even though. Based on the above research, this paper, with reference to the important indicators affecting the winners and losers of tennis matches, explores the important in-match indicators affecting the real-time state of the athletes and their contributions during the conduct of tennis matches through the factor analysis model. Combining the CRITIC method and the rank-sum ratio method to quantify their momentum, the real-time portrayal of the athlete's in-game state. This will help athletes and coaches to make timely adjustments and formulate in-game strategies.

2. Data Pre-processing

2.1. Data Sources

The tennis match data for this paper comes from the 2023 Wimbledon Gentlemen's final, which 20-year-old Spanish rising star Carlos Alcaraz defeated 36-year-old Novak Djokovic (https://www.comapmath.com/MCMICM/index.html). The match itself was a remarkable battle [5]. This is the world's top level tennis tournament, so its data has a high analytical value.

The datasheet records 43 data items including time elapsed since start of first point to start of current point, tennis ball speed, serve, player 1 hit an untouchable winning serve, player 1 has an opportunity to win a game player 2 is serving, player 1 missed both serves and lost the point and point victor. However, data cleaning was required due to incomplete data for some of the matches. At the same time, there were jumps in the duration of the matches, which, based on the actual matches, were caused by match suspensions and needed to be fixed.

2.2. Data Cleaning and Conversation

This paper processes the missing values. Ball speed, width of serve, depth of serve, and depth of return data in the raw data all have vacant values. Due to the individual differences caused by personal habits, there are certain statistical patterns in the direction and depth of serve and return of each player, so this paper uses the pluralities of each player's direction of serve, depth of serve, and depth of return to replace the corresponding missing values.

For game times with large intervals, based on the idea of differencing in Moving Average (MA) model, the average of the normal time series is computed as the time interval in the game by performing a difference operation on the normal time series to replace the abnormal time gaps, and a linear transformation is performed on the subsequent data· Eventually, the time span is allowed to return to normal, and obtain the analyzable time series without altering the original time characteristics.

$$\bar{t} = \frac{\sum_{i=1}^{m} \sum_{j=2}^{n} \left(t_{ij} - t_{ij-1}\right)}{m(n-1)} \tag{1}$$

The average time interval for each game was first calculated from the time difference of a large number of normal time series:

$$\Delta t_i = x_{i\alpha} - x_{i\beta} - \bar{t} \tag{2}$$

Then calculate the value of the linear transformation required for the time value of the time span in which the anomaly occurs:

$$t_r = t_0 - \Delta t_i \tag{3}$$

The score data is processed as follows: The columns "p1 score" and "p2 score" use scores of 0-9, 15, 30, 40, and AD, which correspond to two distinct scoring systems. 0 through 9 are one set of scoring systems, while 0, 15, 30, 45, and AD are the other set to facilitate analysis, the values 15, 30, 40, and AD will be converted to 1, 2, 3, and 4, respectively. This conversion standardizes the scores, where values greater than 10 are interpreted as 1 to 3 points.

3. Model of momentum Changes in a Player's Race

3.1. Factor analysis model

3.1.1 Data Processing of Original Technical Indicators

Due to the excessive number of indicators in the data given, in order to simplify the procedure, we decide to use factor analysis to downscale the forty-seven indicators given. After reviewing the literature and analyzing the preprocessed data, we combine and simplify the indicators with the same categories by linear transformation to extract the eight technical indicator features of the game, as shown in Table.1.

Table.1. Indicator Names

Name	Characteristic	Calculation Method
Z1	Serve or not	Statistical result
Z2	Speed	Statistical result
Z3	The probability of issuing an ACE ball in the serve	The cumulative number of the previous p1 ace /accumulating number of previous p1 serve games
Z4	Percentage of break chances for receiver	The cumulative number of the previous p1 break point /accumulating number of previous p2 serve games
Z5	Elapsed time	Statistical result
Z6	Leading score	p1 points won - p2 points won
Z7	Percentage of break chances for server	The cumulative number of the previous p2 break point /accumulating number of previous p1 serve games
Z8	Ratio of double serve errors	The cumulative number of the previous p1 double fault /accumulating number of previous p1 serve games

