Research on Multi-stage Production Process Decision-making Problem based on Dynamic Programming Model

Jiaming Huang

Department of Reading Academy, Nanjing University of Information Science and Technology, Nanjing, Jiangsu, China

Abstract. With the rapid advancement and widespread adoption of 5G technology in recent years, the global market for smartphones and other terminal electronic devices has grown significantly, leading to heightened quality demands for spare parts from enterprises. This paper focuses on addressing problems related to product inspection and disassembly in the production process to ensure the competitiveness of enterprises in the market and the reliability of their products. Therefore, this paper establishes a dynamic programming model to determine whether the spare parts and products suppliers provide meet quality standards with the least number of inspections possible. The dynamic programming model and optimization decision-making method proposed in this paper provide a theoretical framework and practical guidance for enterprises to reduce the number of inspections and inspection costs while maintaining product quality.

Keywords: Sampling Inspection; Dynamic Programming; Cost-Benefit; Multi-Stage Optimization.

1. Introduction

1.1 Background

With the arrival of the 5G industrial Internet, the electronic devices market has ushered in new development opportunities. Terminal electronic devices represented by mobile phones are rapidly penetrating people's daily lives by working at an unprecedented rate. At the same time, as the scope of the electronic devices market continues to expand, the enterprise's requirements for the quality of spare parts and finished products continue to improve, which also puts forward higher requirements for electronic spare parts suppliers [1].

In producing a best-selling electronic device, an enterprise needs to purchase a variety of spare parts and assemble them into a finished product. During the assembly process, if one of the parts fails, the finished product must fail; even if all the parts pass, the finished product may not pass. For the unqualified product, the enterprise can choose to scrap or dismantle; the dismantling process will not damage the spare parts, but it will need to spend additional dismantling costs. For example, an existing supplier claims that a batch of spare parts will not exceed a nominal value. The enterprise is prepared to use sampling inspection to decide whether to accept spare parts from the supplier and bear the cost of testing in the interim. Therefore, it is essential to design an optimal decision for the enterprise with as few tests as possible and at the lowest possible cost to ensure product quality and control costs.

1.2 Research Question

In response to the requirements, this paper establishes a mathematical model and addresses the following four problems:

(1) The supplier claims that the defective rate of a batch of spare parts (spare part 1 or spare part 2) is not more than 10%, and the enterprise intends to use the sampling inspection to decide whether to accept the batch of spare parts. Hence, this paper will design a sampling detection scheme to minimize the number of tests and give the specific detection scheme and its results to meet the following conditions:

1) At the 95% confidence level, if the results indicate that the defective rate of the spare parts exceeds the nominal value, then the spare parts will be rejected;

2) At the 90% confidence level, if the results indicate that the defective rate of the spare parts does not exceed the nominal value, then the spare parts will be accepted.

(2) This paper gives the information in Table 1 about the defective rate of spare parts (spare part 1 and/or spare part 2) and finished products encountered by the enterprise in the production process. This paper will make the following decisions for the production phase of the enterprise and give the specific decision-making basis and the results of the indicators:

Table 1. Situations encountered by enterprises in production (Question 2)

Situation	Spare part 1			Spare part 2			Finished product				Unqualified product	
	Defective rate	Unit cost of purchase	Testing cost	Defective rate	Unit cost of purchase	Testing cost	Defective rate	Assembly cost	Testing cost	Market price	Exchange loss	Disassembly costs
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

1) Spare parts testing decision: decide whether to test the spare parts. If testing is not conducted, the spare parts will go directly to the assembly process; if testing is conducted, discard all detected substandard parts.

2) Finished product testing decision: decide whether to test the finished product after assembly. If testing is not conducted, the finished product will enter the market directly; if testing is conducted, only qualified finished products are allowed to enter the market.

3) Unqualified product disposal decision: Decide whether to disassemble the detected unqualified finished product. If no disassembly is conducted, these unqualified products will be discarded directly; if disassembly is conducted, steps 1) and 2) will be repeated for the disassembled spare parts.

4) Returned unqualified products processing decision: For the unqualified products returned by the user, the enterprise will exchange them unconditionally and bear the related exchange losses, such as logistics costs and impacts on the enterprise's reputation. Repeat step 3) for returned unqualified products.

