The Continuity or Discreteness of Space-time

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Abstract. General relativity and quantum field theory have always held opposite views on the continuum and the dispersion of space-time. Scientists have debated this for nearly a century. This paper reviews the basis of general relativity and quantum field theory on the continuity and discreteness of space-time in a unified viewpoint, describing what both stand for in this discussion point and their respective experimental evidence to illustrate their plausibility. By using mathematical reasoning and figures to help describe and apply string theory's ideas, this paper describes the universe as a wave. Based on this, a new conjecture of the unified field theory is obtained: space-time is described as a piecewise function, which presents different states under different conditions. In the normal state, the continuous wave appears, and in the energy exchange with the outside world, it is transformed into a discrete particle state. Space-time is similar to the waves described in string theory, but it is more similar to electromagnetic waves, taking on an ever-changing form.

Keywords: General relativity Quantum mechanics; continuum; discrete; string and unified field theory.

1. Introduction

In the 20th century, it was known that our universe is expanding, and the measurement of the distance-redshift relation provides evidence for it. Due to the expanding of the Universe, objects become farther and farther away from each other in space [1]. The expansion of the universe can be described by a rubber band. One can mark the midpoint of the rubber band and make a mark on each unit length from the midpoint. Stretching the two ends of the rubber band, you will find that the other points marked are farther and farther away from the midpoint, and the point that is farther from the initial midpoint, the faster the displacement rate. On a macro scale, it's clear that the rubber band is getting longer. At the micro level, the distance between the particles that make up the rubber band becomes larger, without the particles themselves changing. So, from a practical point of view, this paper can't be sure whether the expansion of the rubber band is continuous or discrete. Space-time, as the "base" of the universe, also faces such a problem. In the fundamental theory of space-time, represented by general relativity, the space-time is treated as continuous, while in the viewpoint of micro-physics, represented by quantum field theory, one in general believes that space-time is a dynamic physical system with intrinsic quantum structure, which should be discreteness [2]. This also involves the problem that the four fundamental forces of nature cannot be unified. Quantum mechanics upset many physicists, and Einstein called the "uncertainty" principle a ghost, which has destroyed his unified theory [3]. Einstein's position on a unified field theory was strict and clear, but his wish has not yet been fulfilled [4]. After Einstein's death, a unified field theory became a lifelong ambition of many physicists.

Solving this problem head-on is very difficult and requires a lot of computational reasoning and experimentation. However, the true meaning of the unified field theory may lie in another angle beyond our usual understanding. This article reviews this issue, which begins with the Hubble redshift and the cosmological redshift, the fact that space-time is expanding. This then leads to the question of whether space-time expansion occurs within the Planck length. The rationality of general relativity and quantum field theory will be reviewed in turn. The problem is described mathematically, and a situation of "continuous and discrete coexistence" which does not exist in mathematics but can be realized is also described. Finally, from this point of view, the universe feasible scheme of unifying the field is proposed.

2. Theoretical Basis

2.1. Doppler Effect of Light and Hubble Redshift

The Doppler effect is a phenomenon related to the velocity of the wave source and the observer. If there is a relative velocity between them, the frequency received by the observer is different from the frequency emitted by the wave source. Depending on the velocities of the source and the observer, the Doppler effect can be divided into four different cases, which can be expressed by the listed formulas, respectively.

The wave source moves close to the observer, and the observer does not move:

$$f_o = \frac{v}{\lambda_0} = \frac{v}{\frac{v-v_S}{f_S}} = \frac{vf_S}{v-v_S} \tag{1}$$

Where f_o is the frequency received by the observer, v is the velocity of the wave, λ_0 is the wavelength, v_S is the velocity of the moving wave source, f_S is the frequency released by the source.

The wave source moves away from the observer, and the observer does not move:

$$f_o = \frac{v}{\lambda_0} = \frac{v}{\frac{v+v_S}{f_S}} = \frac{vf_S}{v+v_S} \tag{2}$$

The observer moves close to the wave source, and the wave source does not move:

$$f_o = \frac{v+v_o}{\lambda_0} = \frac{v+v_o}{\frac{v}{f_S}} = \frac{(v+v_o)f_S}{v} \tag{3}$$

The observer moves away from the wave source and the source does not move:

$$f_o = \frac{v-v_o}{\lambda_0} = \frac{v-v_o}{\frac{v}{f_S}} = \frac{(v-v_o)f_S}{v} \tag{4}$$

In the following discussions, let's focus on the cases 1 and 2. When the wave source moves close to the observer, the wavelength becomes shorter, and the frequency received by the observer becomes larger, as described by Eq. (1). As the wave source moves away from the observer, the wavelength becomes longer and the frequency received by the observer becomes smaller, as shown in Eq. (2).

