Aircraft Collision Risk Analysis Based on EVENT Improved Modeling

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Abstract. In order to reduce the risk of collision between aircraft and improve the efficiency of airspace resource utilization, this paper proposes an improved EVENT model. The improved EVENT model is based on the rectangular collision template, which is a combination of a cylinder and two cones, so that the improved collision template is more consistent with the real form of the aircraft. Meanwhile, based on the traditional EVENT model, the collision risk model is established in the same height longitudinal plane, the same height lateral plane and vertical plane, and the calculation method of collision risk in different directions is proposed. Finally, taking the Airbus A380 model as an example, the relevant parameters are brought into the Excel table for calculation, and then the magnitude of the collision risk of the improved EVENT model is compared with the collision risk of the traditional EVENT model and the collision risk of the safety target under the ICAO standard. The results show that the collision risk calculated by the improved EVENT model is 9.0% higher than that calculated by the traditional EVENT model, and all of them meet the collision risk requirements of ICAO, so the model can be used to evaluate the aircraft collision risk, and the calculation results are more accurate.

Keywords: collision risk, improved EXENT model, mathematical modeling, example validation.

1. Chapter 1 Preface

1.1. Background and significance of the research

1.1.1. Research background

Throughout the history of world aviation, since the invention of the aircraft in the early twentieth century, there have been many cases of aircraft collisions: in 1978, Pacific Southwest Airlines Flight 182 collided with a private Cessna light aircraft; in 2002, a Russian Baskerville Airlines TU-154M collided with a DHL Express Boeing 757-200SF freighter, with no survivors; In 2006, Gore Air Flight 1907 collided with a Legacy business jet; in 2022, two U.S. military planes collided during an air show, collision and exploding quickly and starting a huge fire. The casualties and property damage caused by aircraft colliding with each other are enormous, and the consequences can be imagined to be very serious.

In the second decade of the 21st century, China's air transportation industry boomed. Prior to the epidemic, passenger demand for domestic civil aviation was growing at a rate of 14% per year. [1] In 2020, both domestic and international air transportation markets suffered a severe blow as a result of Covid-19. However, after the epidemic was brought under control in 2021, the air transportation industry recovered somewhat, with passenger traffic returning to 5.5% growth. This shows that after the end of the epidemic, China's air transportation industry is destined to have a bright future without restrictions. As the air transportation industry continues to develop, aircraft traffic in the airspace is bound to increase, and the higher density of aircraft will lead to a higher risk of aircraft collision within a certain range. Therefore, the study of aircraft collision risk is necessary, and it has become a hot spot for research.

1.1.2. Research significance

The current high cost of airport construction fuel is a side effect of the rising price of aviation fuel, and at the same time, flight delays have occurred time and again. Reducing the standard of safety separation between airplanes can make these problems effectively solved. And the assessment of flight interval safety provides a basis for the research of reducing the safety separation between airplanes and the risk of aircraft collision. The current flight separation safety assessment models contain REICH model, cross-course model, probabilistic model, stochastic analysis model, EVENT model, RASRAM model and so on. The focus of this paper is to study the aircraft collision risk by improving the EVENT model, to establish the collision models of same-height longitudinal, same-height lateral, and vertical directions, and to derive the formula for calculating the collision risk of aircraft in different directions. The improved EVENT model is conducive to reducing the collision risk between aircrafts and improving the efficiency of airspace utilization, which is of some reference value for improving the overall operational efficiency of the domestic civil aviation industry.

1.2. Domestic and international research status

1.2.1. Current status of international research

In 1966, P. G. Reich proposed the Reich model, [2] the earliest model related to collision risk assessment, based on parallel route flights in the North Atlantic Ocean. The initial Reich model had too many constraints and was difficult to obtain and calculate the parameters. In 1993, G. J. Bakker et al [3] used the random sampling method to calculate the collision risk by solving the mathematical method of partial differential equations, and at the same time analyzed the problem of boundary intersections by applying the Markov process. In 1996, D. Anderson et al [4] focused the research scope to focus on intersecting flight paths, while assuming that no control is applied to the aircraft when it encounters collision risk, and established a collision risk model based on conflict areas. In 2003, Peter. Brooker proposed the Event collision risk model on the basis of Reich's model, [5] solved the problem that the key parameters of Reich's model are not obvious, and it itself is not easy to add other functions, while evaluating the collision risk of lateral separation. In 2007, Zhang Zhaoning et al. applied the basic principle of REICH model to study the lateral collision risk under the condition of VOR-based navigation, thus applying the REICH model to the study of collision risk under the condition of navigation equipment in land airspace [21].In 2009, Zhang Wei, on the basis of Peter Brooker's combined the reliability of controllers and pilots, and further improved the model of collision risk of parallel tracks from the perspective of controllers' own factors, software factors, hardware factors and environmental factors and pilots' cultural quality, psychological quality, physiological quality, flight skill level and flight attitude, and used fuzzy comprehensive evaluation to derive a model of controllers' and pilots' reliability [22].In 2010 Zhang Zhaoning conducted a preliminary study on the positioning errors of aircraft on adjacent altitude layers of the same trajectory and their collision risk under CNS performance conditions, and considered the vertical overlap probability due to surveillance and communication performance in addition to the vertical overlap probability due to technical errors in navigation, and finally derived the total vertical overlap probability based on CNS performance [23].

1.2.2. Current status of domestic research

Domestic research on this field started late. In 2007, Zhang Xiaoyan [6] improved the traditional rectangular collision template in Reich model into a cylinder, calculated its lateral safety distance, and made it into a software. In 2008, Xu Xiaohao et al [7], when analyzing the collision risk of parallel flight paths, used the cylinder Event collision template, and simultaneously analyzed the same-direction and reverse flight scenarios of the collision probability. In 2011, Dai Fuqing et al [8], when studying the problem of collision risk in adjacent altitude layers, proposed an ellipsoidal collision template according to the limitation of the aircraft pitch angle, which greatly reduced the calculated collision risk. In 2015, Zhang Zhaoning et al [9], when studying the collision risk in free flight, considered the influence of the CNS performance on the collision risk, and proposed a sphere collision box, while proposing the altitude relieved and the directional relief two relief strategies. In 2019, Yang Shuo [10] established a collision risk assessment model based on four-dimensional coordinates by considering the factors of position and velocity. In 2019, Pan Weijun et al [11] improved the Reich model collision template into a cone by taking into account the current RECAT contrail separation standard, while using the data of the Airbus A330-200 to do the algorithmic example analysis. In 2021 In 2021, Zhen Ran et al [12] improved the collision box model into a cylindrical fuselage and rectangular wing combination shape. In 2021, Chen Ken et al [13] improved the Event model by proposing a collision box with two spliced quadrangular cones. In 2022, Yue Ruiyuan et al [14], while analyzing the velocity vector distribution of an aircraft, proposed that the velocity change in the vertical direction is minimized in the process of flight, and based on this feature, proposed the two spliced elliptical cones as collision box to calculate the collision risk in vertical direction.

1.3. Main content of the research

The research content of this paper is shown in Figure 1.