Before conducting factor analysis, the selected indicators need to be transformed into the same trend. Through observing the selected technical indicators, we can find that the match time, the frequency of facing break points on serve and the frequency of double faults on serve have a negative impact on the momentum of the tennis player, which is treated as a negative indicator. The other five indicators are treated as a positive indicator. According to the formula (4), we process the data.

$$\begin{cases} \hat{x}_{ik} = \dfrac{x_{ik} - x_{imin}}{x_{imax} - x_{imin}} \\ \hat{y}_{ik} = \dfrac{y_{ik} - y_{imin}}{y_{imax} - y_{imin}} \end{cases} \tag{4}$$

Where $\hat{x}_{ik}$ is the Normalized processing value.

3.1.2 Data Dimensionality Reduction Model Based on Factor Analysis

The analysis of comprehensive evaluations in tennis is challenging due to the sheer volume of match data. This study employs factor analysis to condense original indicators and reduce variables effectively. Factor analysis explores the underlying structure by examining interrelationships among numerous match data points, using hypothetical variables to categorize complex relationships into cohesive factors.

During dimensionality reduction, the method calculates and analyzes the variance contributions among each data variable to represent the original system as a new comprehensive factor combination [6]. The model can be expressed as formula (5).

$$\begin{cases} x_1 = a_{11}F_1 + a_{12}F_2 + \cdots + a_{1m}F_m + \varepsilon_1 \\ x_2 = a_{21}F_1 + a_{22}F_2 + \cdots + a_{2m}F_m + \varepsilon_2 \\ \quad\cdots \\ x_p = a_{p1}F_1 + a_{p2}F_2 + \cdots + a_{pm}F_m + \varepsilon_3 \end{cases} \tag{5}$$

Where $X = (x_1, x_2, \dots, x_p)^T$ is an observable random vector representing continuous attributes in tennis match data. The following expectations and covariance structures apply.

$$E_{(X)} = 0 \tag{6}$$

$$\mathrm{Cov}(X) = \Lambda\Lambda^T + \mathrm{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2) \tag{7}$$

The factors $F = (F_1, F_2, \dots, F_M)^T$(where m<p) are unobservable random vectors, derived from linear combinations of original indicators, satisfying:

$$E(F) = 0 \tag{8}$$

$$Cov(F) = 1 \tag{9}$$

The errors $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_p)^T$ are independent of F and each other, with:

$$E(\varepsilon) = 0 \tag{10}$$

In matrix form, the relationship between X, F, and ε is represented as:

$$X = AF + \varepsilon \tag{11}$$

where $A = (a_{ij})_{p*m}$. This yields the factor score model:

$$\begin{cases} F_1 = \omega_{11}x_1 + \omega_{12}x_2 + \cdots + \omega_{1p}x_p \\ F_2 = \omega_{21}x_1 + \omega_{22}x_2 + \cdots + \omega_{2p}x_p \\ \dots \\ F_m = \omega_{m1}x_1 + \omega_{m2}x_2 + \cdots + \omega_{mp}x_p \end{cases} \tag{12}$$

This approach facilitates a structured analysis of extensive tennis match data, offering a concise representation of complex relationships through comprehensive factors.

3.1.3 Solutions of the Factor Analysis Model

Firstly, KMO and Bartlett's test were performed on the eight-feature data and the results were as shown in Table.2.

Table.2. KMO and Bartlett's Test

KMO	Bartlett's Sphericity		
	Approximate	Degrees of freedom	Significance
0.676	2085.919	28	0

The Kaiser-Meyer-Olkin (KMO) measure yielded a value of 0.676, surpassing the threshold of 0.6, indicating significant correlation among the selected technical indicators. Additionally, Bartlett's test of sphericity returned a P-value of 0.000, below the critical threshold of 0.005, thereby rejecting the null hypothesis and confirming the presence of correlations among variables, aligning with the prerequisites for conducting factor analysis. Thus, the suitability of factor analysis is affirmed.