Similarly, light, as an electromagnetic wave, also has a Doppler effect. For the Doppler effect of light, there is the following formula. When the light source moves close to observer, one has

$$f_o = \frac{c}{\lambda_0} = \frac{c}{\frac{c-v_S}{f_S}} = \frac{cf_S}{c-v_S} \tag{5}$$

Where c is the speed of light, and c＝299792458 m/s. If the light source moves away from observer, the one has

$$f_o = \frac{c}{\lambda_0} = \frac{c}{\frac{c+c_S}{f_S}} = \frac{cf_S}{c+v_S} \tag{6}$$

In the first case, the wavelength of the light gets shorter, and the color of the light moves toward the blue, which is the blue-shift. In the second case, the wavelength of the medium light becomes longer, that is, the color of the light moves toward red, which is redshift. And scientists have observed that the spectra of stars in the distant galaxies in the universe are turning red. And the farther away the star is from us, the faster it gets redder. The redder the spectrum, the longer the wavelength, and the inverse Doppler effect suggests that these stars are all moving away from us. This is part of the Hubble redshift - the Doppler redshift of celestial objects.

The other part is called cosmological redshift, and it's caused by the electromagnetic waves themselves being stretched out. The former points to a widening of the distance between the observer and the light source, and the latter points to a widening of the distance between each segment of the electromagnetic wave, either pointing to an expansion of space-time [5].

In physics, however, there is a concept called the Planck length. It represents the smallest measurable length in physics, below one Planck length, where the existing laws of physics no longer apply. The “space-time is expanding” deduced by the previous two is also based on the laws of physics, that is, this paper cannot be sure that space-time is still expanding below the Planck length scale.

For the classical field theory and relativity theory, space-time is different from the specific “matter”, which cannot be quantized. With quantum field theory holds that all physical quantities, including space-time, can be quantized. If the quantum gravity theory is correct, the space-time is coming in discrete pieces [6].

2.2. Based on the Relativistic System, the Continuity of Space-time

General relativity combines “time” and “space” into a four-dimensional structure called “space-time.” The shape of space-time is a Riemannian geometry whose curvature is changed by the uneven distribution of matter and energy. Therefore, the trajectory of objects moving through space-time is not a straight line, but a curved motion along the curved surface of space-time. So, in general relativity, a whole range of energies and matter are related to the curvature of space-time. Including a fundamental force on objects, gravity is also defined as the curvature of space-time, and the movement of objects under the action of gravity is also "pulled" by the curvature of space-time.

Under general relativity, space-time is a mathematical problem. While in mathematics, this paper deal with a series of data and must consider whether it is continuous or discrete. Discrete data or variables can only take specific, separated values, while continuous data or variables can take any value within a given interval. The creator of Riemannian geometry, Riemann, emphasized that his model is formed by the continuous motion of points in the mathematical sense into higher dimensional geometry [7]. Since the concept of "curvature" is applied in the space-time concept of general relativity, the definition of curvature is as follows:

$$K = \lim_{dS \to 0} \frac{da}{dS} K = \lim_{dS \to 0} \frac{da}{dS} \tag{7}$$

where K represents the curvature, da is the change in tangent direction angle, dS is change in arc length.

The curvature K equals the derivative of tangent direction angleαwith respect to arc length S. The original function of this derivative is continuous. Thus, in general relativity, Einstein further reasoned about space-time as a continuous whole. Therefore, the subsequent derivation of the theory is based on the "space-time continuum".

The first evidence for general relativity is the precession of Mercury's perihelion. When Mercury reaches perihelion, its actual orbit deviates slightly from the orbit predicted by Newtonian mechanics, a deviation known as precession. According to the general theory of relativity, which is different from Newton's theory, there is a slight difference between Newton-Kepler's law and the movement of planets in their orbits. When a planet moves from one perihelion to the next, the sun and planet move toward each other. The angle that the diameter passes through is greater than the angle of a complete rotation. This difference is given by the following expression [8].

$$+\frac{24\pi^3 a^3}{T^2 c^2 (1-e)} \quad . \tag{8}$$

In this expression, a represents the semi-major axis of the ellipse, e represents the eccentricity of the ellipse, c represents the speed of light, and T represents the orbital period of the planet. This fully explains the 43-arc-second deviation of Newton's law from the actual observation, or the observation of Mercury's precession can be strong evidence for general relativity.