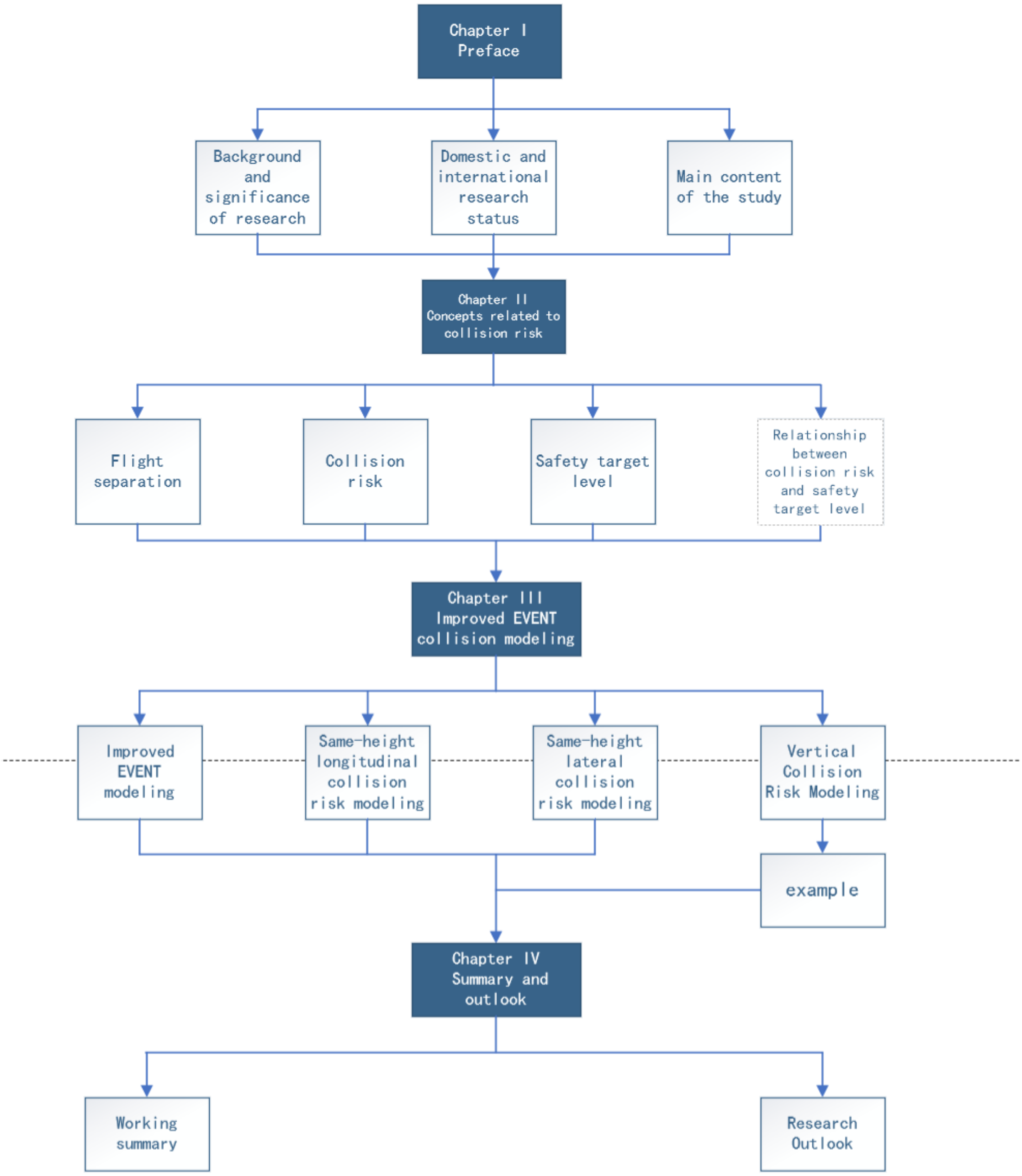

Figure 1. Content and Structure

Chapter 1 describes the research background, research significance and research content of the aircraft collision problem, and provides a brief overview of domestic and international research on collision risk assessment modeling.

Chapter 2 introduces concepts related to collision risk assessment, such as flight separation, collision risk and safety target level, and the relationship between collision risk and safety target level.

Chapter 3 improves the traditional EVENT model collision template, establishes the collision risk model on the longitudinal plane at the same height, the lateral plane at the same height and the vertical plane, and at the same time, using mathematical knowledge, establishes the calculation formula of the collision risk in different directions. And take Airbus A380 as an example to calculate and analyze the collision risk of aircraft in vertical direction.

Chapter 4 summarizes and looks forward to the research.

2. Chapter 2 Concepts related to collision risk

In order to better understand the concepts related to collision risk assessment, this chapter will provide an introduction to the concepts of flight separation, collision risk, and safety target level.

2.1. Flight separation

Flight separation is the distance that air traffic control requires between aircraft to maintain in order to reduce the risk of mutual collision between aircraft operating in the same airspace. [15] Flight separation can be divided into horizontal separation and vertical separation, where horizontal separation can be further divided into vertical separation and lateral separation.

2.1.1. Longitudinal separation

Longitudinal separation refers to the separation between aircraft on the same trajectory and at the same altitude level. In common parlance, this means that aircraft are separated from each other by a specified distance before and after, as shown in Figure 2. Meanwhile, longitudinal separation can be divided into longitudinal time separation and longitudinal distance separation. In procedural control, longitudinal time separation is often used; in radar control, longitudinal distance separation is often used.

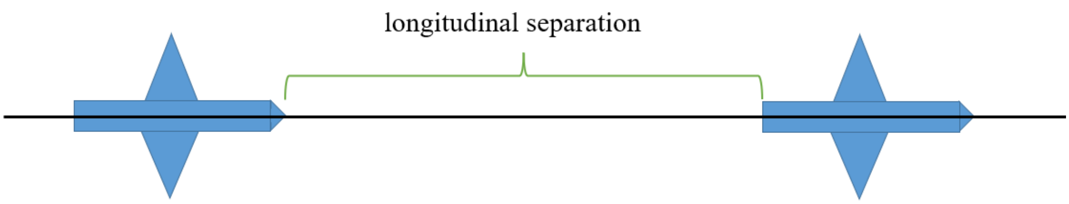

Figure 2. Longitudinal Separation

2.1.2. Lateral separation

In parallel or intersecting routes, lateral separation refers to the distance between aircraft positions on the same altitude level; in parallel routes, lateral separation refers to the distance between two parallel routes, as shown in Figure 3.