(3) For a production process containing *m* processes and *n* spare parts, known as the defective rate of spare parts, semi-finished products, and finished products, repeat the decision-making program of the production process in problem two. In this paper, for the above case, we give the situation of a company encountered in producing a production process containing two processes and eight spare parts, as shown in Fig. 1 and Table 2. For this purpose, the paper will provide a specific decision scheme, the basis of the decision, and related indicators.

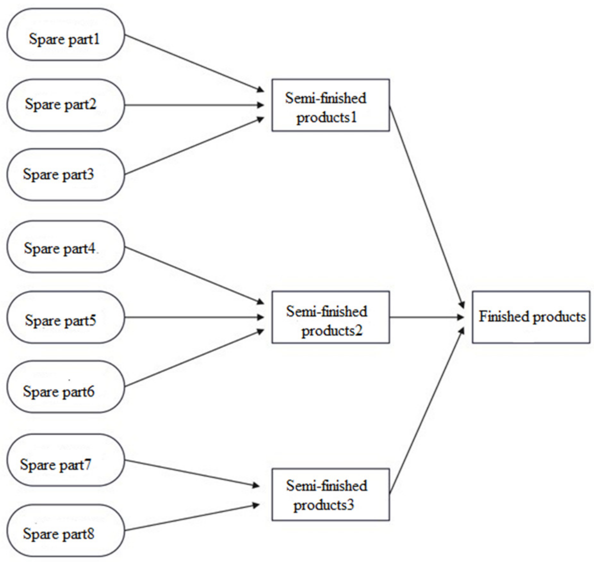

Fig 1. Assembly of two processes and eight spare parts

Table 2. Situations encountered by enterprises in production (Question 3)

Spare part	Defective rate	Unit cost of purchase	Testing cost	Semi-finished products	Defective rate	Assembly cost	Testing cost	Disassembly cost
1	10%	2	1	1	10%	8	4	6
2	10%	8	1	2	10%	8	4	6
3	10%	12	2	3	10%	8	4	6
4	10%	2	1					
5	10%	8	1	**Finished product**	10%	8	6	10
6	10%	12	2					
7	10%	8	1		**Market price**		**Exchange loss**	
8	10%	12	2	**Finished product**	200		40	

(4) Assuming that the defective rates of spare parts, semi-finished products, and finished products in Questions 2 and 3 are all obtained by sampling and testing methods, this paper will restate Questions 2 and 3 based on this.

2. Model Assumption and Notation Description

2.1 Model Assumption

To facilitate the establishment of the model and the feasibility of the model, this paper makes the following assumptions based on objective facts:

1. Assuming the defective rates of spare parts, semi-finished products, and finished products are known and fixed in the production process.
2. Assuming the quality of each spare part is independent of the production process, the defective rate between different spare parts does not affect each other.
3. Assuming the defective rate is stable at each production stage, no large-scale quality fluctuations occur during production.
4. Ignoring the effect of tested yields on sales volume, calculate the in-process cost as the expected cost of a product.
5. Assuming the customer receives an unqualified finished product after the first exchange and does not consider the cost of producing the product a second time.

2.2 Notation Description

To facilitate the modeling and solving process, this paper describes the critical notations used below:

3. Modeling and solving Question 1

3.1 Question Analysis

Table 3. This paper describes the critical notations used

Notation	Notation Description
p_0	Nominal defective rate
p_1	Alternative hypothesized defective rate
Z_α	The critical value corresponding to significance level α in the standard normal distribution
Z_β	The critical value corresponding to test power $1-\beta$ in the standard normal distribution
p_n	Defective rate of spare parts, semi-finished or finished products
W_n	Total cost of each stage

To address question 1, this paper requires the design of a sampling and testing scheme to reduce the number of inspections and to determine whether the defective rate of parts exceeds the nominal value at different confidence levels.

According to the requirements of saving inspection workload and inspection cost, this paper selects the sequential sampling inspection plan that can save the average sample size. First of all, since the sample size of the enterprise sampling is large, it can be modeled using the normal distribution approximation. Then, this paper calculates the minimum sample size under different confidence levels according to the critical value. Next, this paper generates the sample defective rate by randomly simulating the data through binomial distribution and calculates the test statistic. Finally, this paper compares the test statistic with the critical value to determine whether to accept the batch of parts.

3.2 Establishment of the Statistical Testing Model

The sampling plan is derived according to the principle of statistical sampling inspection, based on the inherent characteristics of spare parts and the need to save the inspection workload and inspection costs. This paper chose the minimum sampling size of the study to save the average sample size of the sequential sampling inspection plan [2].