Another strong piece of evidence available for general relativity is black holes. In July 26, 2018, Max Plank Institute for Astronomy dispatched that when they observed and analyzed the orbit of a star around the central super-massive black hole of the Milky Way, they found what general relativity predicts is consistent with this. Data from stars passing closely to the central mass have ruled out all other known astronomical objects that could provide the mass. These data show that the center of our galaxy is truly a black hole, a region with a mass concentrated that even light cannot escape that region's gravitational pull; matter and light can fall in, but cannot get out again [9].

Black holes are an extreme case of the gravitational field in general relativity. Due to its resting mass and carrying too much energy, its escape velocity is greater than the speed of light, resulting in many special astronomical phenomena. At the time of the development of general relativity, scientists had not yet observed black holes and could not analyze their data, so the theory was not fully proven at that time, while the results of the Max Planck Radio Observatory in Germany mentioned in the previous paragraph can be used as a strong support for general relativity.

In the same way, since black holes, such an extreme case, and its related predictions have been confirmed, general relativity has been confirmed, which also shows that there is evidence for space-time. Space-time, when taken as a whole, exhibits characteristics consistent with the experimental observations, which means that according to the information this paper has so far, "its series of data can be regarded as continuous" is feasible.

2.3. Based on the Quantum Field Theory,the Discreteness of Space-time

Before quantum mechanics and general relativity, physicist Max Planck imagined an extreme case. In modern terms, the mass of a black hole whose Schwarzschild radius is equal to its Compton wavelength is defined as a Planck mass. The Schwarzschild radius is:

$$r = \frac{2GM_P}{c^2} \tag{9}$$

where G is the Gravitational constant, M_P is the mass of the black hole, and c is the speed of light. The Compton wavelength is:

$$\lambda = \frac{h}{mc} \tag{10}$$

where h is the Plank constant, m is the mass of the electron, and c is the speed of light. Plank's imagine is when $r = \lambda$,

$$\frac{2GM_p}{c^2} = \frac{h}{mc} \tag{11}$$

So a Plank mass is:

$$M_p = \frac{hc}{2mG} \tag{12}$$

This equals 2.176×10^{-8} kg, while a Plank length is defined as the Schwarzschild radius of this black hole, which equals 1.616×10^{-35} m [10]. To determine the position of an object, this paper must measure the reflection of light on it. If this paper wants to increase the accuracy to measure smaller objects, this paper use photons with shorter wavelengths, which means this paper must use photons with higher energy. When the energy of these photons is high enough, they collide with the object and collapse to produce a black hole. The black hole's escape velocity is greater than the speed of light, causing the photons used in the measurements to be swallowed up. Through the calculation, this paper can see that the above problems will occur when the measurement accuracy reaches below the Planck length. Since this involves relativity and quantum mechanics, all measurements and calculations below Planck length are invalid. According to special relativity, the limit speed in the universe is the speed of light, so the time it takes the speed of light to travel a fixed distance is the shortest time to travel that distance. The time it takes to travel the Planck length at the speed of light is Planck time. When time is less than Planck time and length is less than Planck length, all the existing laws of physics for space and time will break down.

Quantum field theory was based on a series of theories from Planck, it combines quantum mechanics, special relativity, and classical field theory to create an alternative field theory of space-time. In the process of quantization of the classical field, although the field written in the Lagrange is spatially continuous, after quantization, the state space is discrete. From this, this paper can find that this is contrary to the "space-time is continuous" according to the general relativity theory mentioned before.

As mentioned above, general relativity has been supported by observations and experiments, but now there's experimental evidence for quantum field theory, too. Researchers at the University of Innsbruck in Austria and the Institute for Quantum Optics and Quantum Information at the Austrian Academy of Sciences have experimentally confirmed the predictions of quantum field theory for the first time by using a new method. In an ion trap quantum simulator with 51 particles, they used particles to mimic real materials and conducted experiments in a controlled environment. These quanta are strongly entangled. According to the uncertainty principle, observations of strongly entangled particles produce random results. Those with large fluctuations in the measurement are marked in orange-red which is called "hot", and those with an increased probability of a particular outcome are in blue which is called "cold". Quantum field theory predicts that the subregions of a system consisting of many entangled particles can be assigned a temperature distribution, and these distributions can be used to derive the degree of entanglement of the quantum. The temperature profile obtained by the researchers shows that particles that entangle strongly with the environment are "hot", while particles that entangle little are "cold", which conformed to the theory's prediction [11].