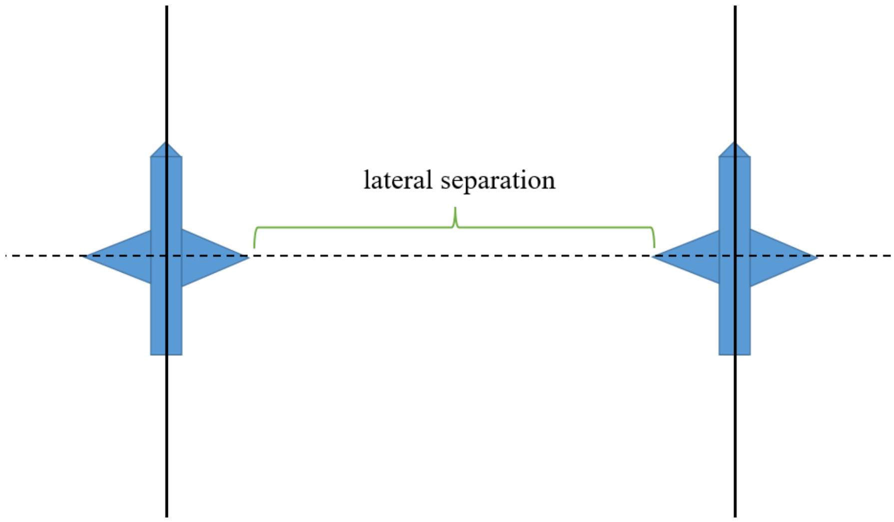

Figure 3. Lateral Separation

2.1.3. Vertical separation

Vertical separation refers to the distance between aircraft at different altitude levels. In layman's terms, this means that aircraft are separated from each other by a specified distance up and down the vertical, as shown in figure 4.

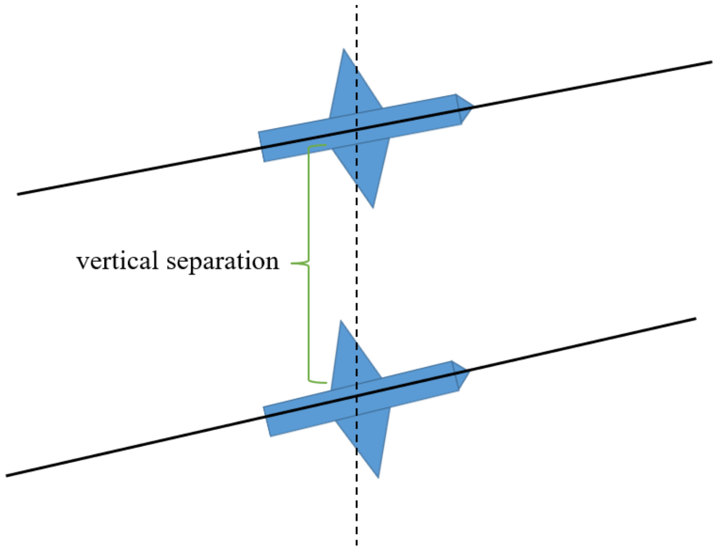

Figure 4. Vertical Separation

2.2. Collision risk

Collision risk is the probability value of an aircraft collision during flight due to various errors. [16] It is expressed as the number of flight collisions per unit of time in terms of times/hour. Collision risk can be categorized into three types, namely flight conflict, hazardous approach and flight collision.

2.3. Safety target level

The ICAO defined cut-off value for collision risk is the safety target level, which is 1.5×10^{-8} accidents/hour in total, with 5×10^{-9}accidents/hour corresponding to the longitudinal, lateral and vertical directions. [17]

2.4. Relationship between collision risk and safety target level

By comparing the minimum distance between the two vehicles in the flight process and the safety target level, the flight collision probability value is obtained, and the larger value of the safety target level will lead to the larger value of the flight collision probability; by comparing the safe flight separation standard with the value of the flight collision probability, the safety target level is obtained, and the value of the flight probability value will be adjusted to a smaller value if the value of the safety target level is too large. Therefore, collision risk and safety target level are positive feedback and negative feedback relationship.

3. Chapter 3 Improved EVENT collision modeling

3.1. Improved EVENT modeling

The improved EVENT model is obtained by improving the traditional EVENT model.

3.1.1. Traditional EVENT modeling

The traditional EVENT collision model considers the length, width and height of a rectangular body as twice the length of the fuselage, twice the width of the wings and twice the height of the body, i.e., $2\lambda_x$, $2\lambda_y$, $2\lambda_z$. Imagine that aircraft A is in the rectangular body, and consider the rectangular body as its collision template. Define the other airplane that may collide with airplane A as B, and regard airplane B as a mass point, then the problem of mutual collision between airplane A and airplane B is transformed into the problem of positional relationship between the rectangular body and the mass point. Peter Brooker establishes a spatial right-angle coordinate system at the point B, and expresses the x-axis, y-axis and z-axis as the longitudinal, lateral and vertical direction of the airplane, and sets the xBy plane as the vertical interval layer, then the xBy plane is the vertical interval layer. plane is set as the vertical separation layer, then the traditional EVENT collision model is shown in Figure 5.

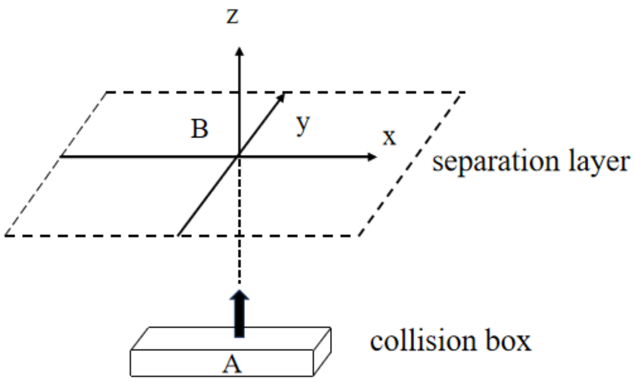

Figure 5. Traditional EVENT Modeling

If Aircraft A is yawed due to various factors, such as inaccurate navigation, Aircraft A has altered its original flight path, making it possible for it to cross the vertical separation layer. Considering the rectangle CDEF as its expanding collision box, the area of the shaded part JGDHIF is the range of aircraft A's activity. According to the literature [5], if aircraft B happens to enter the range of aircraft A's activity at this time, it is considered that aircraft A collides with aircraft B, and there is a risk of vertical collision between the two aircraft. As shown in Figure 6.

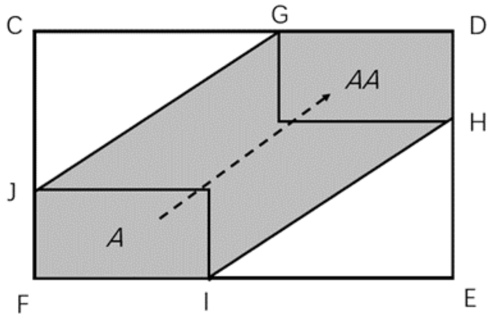

Figure 6. Expanded Collision Box

In this case, collision risk = frequency of collision box crossing the separation vertically * probability that aircraft B is located inside the expanding collision box, [5]

$$N_z = 2 \times P_z\left(S_z\right) \times E\left(0\right) \times P_y\left(0\right) \times \frac{\lambda_x}{S_x} \times \frac{u_z}{2\lambda_z} \times \left(1 + \frac{2u_x\lambda_z}{2u_z\lambda_x}\right) \times \left(1 + \frac{2u_y\lambda_z}{2u_z\lambda_y}\right)$$

Nc is defined as the collision risk value of an airplane in the vertical direction, $P_z(S_z)$ is defined as the vertical overlap probability, E(0) is defined as the longitudinal proximity rate, $P_z(0)$ is defined as the probability of two airplanes overlapping laterally at the same altitude, S_a is defined as the longitudinal spacing criterion of the airplanes, $\lambda_x, \lambda_y, \lambda_z$are defined as the airplane′s fuselage length, wing length, and airframe altitude, respectively, and u_x, u_y, u_z are defined as the relative velocities of the two airplanes in the longitudinal, lateral and vertical directions, respectively.