Currently, an enterprise needs to sample a batch of spare parts (spare part 1 or spare part 2) to determine whether to accept the batch of spare parts. According to the sequential sampling inspection plan with as few tests as possible, this paper adopts the method of hypothesis testing to solve the question. Assume that the defective rate of each spare part is p and the nominal defective rate is p_0

For (1) the statistical testing concluded that the defective rate is more than 10%, but in fact, it is not more than that, this is the first type of error. The original hypothesis is H_0 : defective rate ≤ 10%, the alternative hypothesis H_1 : defective rate > 10%.

For (2) the statistical testing concluded that the defective rate is not more than 10%, but in fact, it is more than that, this is the second type of error. The original hypothesis is H_0 : defective rate ≤ 10%, the alternative hypothesis is H_1 : defective rate > 10%.

• The original hypothesis H_0 : the defective rate of spare parts p does not exceed the nominal value p_0 (10%).

• The alternative hypothesis H_1 : the defective rate of spare parts p exceeds the nominal value p_0 (10%).

In this paper, we set confidence level α and test power $1-\beta$. Confidence level α indicates the probability of rejecting the original hypothesis, while $1-\beta$ indicates the probability of rejecting the original hypothesis under the actual defective rate p_1 .

Assuming that n spare parts are taken from the total number of N spare parts for testing, among which the number of defective parts is X. According to the above assumptions, this paper utilizes the normality and the normalcy to detect the defective parts. According to the above assumptions, this paper uses the normal distribution approximation to model the defective rate of spare parts:

$$X \sim \mathcal{N}\big(np, np(1-p)\big)$$

3.3 Solution of the Normal Distribution Model

According to the requirements of question 1, this paper sets the significant level α:

- At 95% confidence level, the significant level is $\alpha = 0.05$.
- At 90% confidence level, the significant level is $\alpha = 0.10$.

According to the standard normal distribution table with common values, this paper can find out the range of the $Z-$ value corresponding to the rejection condition is 1.6~1.7; the range of the $Z-$ value corresponding to the acceptance condition is 1.2~1.3; and the range of the $Z-$value corresponding to the test power is 0.8~0.9.

To better find and determine the range of critical values of the standard normal distribution, this paper plots the probability density function of the standard normal distribution in Python for 95% confidence level and 90% confidence level, respectively:

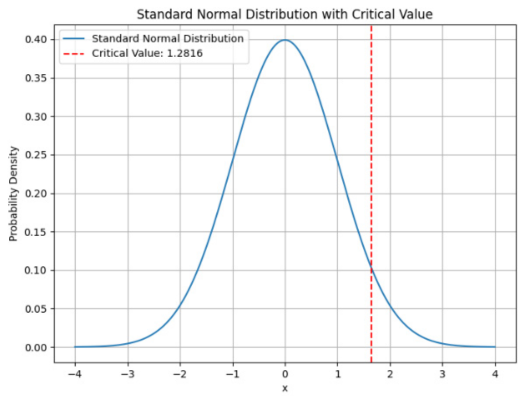

Fig 2. Plot of standard normal distribution at 95% confidence level

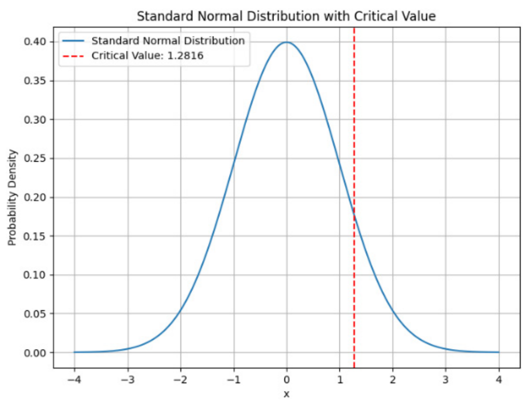

Fig 3. Plot of standard normal distribution at 90% confidence level

According to the standard normal distribution probability density function and normal cumulative distribution function, this paper can calculate the critical value of the standard normal distribution:

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt \tag{1}$$

Where:

• The $Z-$value corresponding to the rejection condition: $Z_1=\Phi^{-1}(1-\alpha_1)=\Phi^{-1}(1-0.05)=\Phi^{-1}(0.95)\approx 1.645$