Table.3. Common Factor Variance

Element	Explanatory Rate of Variance before Rotation			Explanatory Rate of Variance after Rotation		
	Characteristic Root	Variance ExplainedRate(%)	CumulativeVariance Explained (%)	Characteristic Root	Variance Explained Rate (%)	CumulativeVariance Explained (%)
1	3.892	48.656	48.656	273.737	34.217	34.217
2	1.339	16.734	65.39	161.874	20.234	54.451
3	1.182	14.775	80.165	113.912	14.239	68.69
4	0.732	9.15	89.315	100.421	12.553	81.243
5	0.51	6.374	95.689	100.147	12.518	93.761
6	0.247	3.092	98.781	32.577	4.072	97.834
7	0.078	0.977	99.758	14.934	1.867	99.7
8	0.019	0.242	100	2.398	0.3	100

Next, we computed the variance explained by the common factors and summarized the results in Table.3. The characteristic roots of the three primary factors exceeded 1, cumulatively explaining

80.165% of the total variance. Examination of the eigenvalue scree plot revealed a noticeable decline in eigenvalues beyond the third component, suggesting diminishing explanatory power. Consequently, we retained the first three factors as primary factors, while categorizing the subsequent factors as secondary (refer to Figure 1).

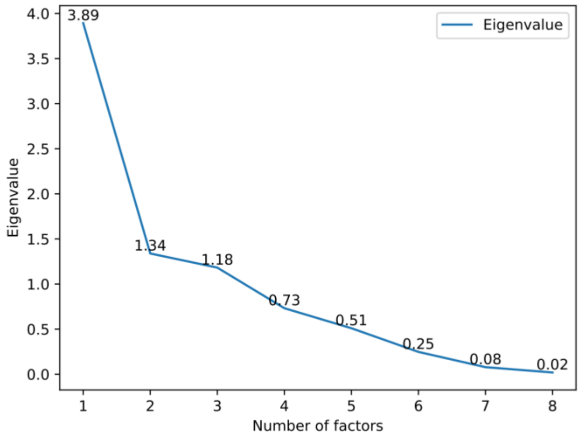

Figure 1. Gravel Chart

Due to the non-uniqueness of the factor loading matrix, the initial variables comprising the first factor in the component matrix exhibit high loadings, indicating substantial influence of factors on variables. However, this high loading complicates the interpretation of underlying constructs. To enhance interpretability and conceptual clarity of the latent factors, factor rotation using the method of maximum variance was performed. This rotation aims to align the loadings of original variables towards 1 on their respective factors, while loadings on other factors tend towards 0. The rotated factor loading matrix, depicted in Figure 2, facilitates the interpretation and naming of latent factors.

	F_1	F_2	F_3
z_1	0.018	0.056	-0.005
z_2	0.007	-0.027	-0.006
z_3	0.149	0.282	0.946
z_4	-0.493	-0.607	-0.349
z_5	0.83	0.446	0.229
z_6	-0.941	0.054	-0.073
z_7	0.159	0.953	0.246
z_8	0.934	0.238	0.07

Figure 2. Neural network structure

Post-rotation, the component matrix reveals that the first factor comprises variables such as match time and trailing score (with leading score negatively loaded), along with the probability of issuing a double fault on serve, all exhibiting significant loadings. These variables collectively characterize what we term as negative state factors. The second factor prominently loads variables related to the probability of facing a break chance in the receiving set and facing a break point on serve, defining what we identify as the break factor. Finally, the third factor shows a strong loading for variables associated with the probability of an ACE being issued on serve, hence termed the serving factor.

Subsequently, the factor score matrix was computed using the regression method [7], as shown in Tabel.4. Thus, the factor analysis model can be succinctly articulated as formula (13):

$$\begin{cases} F_1 = -0.007z_1 - 0.014z_2 - 0.056z_3 + 0.129z_4 + 0.259z_5 - 0.457z_6 - 0.116z_7 + 0.477z_8 \\ F_2 = -0.073z_1 + 0.028z_2 - 0.288z_3 + 0.025z_4 - 0.024z_5 - 0.006z_6 + 1.2z_7 - 0.128z_8 \\ F_3 = 0.032z_1 + 0.001z_2 + 1.181z_3 + 0.117z_4 - 0.041z_5 + 0.03z_6 - 0.255z_7 - 0017z_8 \end{cases} \quad (13)$$