Similarly, the algorithm for the collision risk value of an airplane in the longitudinal and lateral directions is as follows

$$N_x = 2\times P_x\left(S_x\right)\times E\left(0\right)\times P_z\left(0\right)\times\frac{\lambda_y}{S_y}\times\frac{u_x}{2\lambda_x}\times\left(1+\frac{2u_y\lambda_x}{2u_x\lambda_y}\right)\times\left(1+\frac{2u_z\lambda_x}{2u_x\lambda_z}\right)$$

$$N_y = 2\times P_y\left(S_y\right)\times E\left(0\right)\times P_x\left(0\right)\times\frac{\lambda_z}{S_z}\times\frac{u_y}{2\lambda_y}\times\left(1+\frac{2u_z\lambda_y}{2u_y\lambda_z}\right)\times\left(1+\frac{2u_x\lambda_y}{2u_y\lambda_x}\right)$$

$P_x(S_x)$ *and* $P_y(S_y)$ are defined as the longitudinal overlap probability and the lateral overlap probability, respectively, $P_x(0)$ and $P_z(0)$ are defined as the probability that two airplanes will overlap longitudinally and vertically at the same altitude, respectively, and S_y and S_x are defined as the lateral spacing criterion and the vertical spacing criterion, respectively.

3.1.2. Improved EVENT modeling

By examining the traditional EVENT collision model, we can find the following drawbacks:

(1) The shape of the collision template does not conform to the actual situation. The rectangular body can't fully show the real shape of the airplane, and putting the airplane into the rectangular body will cause the space gap to be large, which will cause the calculation results to be inaccurate when carrying out the study of collision risk.

(2) The impact of human factors on collision risk is not considered. Reaction delay time between controllers and pilots, the possibility of miscommunication when controllers are in command, pilot errors when flying the airplane and communication delays when an accident occurs, to name a few. [18]

In this chapter, the traditional EVENT model will be improved to address the first drawback. The traditional EVENT collision model uses a rectangular body as the collision template, which cannot fully represent the real shape of the aircraft, there are redundant areas in the calculation, the results are inaccurate, and the resulting collision probability is large, which indicates that the airspace resources cannot be fully utilized. So it is very necessary to propose an improved EVENT model suitable for today's civil aircraft. In this paper, we propose to replace the original rectangular body collision template with a combination body collision template of a cylinder and two square cones, set the length of the aircraft fuselage as λ_x, the length of the aircraft wing as λ_y, and the height of the aircraft body as λ_z, and establish the improved model centered on the airplane A. Then the length of the cylinder in this combination is $2\lambda_x$, the length of the line connecting the vertices of the two square cones is $2\lambda_y$, and the height of the cylinder is $2\lambda_z$, as shown in Fig. 7; at the same time, this paper assumes that the base area of the square cones is equal to that of the cylinder; and assuming that the bus of the square cones is twice the radius of the circle of the base, then the side projection of the square cones is an equilateral triangle, i.e., the top angle is 60°. The assumption of the angle is also in line with the current civil aviation airliner wing slender, from the wing root to near the wing tip width of the overall gradual narrowing of the characteristics.

Figure 7. Collision Model

In this paper, we define the other airplanes that have the potential to collide with airplane A as B, and establish a spatial right-angled coordinate system with airplane B as the origin. Define the longitudinal plane, lateral plane and vertical plane as xBz plane, yBz plane and xBy plane respectively. The collision risk of the aircraft flying in the longitudinal plane at the same altitude, the lateral plane at the same altitude, and the vertical plane are investigated separately, and the projections of their three planes on the coordinate axes are calculated and analyzed. The projection of the collision template box is firstly derived, as shown in Figure 8.

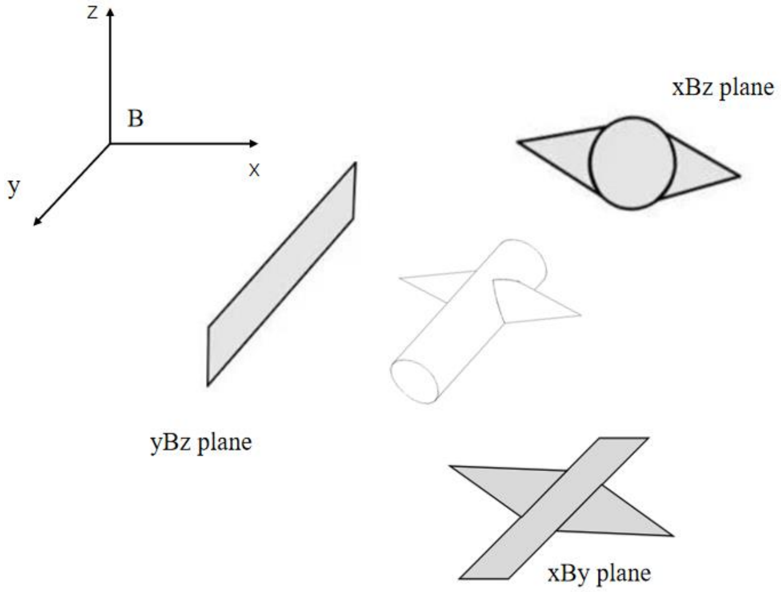

Figure 8. Collision Model Projection

3.2. Same-height longitudinal collision risk modeling

In the improved EVENT model proposed above, an aircraft collision between longitudinal directions at the same altitude can be considered as a collision where the collision template passes longitudinally through the spacer layer, as shown in Figure 9.

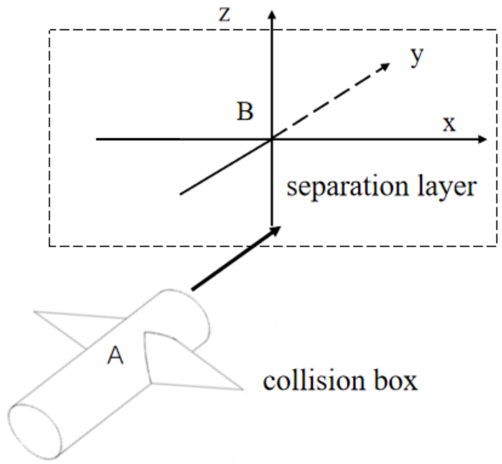

Figure 9. Same-height Longitudinal Collision Risk Modeling

The collision template leaves a projected area in the spacer layer as it passes through it, as shown in Figure 10.