• The $Z-$value corresponding to the acceptance condition: $Z_2=\Phi^{-1}(1-\alpha_2)=\Phi^{-1}(1-0.10)=\Phi^{-1}(0.90)\approx 1.282$

• The $Z-$value corresponding to the power of the test: $Z_\beta=\Phi^{-1}(1-\beta)=\Phi^{-1}(1-0.2)=\Phi^{-1}(0.8)\approx 0.842$

According to the nature of normal distribution, the sample size n can be calculated by the following formula:

$$n=\left(\frac{Z_\alpha\sqrt{p_0(1-p_0)}+Z_\beta\sqrt{p_1(1-p_1)}}{p_1-p_0}\right)^2 \tag{2}$$

Where：

• At 95% confidence level, the sample size $n_1\approx 253$.

• At 90% confidence level, the sample size $n_2\approx 188$。

3.4 Test of the Binomial Distribution Model

According to the central limit theorem, when n is large, the binomial distribution can be approximated by a normal distribution. As a result, this paper uses the binomial distribution in MATLAB to simulate the number of defective products produced to test the model.

$$X\sim Binomial(n,p)\approx Normal\left(np,\sqrt{np(1-p)}\right)$$

Firstly, this paper establishes a random number generator for a binomial distribution with a defect rate of 10%:

• Assuming n_{01} is the sample size n_1 corresponding to the rejection condition at a 95% confidence level.

• Assuming n_{02} is the sample size n_2 corresponding to the rejection condition at a 90% confidence level.

Next, assuming the defect rate for each component is p, the probability of having k defective items in n samples follow a binomial distribution. Based on the formula, the defect rate of the test samples can be obtained:

$$P(X=k)=(\frac{n}{k})p^k(1-p)^{n-k} \tag{3}$$

Then, according to the properties of the binomial distribution, this paper uses the normal distribution approximation to calculate the sample defect rate $\hat{p}$:

$$\hat{p}=\frac{k}{n} \tag{4}$$

The test is performed according to the standard normal distribution approximation and the following test statistic is used:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{5}$$

Finally, this paper compares Z with the critical value of the standard normal distribution:

• At the 95% confidence level, if $Z > Z_1$, the null hypothesis is rejected, indicating that the defect rate exceeds the nominal value.

• At the 90% confidence level, if $Z < Z_2$, the null hypothesis is accepted, indicating that the defect rate does not exceed the nominal value.

3.5 Decision Method

In the case of the sequential sampling inspection plan, when the defective rate of spare parts does not exceed the nominal value, the minimum sample size required by the enterprise for sampling and testing for different confidence levels is as follows:

• At the 95% confidence level, the minimum sample size required by the enterprise for sampling and testing is 253.

• At the 90% confidence level, the minimum sample size required by the enterprise for sampling and testing is 188.

4. Modeling and Solving Question 2

4.1 Question Analysis

For question 2, given that the defective rates of spare parts and finished products are known, a company needs to face different decision-making stages to increase productivity, reduce defective rates, and minimize associated costs. The question requires making decisions for the production phase of the enterprise and giving specific decision bases and indicator results for the given situation.

Based on the four steps of the product generation phase, this paper draws a flow chart of inspection and disassembly, as shown in Fig. 4:

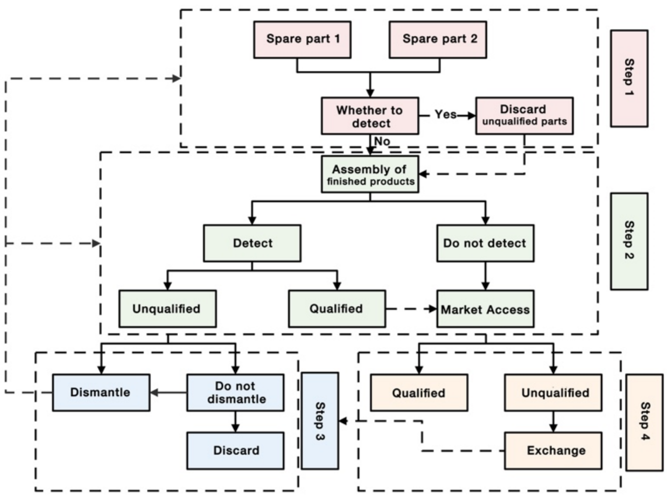

Fig 4. Inspection and dismantle flowchart (Question 2)

First, this paper needs to preprocess the data in the table and analyze the outliers. According to the data in the table, each stage produces an inevitable cost loss and gain, so this paper constructs the cost-benefit model to calculate the total cost of each decision. Next, this paper adopts the 0-1 variable to define whether the state of each stage is tested or dismantled and establishes the state transfer equation and the cost function of each stage according to the requirements at the same time to establish the global objective function to minimize the total cost of each stage.