3. Based on the Rationality of both Theories, the Assumption of the Unified Field

The above two theories, it is obvious that their descriptions of space-time are contradictory, a data or variable theoretically cannot be both continuous and discrete. However, various experimental results have shown that both predictions are correct, indicating that the same data can lead to two diametrically opposite results, which is normally considered impossible. The construction of the unified field theory is to realize the impossible event in the conventional sense under the condition of two opposite results. According to the conventional thinking, there is no solution. So, there are two possibilities. One is that at least one of the two is not perfect, and the experimental result is only a temporary unexplained phenomenon, which happens to be close to the prediction; The second is that these two theories show contradictory characteristics in a specific Angle, but they are generally similar, but the two are not well linked. As for the first, further experiments may be needed to confirm it, and many derivative theories may be disproved. Regarding the second, guesses will be made below.

3.1. The Mathematical Statement of the Proposition

In Fig.1, this paper describes discrete quantum mechanics with red lines and continuous general relativity with blue lines.

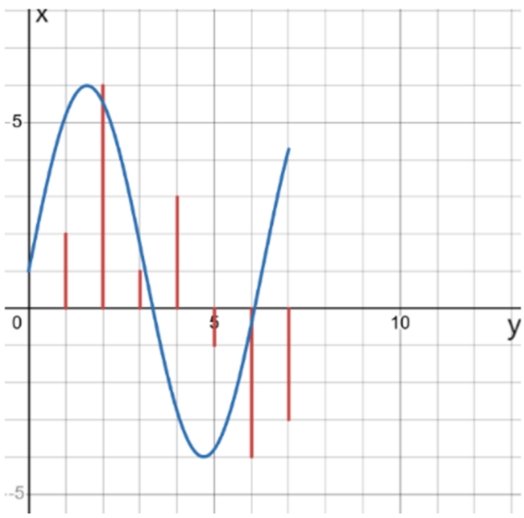

Fig. 1 Abstract expressions of general relativity and quantum field theory

The discrete quantum mechanics is represented by red lines, and the continuous general relativity is described by blue lines.

Einstein stated in his special theory of relativity that our universe is 4-dimensional (although string theory also proposes 10-dimensional space, for which no evidence has yet been found), so this paper can't limit ourselves to three dimensions when this paper talk about space-time. Space-time, at least for the time being, can be a four-element function with four azimuth vectors as the independent variables.

Quantum mechanics is like a summation symbol for discrete data, while general relativity is like an integral symbol for continuous data. Here, this paper describes the former as a summation over this quaternion function $\sum f(\hat{x},\hat{y},\hat{z},\hat{t})$; the latter is described as an integral over this quaternion function $\int\int\int\int \{f(\hat{x},\hat{y},\hat{z},\hat{t})\,\partial x\,\partial y\,\partial z\,\partial t$. Then this paper should find a case that:

$$\sum f(\hat{x},\hat{y},\hat{z},\hat{t}) = \int\int\int\int \{f(\hat{x},\hat{y},\hat{z},\hat{t})\,\partial x\,\partial y\,\partial z\,\partial t \quad (13)$$

Theoretically, this equation works easily when the left summation part has a step size of 0. However, due to the Planck length mentioned above, the smallest possible step size is a Planck length, which cannot be equated with 0. So it's going to be mathematically very complicated to make this equation work.

3.2. The Scope of Application of the Theory is Limited

In mathematics, there is a relatively complex function to deal with - the piecewise function. Let us list two functions: y=x and y=x!. In Fig.2, this paper displays the curves of the two functions, while the former function is depicted in the left panel and the second function is displayed in the right panel.