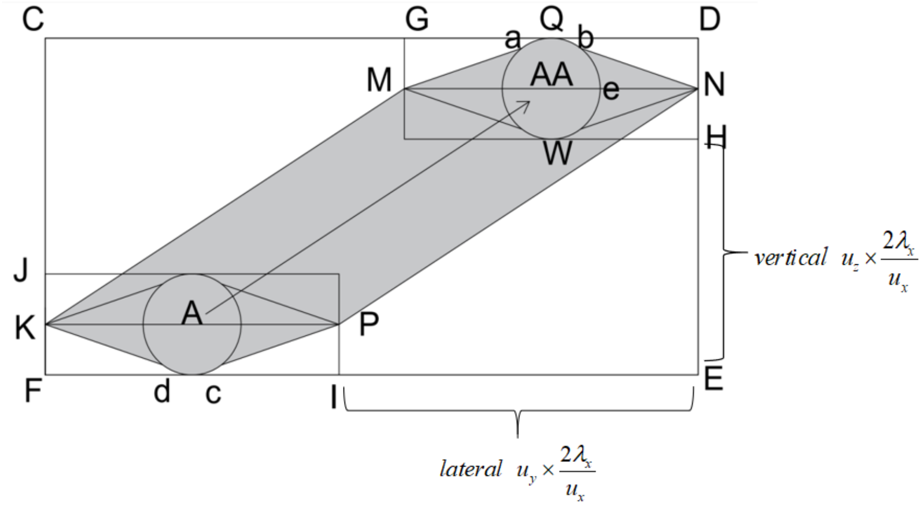

Figure 10. Longitudinal Expansion of the Collision Modeling

Let the area of the rectangle be S▭CDEF and the area of the shaded portion MabNPcdK be S_1. The probability of collision between aircraft A and aircraft B in the longitudinal plane at the same altitude is the ratio of the area of the shaded portion to the area of the rectangle, which is defined to be P_1. As can be seen in the figure, IE and CG are considered to be distances resulting from the lateral flight of aircraft A at the same altitude, and HE and CJ are considered to be the collision template upward distance resulting from crossing the longitudinal collision layer.

Assuming that the time for the collision template to cross the collision layer is t, then we have

$$t = \frac{2\lambda_x}{u_x}$$

From this we can also calculate HE (CJ) and IE (CG) as follows

$$HE = CJ = u_z \times \frac{2\lambda_x}{u_x}$$

$$IE = CG = u_y \times \frac{2\lambda_x}{u_x}$$

And because DH and JF are twice the height of the airplane's fuselage, so we get

$$DH = JF = 2\lambda_z$$

Also, since FI and GD are twice the length of the airplane's wing, it follows that

$$FI = GD = 2\lambda_y$$

From the above known equations, it is not difficult to find out that

$$S_{\square CDEF} = (DH + HE) \times (FI + IE) = (2\lambda_z + u_z \times \frac{2\lambda_x}{u_x}) \times (2\lambda_y + u_y \times \frac{2\lambda_x}{u_x})$$

The area of the shaded portion MabNPcdK can be viewed as the area of the parallelogram MNPK plus the area of the shaded portion MabN and the shaded portion PcdK.

The area of the parallelogram MNPK can be expressed as

$$S_{\square MNPK} = GD \times HE = 2\lambda_y \times u_z \times \frac{2\lambda_x}{u_x}$$

The sum of the latter two areas is a complete frontal area, as shown in Figure 11. Approximating the figure beN as a triangle, this frontal area can be approximated as the area of a complete circle plus four congruent triangles.

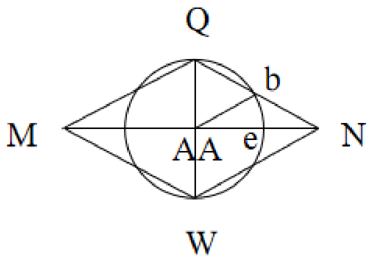

Figure 11. Front View

Because the radius of the circle is λ_z, then the area of the circle can be expressed as

$$S_{circle} = \pi \times \lambda_z^2$$

The improved collision template consists of a cylinder and two square cones combined, the base of the square cone is circular with radius λ_z, i.e., QAA=λ_z. From the assumptions, it can be seen that △QNW is an equilateral triangle, and from the geometric relationship, it is easy to obtain that the points b and e are the midpoints of the line segment QN and the line segment AAN, respectively, and that △beN and △QAAN are similar triangles, and therefore we get

$$S_{\triangle}ebN = \frac{1}{4} \times S_{\triangle}QNAA$$

Also since line QAA and line NAA are half of line QW and line NM respectively and △QAAN is a right triangle, we get

$$S_{\triangle}QNAA = \frac{1}{2} \times QAA \times NAA = \frac{1}{2} \times \frac{1}{2} \times DH \times \frac{1}{2} GD = \frac{1}{8} \times DH \times GD$$

So the area of △beN can be expressed as

$$S_{\triangle}ebN = \frac{1}{4} \times \frac{1}{8} \times DH \times GD = \frac{1}{32} \times DH \times GD = \frac{1}{32} \times 2\lambda_z \times 2\lambda_y = \frac{1}{8} \times \lambda_z \times \lambda_y$$

Then the area of the front side can be expressed as

$$S_{front} = S_{circle} + 4 \times S_{\triangle}\text{ebN} = \pi \times \lambda_z^2 + 4 \times \frac{1}{8} \times \lambda_z \times \lambda_y = \pi \times \lambda_z^2 + \frac{1}{2} \times \lambda_z \times \lambda_y$$

And the shaded area is expressed as follows

$$S_1 = S_{front} + S_{\square MNPK} = \pi \times \lambda_z^2 + \frac{1}{2} \times \lambda_z \times \lambda_y + 2\lambda_y \times u_z \times \frac{2\lambda_x}{u_x}$$

So, the probability of collision between airplane A and airplane B in the longitudinal plane at the same altitude is as follows.

$$P_1 = \frac{S_1}{S_{\square CDEF}} = \frac{\pi \times \lambda_z^2 + \frac{1}{2} \times \lambda_z \times \lambda_y + 2\lambda_y \times u_z \times \frac{2\lambda_x}{u_x}}{(2\lambda_z + u_z \times \frac{2\lambda_x}{u_x}) \times (2\lambda_y + u_y \times \frac{2\lambda_x}{u_x})}$$

The algorithm of the traditional EVENT collision model shows that the probability of an airplane's risk of collision in the longitudinal direction is as follows.

$$N_x = 2 \times P_x(S_x) \times E(0) \times P_z(0) \times \frac{\lambda_y}{S_y} \times \frac{u_x}{2\lambda_x} \times \left(1 + \frac{2u_y\lambda_x}{2u_x\lambda_y}\right) \times \left(1 + \frac{2u_z\lambda_x}{2u_x\lambda_z}\right)$$

Then the probability value of the same height longitudinal collision risk in the improved EVENT model is as follows.

$$N_x^{'} = N_x \times P_1$$

3.3. Same-height lateral collision risk modeling

In the improved EVENT model proposed above, the collision of an airplane between laterals at the same altitude can be considered as a case where the collision template passes laterally through the spacer layer, as shown in Figure 12.