Due to the need to optimize the multi-stage decision-making process, this paper adopts the establishment of a dynamic programming model to solve the above-established model. After deriving the dynamic programming recursive relationship equation, this paper uses backward computation and forward solving to derive the final global optimal solution so that it can finally give the rational decision basis and the minimum cost for each situation.

4.2 Data Analysis

To address the situation encountered by the company in production and to identify outliers in the data in the table provided in Question 2, this paper utilizes Python to preprocess the data in Table 2.

First, the paper plotted a box-line plot of spare parts and finished product assembly costs, as shown in Fig. 5, to visualize the data distribution and identify potential outliers. Next, these outliers are pinpointed and labelled by calculating the statistical characteristics of each data point. Finally, a filtering algorithm is applied to remove these outliers to generate a more accurate cleaned dataset.

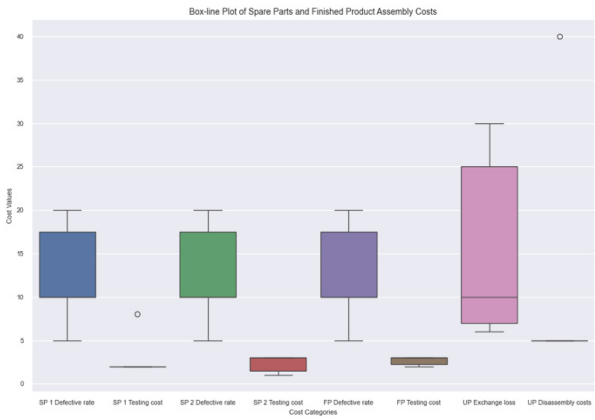

Fig 5. Box-line plot of spare parts and finished product assembly costs

By analyzing the box-line plot of spare parts and finished product assembly costs, the data of the sixth entry does have a significant difference compared with the other entries, which may be related to some exceptional circumstances in the actual production process. The specific differences and reasons can be summarized as follows:

(1) The defective rate of spare part 1 and spare part 2 in the sixth data entry is only 5%, while the defective rate in most other cases is between 10% and 20%. This may indicate that in this case, the enterprise has adopted more stringent quality control measures to ensure higher product quality. In addition, it may also be due to the excellent manufacturing process and material selection of the spare parts in the batch itself, which resulted in a significantly higher quality than the other batches.

(2) The dismantling cost in the sixth figure is 40, while in all other cases, it is 5. This anomaly may be due to the complexity of the product structure, materials, or dismantling process. For example, certain products may require more labor and time to dismantle after failure, or the dismantling process may involve higher safety risks and unique technical requirements. In addition, the design of a particular product may result in a more cumbersome dismantling process requiring specialized equipment and technical support, significantly increasing the dismantling cost.

(3) During the production process, the enterprise may have made adjustments to its production strategy in the batch, and these adjustments may include optimizing the quality inspection process, introducing advanced technologies, or renegotiating contract terms with suppliers. In addition, specific orders or products for particular markets may also result in significant increases in

dismantling costs and inspection costs, as these products may require more sophisticated dismantling processes and finer quality control measures to ensure meeting specific standards and specifications.

4.3 Establishment of the Cost-benefit Model

This question involves optimizing multiple decisions in the production phase to reduce overall costs and increase productivity. Since each stage requires a decision on whether to test or dismantle, and each incurs certain costs and benefits, this paper calculates the total cost of each decision by constructing a cost-benefit model:

$$\text{Total cost} = \sum_{i=1}^{n} \begin{pmatrix} \text{Cost of spare parts} + \text{Replacement loss} + \text{Cost of parts} + \\ \text{Cost of finished product whether tested} + \\ \text{Cost of dismantle} - \text{Recovery value of spare parts} \end{pmatrix} \tag{6}$$

For the different variables affected by various decisions at each stage of the production process, this paper defines the defect rate of spare part 1 as p_1, the defect rate of spare part 2 as p_2, the defect rate of the final product as p_3, and the defect rate of the final product after inspection of the components as $p_3^{'}$. The assembly cost is N, and the inspection costs for component 1, component 2, and the final product are C_1, C_2, C_3, respectively. The purchase cost of component 1 is L_1, the purchase cost of component 2 is L_2, the disassembly cost for defective final products detected during inspection is D_1, the disassembly cost for returned defective final products is D_2, and the replacement loss is M. The salvage values of component 1 and component 2 are V_1 and V_2, respectively. The costs for the four stages are W_1, W_2, W_3, W_4, respectively.