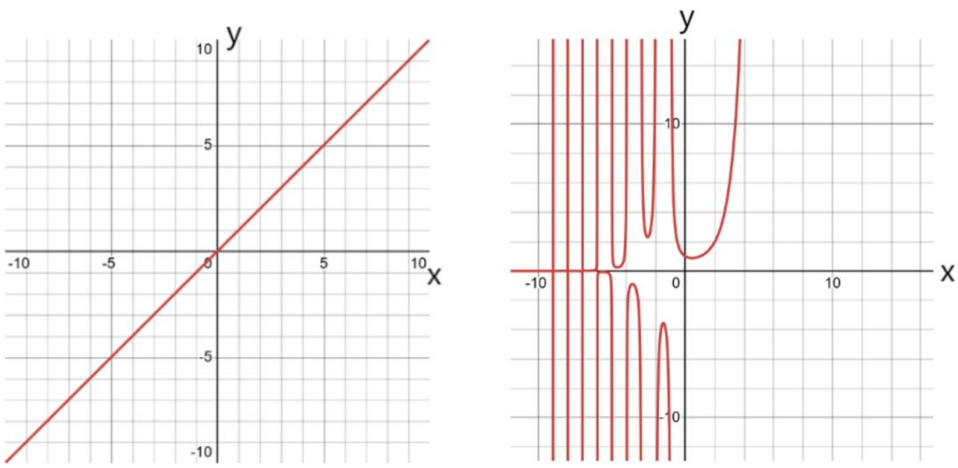

Fig. 2 The curves of the two functions, : y=x (left panel) and y=x! (right panel), respectively.

It can be seen that there is no similarity between the two images. Now let's imagine that in the same function, the expression is different in different intervals, so that the above two very different images are used to represent the same function.

$$\begin{aligned} &\text{when } x \le 0,\ y = x, \\ &\text{when } x > 0,\ y = x! \end{aligned} \quad (14)$$

This function is illustrated in Fig.3.

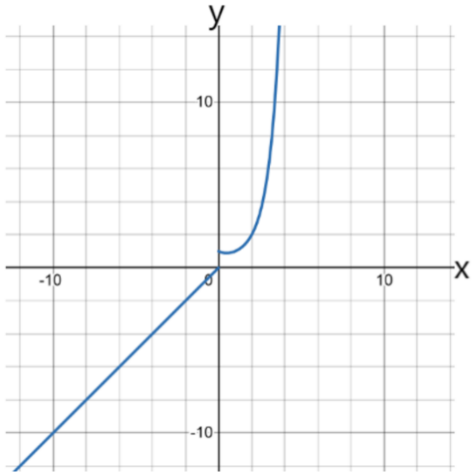

Fig. 3 The piecewise function in Eq.(14).

In the same way, space-time shows the shape of general relativity in the "macro" interval and quantum field theory in the "micro" interval. In other words, the nature of the unified field is "disunity", and if this paper look at it on a large scale, it will show the shape described by general relativity, and it is continuous. At small scales, if you cut it very thin and put it down to the size this paper can see, it will show the shape described by quantum field theory.

Here this paper find that the above description is similar to the wave-particle duality of light. That is, under event a, it appears as form A; Under event b, it appears as form B. What affects its "real" shape is the type of event, similar to how light shows wave patterns, and also shows photons. This also suggests that both general relativity and quantum field theory have limitations. General relativity only holds if event a is observed from a macroscopic perspective; quantum field theory, on the other hand, only holds for event b, which is observed from a microscopic perspective. Both are true, but both are limited. This could also explain how two kinds of experimental data that predicted opposite results can appear at the same time.

The specific shape of events a and b may also be similar to electromagnetic waves. When the "space-time particle" has an energy exchange with the outside world, it transforms into a quantum state. At the macro level, when the "particles" do not exchange, they appear as continuous waves. When space-time interacts with the objects attached to it, the "particles" that make it up do not change. In the quantum state, it shows the characteristics of irregularity; On a large scale, it's ordered.

4. Conclusion

General relativity and quantum mechanics are contrary to the view of the continuity and discreteness of space-time, but both have a basis. this paper cannot decide which one is right and which is wrong until there is a subversive experimental result or theory. It is not impossible to rule out the possibility that a unified field theory can never be arrived at from the front. Perhaps space-time itself is outside of general logic and can manifest two or more different patterns at the same time. In some cases, these patterns may even exhibit opposite sexual characteristics, showing situations that are not possible in pure mathematics.

In string theory, Veneziano envisioned our universe as a fluctuating string. Imagine that this is not a string doing basic harmonic motion, but a string that is oscillating to produce electromagnetic waves. Space-time may indeed resemble electromagnetic waves, or even electromagnetic waves, showing continuity in a general form under the traditional Maxwell equations. In the case of micro-energy exchange, it is quantized, showing discreteness.

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