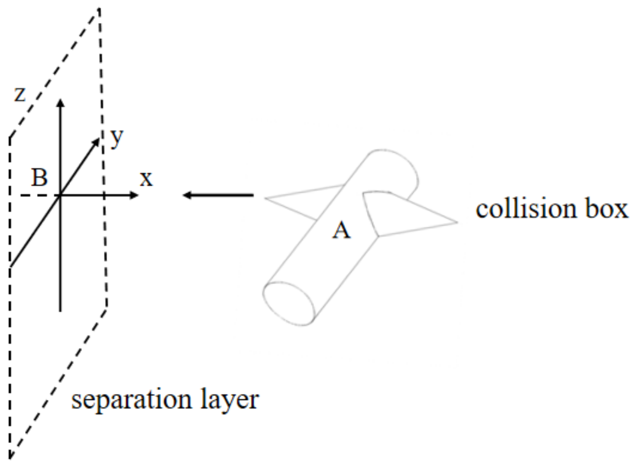

Figure 12. Same-height Longitudinal Collision Risk Modeling

The collision template leaves a projected area in the spacer layer as it passes through it, as shown in Figure 13.

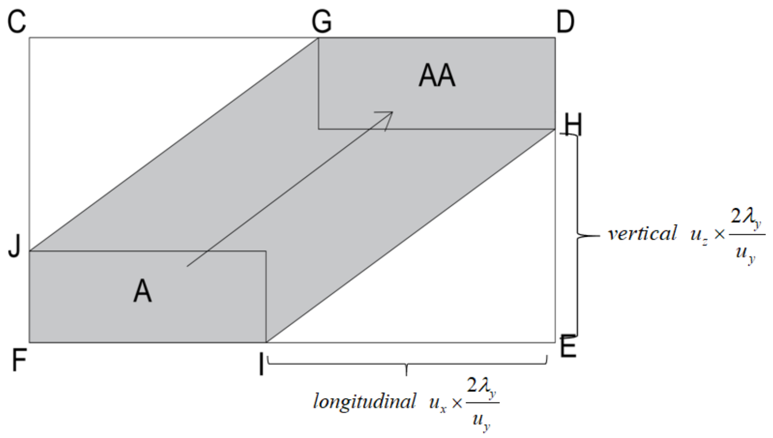

Figure 13. Vertical Expansion Collision Modeling

Let the area of the rectangle be S□CDEF and the area of the shaded portion JGDHIF be S_2, the probability of collision between aircraft A and aircraft B on the lateral plane at the same altitude is the ratio of the area of the shaded portion to the area of the rectangle, which is defined to be P_2. From the figure, it can be seen that IE and CG are considered to be distances generated by aircraft A flying longitudinally at the same altitude, while HE and CJ are considered to be distances generated by collision templates moving through the lateral collision layer resulting in an upward distance.

Assuming that the time for the collision template to cross the collision layer is t, then we have

$$t = \frac{2\lambda_y}{u_y}$$

From this it is also possible to calculate IE (CG) and HE (CJ) as follows.

$$IE = CG = u_x \times \frac{2\lambda_y}{u_y}$$

$$HE = CJ = u_z \times \frac{2\lambda_y}{u_y}$$

Since DH and JF are twice the height of the airplane's fuselage, we can get

$$DH = JF = 2\lambda_z$$

And because GD and FI are twice the length of the airplane's fuselage, it follows that

$$GD = FI = 2\lambda_x$$

From the above known equations, it is not difficult to find out that

$$S_{\square CDEF} = (DH + HE)\times(FI + IE) = (2\lambda_z + u_z \times \frac{2\lambda_y}{u_y})\times(2\lambda_x + u_x \times \frac{2\lambda_y}{u_y})$$

To calculate the area of the shaded part JGDHIF, you can first calculate the area of △CGJ and △HEI, and then subtract the area of the two triangles from the area of the rectangle CDEF, and since the two triangles are congruent, the equation can be obtained as follows.

$$S_{\square HEI} = \frac{1}{2}\times HE \times IE = \frac{1}{2}\times u_z \times \frac{2\lambda_y}{u_y}\times u_x \times \frac{2\lambda_y}{u_y}$$

$$S_2 = S_{\square CDEF} - 2S_{\square HEI} = (2\lambda_z + u_z \times \frac{2\lambda_y}{u_y})\times(2\lambda_x + u_x \times \frac{2\lambda_y}{u_y}) - 2\times\frac{1}{2}\times u_z \times \frac{2\lambda_y}{u_y}\times u_x \times \frac{2\lambda_y}{u_y}$$

So, the probability of collision between airplane A and airplane B on the lateral plane at the same altitude is as follows

$$P_2 = \frac{S_2}{S_{\square CDEF}} = \frac{(2\lambda_z + u_z \times \frac{2\lambda_y}{u_y})\times(2\lambda_x + u_x \times \frac{2\lambda_y}{u_y}) - 2\times\frac{1}{2}\times u_z \times \frac{2\lambda_y}{u_y}\times u_x \times \frac{2\lambda_y}{u_y}}{(2\lambda_z + u_z \times \frac{2\lambda_y}{u_y})\times(2\lambda_x + u_x \times \frac{2\lambda_y}{u_y})}$$

From the equations of the conventional EVENT collision model, it is known that the probability of an airplane's risk of collision in the lateral direction is as follows.

$$N_y = 2\times P_y\left(S_y\right)\times E\left(0\right)\times P_x\left(0\right)\times\frac{\lambda_z}{S_z}\times\frac{u_y}{2\lambda_y}\times\left(1+\frac{2u_z\lambda_y}{2u_y\lambda_z}\right)\times\left(1+\frac{2u_x\lambda_y}{2u_y\lambda_x}\right)$$

Then the probability value of same-height lateral collision risk in the improved EVENT model is as follows.

$$N_y' = N_y \times P_2$$

3.4. Vertical collision risk modeling

In the improved EVENT model proposed above, an airplane collision in a vertical plane can be considered as a case where the collision template passes vertically through the spacer layer, as shown in Figure 14.

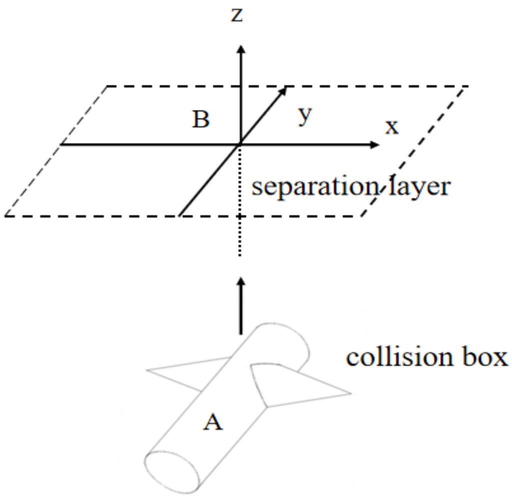

Figure 14. Vertical Collision Risk Modeling

The collision template leaves a projected area in the spacer layer as it passes through it, as shown in Figure 15.