Table.4. Factor Score Matrix

Name	Element		
	Element1	Element2	Element3
Serve or not Positive Term Indicator Processing	-0.007	-0.073	0.032
speed mph Positive Term Indicator Processing	-0.014	0.028	0.001
Probability of serving an ace Positive Term Indicator Processing	-0.056	-0.288	1.181
Probability of a break chance in a service-receiving set negative elapsed time negative Term Indicator Processing	0.259	-0.024	-0.041
Leading Score Positive Term Indicator Processing	-0.457	-0.006	0.03
Probability of facing a break point on serve Negative Term Indicator Processing	-0.116	1.2	-0.255
Probability of a double fault on the serve Negative Term Indicator Processing	0.477	-0.128	-0.017

From Table.4., the three factors are obtained from the linear regression of eight composite indicators with different ratios, so that the values of the three factors can be clearly represented.

3.2. Comprehensive Assessment

3.2.1 CRITIC Weighting

The CRITIC method offers a more effective and objective approach to weighting compared to traditional entropy weight and standard deviation methods. While these conventional methods often focus on the data itself, CRITIC incorporates both the volatility of the data and the correlation between indicators, thus providing a comprehensive measure of the indicators' objective weights [8]. Specifically, CRITIC calculates weights by assessing both the dispersion of the data (using standard deviation to reflect variability) and the degree of correlation between indicators (where a higher correlation implies lower conflict and consequently, lower weight). The resultant weights are derived by normalizing the product of these volatility and conflict measures. This approach enhances the objectivity of the evaluation by considering both the variability and the correlation of the indicators, thus avoiding simplistic direct correlations between importance and value.

To apply the CRITIC method to weight the factors F_1, F_2 and F_3 identified in the previous analysis, the following steps are necessary:

We begin with the original data matrix X, which comprises m objects and an evaluation indicator:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x & \cdots & x_{mn} \end{pmatrix} \tag{14}$$

First, we calculate the information fluctuation for each indicator using the standard deviation formula:

$$S_j = \sqrt{\frac{\sum_{i=1}^{m}\left(x_{ij} - \bar{x}_j\right)^2}{n-1}} \tag{15}$$

where x_{ij} represents the value of the j-th indicator for the i-th object, and $\bar{x}_j$ is the mean of the j-th indicator.

Next, we assess the conflictivity between indicators using the correlation matrix. The correlation coefficient matrix RR is computed as follows:

$$R = \frac{\sum_{j,k=1}^{n}\left(x_{ij} - \bar{x}_j\right)\left(x_{ik} - \bar{x}_k\right)}{\sum_{j,k=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2 \sum_{k=1}^{n}\left(x_{ik} - \bar{x}_k\right)^2} \tag{16}$$

From this correlation matrix, the conflictivity A_j is derived by:

$$A_j = \sum_{i=1}^{n} (1 - r_{ij}) \tag{17}$$

where r_{ij} denotes the correlation coefficient between the i-th and j-th indicators.

Following this, we determine the volume of information C_j for each indicator by multiplying its fluctuation S_j with its conflictivity A_j:

$$C_j = S_j \times A_j \tag{18}$$

The final step involves calculating the weights W_j for each indicator as the percentage of the total information carrying capacity:

$$W_j = \frac{C_j}{\sum_{j=1}^{n} C_j} \tag{19}$$

Applying this methodology, we obtain the weights for the indicators F_1, F_2 and F_3 as 63.943%, 21.950%, and 13.107%, respectively. This approach ensures that the weights reflect both the variability and correlation among indicators, resulting in a more nuanced and objective assessment of their relative importance.

3.2.2 RSR Method Assessing Momentum

The Rank-Sum Ratio (RSR) method provides a robust framework for comprehensive evaluation by ranking various indicators and measuring the average of these ranks as the evaluation standard [9]. This method is particularly useful for evaluating indicators with different units of measurement. In this study, we apply the RSR method to assess a player's momentum, treating all indicators as positive and utilizing a benefit-based evaluation framework.