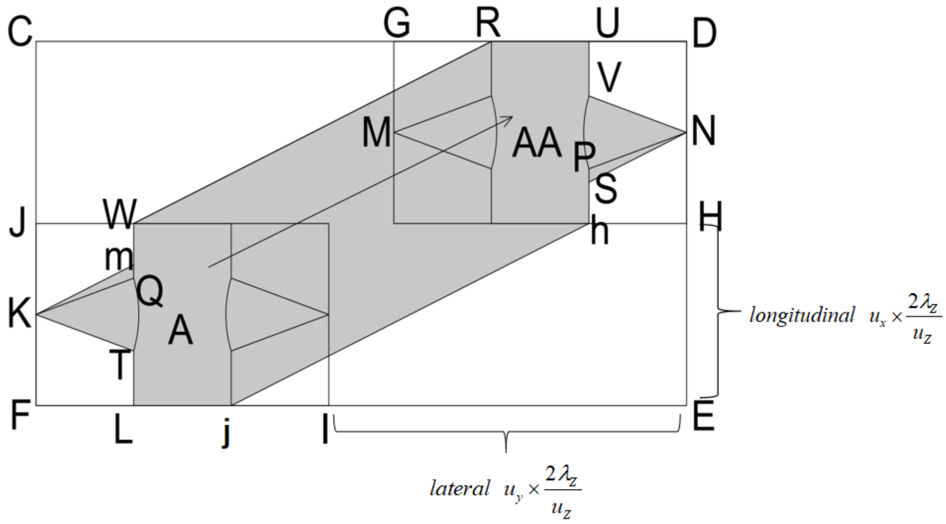

Figure 15. Vertical Expansion Collision Modeling

Let the area of the rectangle be S▫CDEF and the area of the shaded portion KmWRUVNShjL be S_3, the probability of collision between aircraft A and aircraft B in the vertical plane is the ratio of the area of the shaded portion to the area of the rectangle, which is defined to be P_3. From the figure, it can be seen that IE and CG are considered to be distances generated by aircraft A flying in the sideways direction, and HE and CJ are considered to be distances generated by collision templates moving through the vertical collision layer and the longitudinal distance resulting from the vertical movement of the collision template.

Assuming that the time for the collision template to cross the collision layer is t, then we have

$$t = \frac{2\lambda_z}{u_z}$$

From this it is also possible to calculate HE (CJ) and IE (CG)

$$HE = CJ = u_x \times \frac{2\lambda_Z}{u_Z}$$

$$IE = CG = u_y \times \frac{2\lambda_Z}{u_Z}$$

Since DH and JF are twice the length of the fuselage of the airplane, we get

$$DH = JF = 2\lambda_x$$

Also, since FI and GD are twice the length of the airplane's wing, it follows that

$$FI = GD = 2\lambda_y$$

And because RU and Lj are twice the height of the fuselage of the airplane, we get

$$RU = Lj = 2\lambda_z$$

From the above known equations, it is not difficult to find out that

$$S_{\square CDEF} = (GD + CG) \times (DH + HE) = (2\lambda_y + u_y \times \frac{2\lambda_Z}{u_Z}) \times (2\lambda_x + u_x \times \frac{2\lambda_Z}{u_Z})$$

The area of the shaded portion KmWRUVNShjL can be found by subtracting the area of the rectangle CDEF from the area of the two congruent trapezoids CJWR and HEjh, the area of the two congruent trapezoids hSNH and WmKJ, and the two congruent trapezoids UVND and TLKF. In this case, since the point S is very close to P and the point m is very close to the point Q, the area of the trapezoid hSNH is approximated as the area of hPNH and the area of the trapezoid WmKJ is approximated as the area of WQKJ. The area of the shaded portion KmWRUVNShjL is then equal to the area of the rectangle CDEF minus the areas of the two congruent trapezoids CJWR and HEjh and the areas of the four congruent trapezoids UVND, TLKF, hPNH, and WQKJ.

The projection of an airplane on a vertical plane is shown in Figure 16.

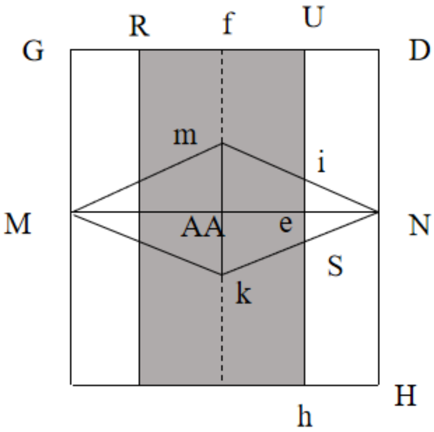

Figure 16. Projection of the Vertical Plane

From the figure, we can see that △eSN is similar to △KAAN, therefore we get

$$\frac{eS}{KAA}=\frac{eN}{AAN}$$

$$eS=\frac{KAA\times eN}{AAN}$$

Since the length of KAA is the radius of the circle and N is the midpoint of DH, the relevant line segment can be expressed as

$$KAA=\frac{1}{2}Km=\frac{1}{2}RU=\lambda_Z$$

$$AAN=\frac{1}{2}GD=\lambda_y$$

$$hH=eN=AAN-AAe=\frac{1}{2}GD-\frac{1}{2}RU=\lambda_y-\lambda_Z$$

$$NH=\frac{1}{2}DH=\lambda_x$$

The length of the line segment eS and the length of hS can be expressed as

$$eS=\frac{KAA\times eN}{AAN}=\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\frac{1}{2}GD}=\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y}$$

$$\mathrm{h}S=he-eS=\frac{1}{2}DH-eS=\lambda_x-\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y}$$

So, the area of trapezoid hSNH can be expressed as

$$S_{\mathrm{hsNH}}=\frac{1}{2}\times hH\times(hS+NH)=\frac{1}{2}\times(\lambda_y-\lambda_Z)\times(2\lambda_x-\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y})$$

The area of trapezoid CJWR can be expressed as

$$S_{\mathrm{CJWR}}=\frac{1}{2}\times CJ\times(JW+CR)=\frac{1}{2}\times CJ\times(JW+CG+GR)=\frac{1}{2}\times u_x\times\frac{2\lambda_Z}{u_Z}\times(2\lambda_y-2\lambda_Z+u_y\times\frac{2\lambda_Z}{u_Z})$$

So, the area of the shaded portion KmWRUVNShjL can be expressed as

$$S_3=S_{\square CDEF}-2\times S_{\mathrm{CJWR}}-4\times S_{\mathrm{hsNH}}=(2\lambda_y+u_y\times\frac{2\lambda_Z}{u_Z})\times(2\lambda_x+u_x\times\frac{2\lambda_Z}{u_Z})-u_x\times\frac{2\lambda_Z}{u_Z}\times(2\lambda_y-2\lambda_Z+u_y\times\frac{2\lambda_Z}{u_Z})-2\times(\lambda_y-\lambda_Z)\times(2\lambda_x-\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y})$$

Then the probability of collision between airplane A and airplane B in the vertical plane is as follows.

$$P_3=\frac{S_3}{S_{\square CDEF}}=\frac{(2\lambda_y+u_y\times\frac{2\lambda_Z}{u_Z})\times(2\lambda_x+u_x\times\frac{2\lambda_Z}{u_Z})-u_x\times\frac{2\lambda_Z}{u_Z}\times(2\lambda_y-2\lambda_Z+u_y\times\frac{2\lambda_Z}{u_Z})-2\times(\lambda_y-\lambda_Z)\times(2\lambda_x-\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y})}{(2\lambda_y+u_y\times\frac{2\lambda_Z}{u_Z})\times(2\lambda_x+u_x\times\frac{2\lambda_Z}{u_Z})}$$