Initially, normalization and homogeneity processing are performed on the data dimensions F_1, F_2, and F_3. Given the presence of both positive and negative indicators, we preprocess these indicators accordingly. Specifically, we adjust for cases where a column of data may have uniform values by modifying the minimum and maximum values as follows:

$$X_{min} = \min(X_{1j}, X_{2j}, \dots, X_{nj}) - 0.0001 \tag{20}$$

$$X_{max} = \max(X_{1j}, X_{2j}, \dots, X_{nj}) + 0.0001 \tag{21}$$

For positive indicators:

$$z_{ij} = \frac{X_{ij} - X_{min}}{X_{max} - X_{min}} \tag{22}$$

For negative indicators:

$$z_{ij} = \frac{X_{max} - X_{ij}}{X_{max} - X_{min}} \tag{23}$$

Subsequently, a data matrix of dimension n×m is constructed, where n represents the number of evaluation objects and m denotes the number of indicators. The matrix is then ranked using two methods: integer rank sum ratio and non-integer rank sum ratio methods.

The integer rank sum ratio method ranks the indicators from smallest to largest for benefit-type indicators and from largest to smallest for cost-type indicators. The average ranks are calculated, leading to the rank matrix (24):

$$R = (R_{ij})_{m\times n} \tag{24}$$

In contrast, the non-integer rank sum ratio method offers a linear correspondence between the rank and the original indicator values, addressing the RSR method's limitations in retaining quantitative information. For benefit-based indicators:

$$R_{ij} = 1 + (n-1)\frac{X_{ij} - \min(X_{1j}, X_{2j}, \ldots, X_{nj})}{\max(X_{1j}, X_{2j}, \ldots, X_{nj}) - \min(X_{1j}, X_{2j}, \ldots, X_{nj})} \tag{25}$$

For cost-based indicators:

$$R_{ij} = 1 + (n-1)\frac{\max(X_{1j}, X_{2j}, \ldots, X_{nj}) - X_{ij}}{\max(X_{1j}, X_{2j}, \ldots, X_{nj}) - \min(X_{1j}, X_{2j}, \ldots, X_{nj})} \tag{26}$$

The correlation coefficient between the non-integer and integer rank sum ratio methods is 0.791 and 0.933, respectively, indicating a stronger alignment with the integer rank sum ratio method. Consequently, the integer rank sum ratio method is preferred.

To calculate the Weighted RSR ($WRSR_i$), given pre-determined weights via the CRITIC method:

$$WRSR_i = \frac{1}{n}\sum_{j=1}^{m} W_j R_{ij} \tag{27}$$

where W_j represents the weight of the j-th indicator and the weight sum equals 1. A higher $WRSR_i$ value signifies a superior evaluation object. Thus, objects are ranked based on their $WRSR_i$ values.

After compiling the rank-based frequency distribution table for WRSR i, we tabulated the frequency (f) for each group, calculated both the cumulative frequency (cf) and the cumulative frequency percentage (p). These values were then converted into probit units to assess the influence factors of F_1, F_2 and F_3 obtained through comprehensive analysis. Subsequently, we performed linear regression with probit values as the independent variable and RSR as the dependent variable [8,9]. The final step involved ranking the subjects based on the estimated $WRSR_i$ values derived from the regression equation.