From the algorithm of the conventional EVENT collision model, the probability of vertical collision risk of an airplane is as follows.

$$N_z=2\times P_z\left(S_z\right)\times E\left(0\right)\times P_y\left(0\right)\times\frac{\lambda_x}{S_x}\times\frac{u_z}{2\lambda_z}\times\left(1+\frac{2u_x\lambda_z}{2u_z\lambda_x}\right)\times\left(1+\frac{2u_y\lambda_z}{2u_z\lambda_y}\right)$$

Then the probability value of vertical collision risk in the improved EVENT model is as follows.

$$N_z^{'}=N_z\times P_3$$

3.5. Example

3.5.1. Determination of parameters

According to the current research, the longitudinal overlap probability $P_x(S_x)$ and lateral overlap probability $P_y(S_y)$ are unknown, while the vertical overlap probability $P_z(S_z)$ is known, so we take the Airbus 737-800 as an example to compute the vertical collision risk between airplanes. It is known that the fuselage length λ_x of the airplane is 72.8 m, the wing length λ_y is 79.8 m, and the height of the fuselage λ_z is 24.1 m. From the literature [19], it is obtained that the longitudinal relative velocity u_x of the airplane is 514 m/s, the lateral relative velocity u_y is 6.43 m/s, and the vertical relative velocity u_z is 0.78 m/s. From the literature [20], it is obtained that the vertical overlap probability $P_z(S_z)$ is 6.6×10^{-6}, the vertical proximity rate $E(0)$ is 0.01, the overlap probability $P_y(0)$ in the lateral direction is 0.043, and the standard value of longitudinal spacing of the aircraft S_x is 10,000m.

Specific data are shown in Table 1.

Table 1. Parameters Required for Calculation

Parameters	Values
λ_x	72.8
λ_y	79.8
λ_z	24.1
u_x	514
u_y	6.43
u_z	0.78
$P_z(S_z)$	6.6×10^{-6}
$E(0)$	0.01
$P_y(0)$	0.043
S_x	10000

3.5.2. Calculus analysis

After substituting the above parameter values into the following equation, we get

$$N_z=2\times P_z\left(S_z\right)\times E\left(0\right)\times P_y\left(0\right)\times\frac{\lambda_x}{S_x}\times\frac{u_z}{2\lambda_z}\times\left(1+\frac{2u_x\lambda_z}{2u_z\lambda_x}\right)\times\left(1+\frac{2u_y\lambda_z}{2u_z\lambda_y}\right)$$

$$P_3=\frac{S_3}{S_{\square CDEF}}=\frac{(2\lambda_y+u_y\times\frac{2\lambda_Z}{u_Z})\times(2\lambda_x+u_x\times\frac{2\lambda_Z}{u_Z})-u_x\times\frac{2\lambda_Z}{u_Z}\times(2\lambda_y-2\lambda_Z+u_y\times\frac{2\lambda_Z}{u_Z})-2\times(\lambda_y-\lambda_Z)\times(2\lambda_x-\frac{\lambda_Z\times(\lambda_y-\lambda_Z)}{\lambda_y})}{(2\lambda_y+u_y\times\frac{2\lambda_Z}{u_Z})\times(2\lambda_x+u_x\times\frac{2\lambda_Z}{u_Z})}$$

$$N_z^{'} = N_z \times P_3$$

Calculate using the EXCEL form to get the results as shown in Figure 17

Nz	5.11373E-10
P3	0.089905092
Nz'	4.59751E-11

Figure 17. Calculation Result of Vertical Collision Risk

The magnitude of the collision risk values, i.e., N_z and N_z', evaluated before and after improvement based on the EVENT model were obtained, respectively, as shown in Table 2.

Table 2. Vertical Collision Risk Assessment Results

Model	Traditional model	Improved model
Collision Risk	5.12×10^{-10}	4.6×10^{-11}
Safety Target Collision Risk	5×10^{-9}	

The safety target collision risk under the ICAO standard is 5×10^{-9} accidents/hour, and from the calculated results, it can be seen that both models meet the safety target collision risk, whether it is the traditional EVENT model or the improved EVENT model, but the collision risk of the improved EVENT model is 9.0% of the collision risk of the traditional EVENT model before the improvement, which also It shows that the improvement of the collision template for the EVENT model is feasible and effective, the calculation results are more accurate, and the utilization of airspace resources is higher.

4. Chapter 4 Summary and outlook

4.1. Working summary

In this paper, based on the traditional EVENT collision model, the collision template is improved, and the traditional rectangular body is improved to a combination of a cylinder and two orthoconical cones, which provides a new solution idea to reduce the redundancy of the space of the collision template and to improve the efficiency of the use of airspace resources.

The specific research is summarized as follows.

(1) This paper describes concepts related to collision risk assessment, such as flight separations, collision risk and safety target levels, and the relationship between collision risk and safety target levels.

(2) In this paper, the traditional EVENT model is optimized and improved in terms of its collision template, and then the collision risk model is established on the same-height longitudinal plane, same-height lateral plane and vertical plane. Using mathematical knowledge, this paper establishes the formula for calculating the collision risk in different directions, and verifies the feasibility of the improved collision model by introducing the data of Airbus A380 model.

(3) In this paper, we have analyzed the collision risk of an aircraft in the vertical plane by using an Excel sheet for the arithmetic example, and compared the collision risk of the improved EVENT model with the collision risk of the traditional EVENT model and the safety target level under the ICAO standard, which proved that the improved EVENT model studied in this paper is effective.

4.2. Research outlook

In this paper, the collision risks on same-height longitudinal, same-height lateral and vertical surfaces are analyzed and studied based on the improved EVENT model, respectively, but there are still deficiencies.

(1) The uncertainty of the angle of the top angle of the positive cone leads to the uncertainty of the projected area in the vertically expanding collision box and the vertically expanding collision box. This paper assumes that the angle of the top of the positive cone is 60 degrees, which is not rigorous enough.

(2) This paper still does not analyze and consider the impact of human factors, on-board collision avoidance systems, on-board equipment error factors, etc. on the risk of collision.

(3) Although this study proposes formulas for the calculation of longitudinal, lateral, and vertical collision risk, due to the limited data, only the vertical collision risk was analyzed in an arithmetic example, which is somewhat incomplete.

Meanwhile, through the research on domestic and international models about aircraft collision risk assessment, the following 2 directions can be studied in depth in the future.

First, in-depth research on the influence of communication, navigation, surveillance and other factors will be conducted, which will in turn enhance the calculation accuracy of the collision model based on CNS performance. At present, the maintenance of trajectory, position and altitude between two airplanes basically relies on the navigation system, but we cannot ignore the important role of communication and surveillance performance on the maintenance of aircraft spacing.

The second is to consider the impact of human factors in air traffic. People are an integral part of the aircraft operation process, including pilots, dispatchers, controllers, maintenance personnel, etc., and each link will have an impact on the safe flight of the aircraft, especially the interaction between controllers and pilots, and the breakdown of the risk factors of each link will help to eliminate the risk of collision of the aircraft at a fixed point.

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