3.3. Outcome

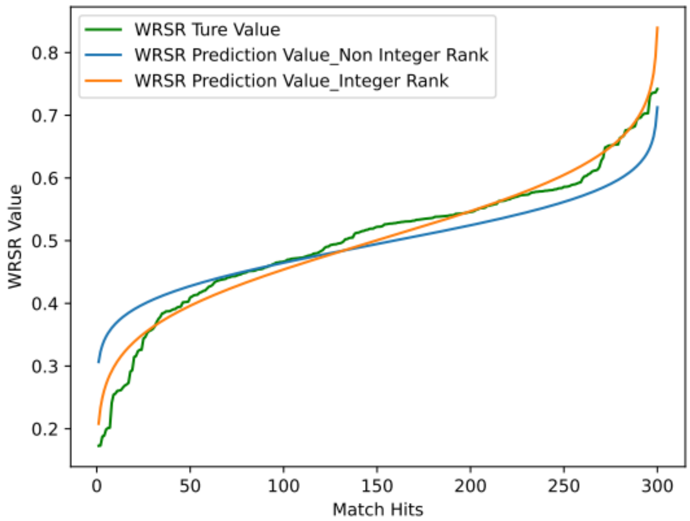

Figure 3. Comparison of the integer rank method and the non-integer rank method

In this paper, the integer rank method and the non-integer rank method are evaluated and compared comprehensively respectively, and the non-integer rank method is finally concluded to have a better fitting effect, as shown in Fig. 3 [10].

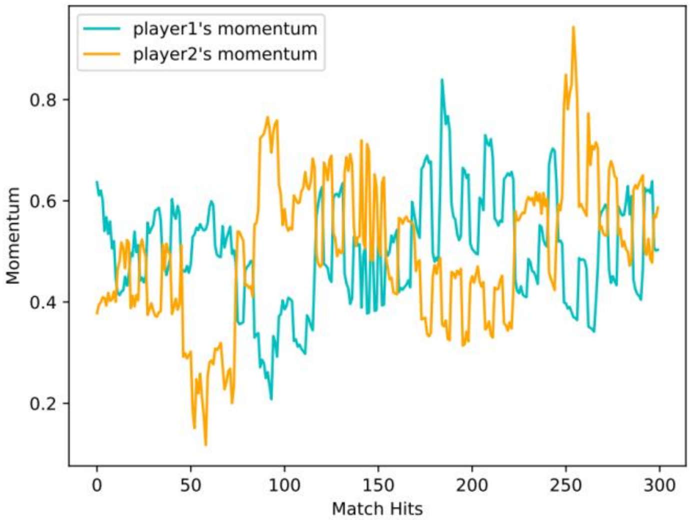

Figure 4. Momentum diagrams of the players in match

Based on the known data of the match and the non-integer rank method described above, we made Figure 4: Mid-match momentum changes of the two players, which describes the power dynamics of the two players in the first match. As can be seen from the figure, the two players were relatively equalized and their momentum fluctuated with each other's performance. Throughout the game, there were about three distinct momentum changes. It is noteworthy that player 1 maintained a superior momentum throughout and eventually secured victory in the match. Fig. 4 depicts the momentum dynamics of the two players in the first game. As can be seen from the figure, both players showed a relatively balanced strength, with their momentum fluctuating according to each other's performance. There are about three distinct momentum changes during the whole game. It is noteworthy that player 1 maintained a superior momentum throughout the game, ultimately securing the victory. This analysis emphasizes the relative advantage that player 1 maintained throughout the game.

Our method more accurately describes the momentum changes of athletes during the game and can point out the most important factors affecting the player's state, which can be used by players and coaches to analyze their own and opponent's state in real time, and make changes in the game strategy.

4. Conclusions

This study offers a novel approach to the real-time analysis of tennis match data by utilizing linear combinations and factor analysis to distill complex datasets into three pivotal technical factors. By applying the CRITIC weighting method and the Rank-Sum-Ratio (RSR) technique, we effectively capture and interpret momentum shifts throughout matches, providing valuable insights into the dynamic performance trends of players.

Our findings make significant contributions to in-competition strategy in tennis, offering players and coaches actionable insights for adjusting tactics in response to real-time momentum changes. Furthermore, we recognize that player performance is influenced by various factors, including the specific characteristics of different tennis venues and the psychological effects of world ranking disparities. These elements can play a crucial role in momentum fluctuations during matches.

Looking ahead, future research should consider additional variables such as venue-specific factors and player rankings to enhance the generalizability of the proposed model. Expanding the scope to include these factors could provide a more comprehensive understanding of momentum dynamics and improve the accuracy of the momentum evaluation system under diverse conditions.

In conclusion, this paper introduces an innovative and reliable methodology for assessing players' in-match status. The proposed approach holds considerable promise for advancing in-match analysis and post-match evaluation in tennis, offering strategic benefits for players, coaches, and sports analysts.

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