For this type of “binary choice” decision, this paper uses a 0-1 variable state S_i to define whether inspection or disassembly is performed at each stage, where the state $S_i = 0$ indicates no inspection or disassembly, and the state $S_i = 1$ indicates inspection or disassembly.

With two known spare parts and finished product defective rates, there are the following stages of the production process where decisions must be made. At the same time, each stage of the decision will affect the next stage, which needs to be defined at the same time as the transfer of state equation:

(1) Spare part detection decision: According to the defective rate of spare parts and the detection cost, this paper decides whether to detect spare parts. If chosen to detect, it can improve the quality and reduce the defective rate of the finished product, and at the same time reduce the cost of the finished product testing and unqualified product dismantling stage, but it may bring higher cost of spare part detection. If chosen not to detect, it may result in a higher defective rate of finished products. The state transfer equation for this stage can be expressed as:

$$S_1 = \begin{cases} 1 & \text{Detect spare part 1 and/or spare part 2} \\ 0 & \text{Do not detect spare parts} \end{cases} \tag{7}$$

The cost function for this stage is:

$$\begin{aligned} W_1 &= W_{\text{Spare part inspection cost}} + W_{\text{Total assembly cost}} \\ &= S_1(p_1 \times C_1 + p_2 \times C_2) + (N + L_1 + L_2) \end{aligned} \tag{8}$$

(2) Finished product detection decision: This paper needs to decide whether to detect the finished product after assembly based on spare part detection. If chosen to detect, the market acceptance and satisfaction of the product will be improved to some extent, but it may lead to a higher cost of finished product detection. If chosen not to detect, the untested finished product may lead to higher market return rates and losses. The state transfer equation for this stage can be expressed as:

$$S_2 = \begin{cases} 1 & \text{Detect finished products} \\ 0 & \text{Do not detect finished products} \end{cases} \tag{9}$$

The expression for the finished product defective rate after detection of spare parts is:

$$p_3^{'} = 1 - (1 - S_1 \times p_1) \times (1 - S_1 \times p_2) \times (1 - p_3) \tag{10}$$

The cost function for this stage is:

$$W_2 = W_{\text{Loss of whether to detect finished products}} = \begin{cases} p_3^{'} \times M & S_2 = 1 \\ C_3 & S_2 = 0 \end{cases} \tag{11}$$

(3) Unqualified product processing decision: This paper needs to choose whether to dismantle the unqualified products detected after finished product detection. If chosen to dismantle, it can improve product quality control and reduce the risk of nonconforming products in the market, but it may bring the cost of dismantling composed of dismantling cost and re-testing cost of dismantled spare parts. If chosen to discard, the cost of disposal and recovery of nonconforming products may be incurred, including the cost of purchasing spare parts and the cost of assembly. The state transfer equation for this stage can be expressed as:

$$S_3 = \begin{cases} 1 & \text{Dismantle unqualified products} \\ 0 & \text{Discard unqualified products} \end{cases} \tag{12}$$

The cost function for this stage is:

$$W_3 = W_{\text{Dismantling cost}} = S_3 \times D_1 \tag{13}$$

(4) Returned nonconforming products processing decision: For nonconforming products returned by users, the enterprise will exchange them unconditionally and bear the related losses. If chosen to dismantle, it may bring the processing cost of repeating the dismantling and testing process. If chosen to discard, it may incur product production costs consisting of spare part purchase costs and assembly costs. The state transfer equation for this stage can be expressed as:

$$S_4 = \begin{cases} 1 & \text{Dismantle returned nonconforming products} \\ 0 & \text{Discard returned nonconforming products} \end{cases} \tag{14}$$

The expressions for the recycling value of spare part 1 and the recycling value of spare part 2 are, respectively:

$$V_1 = \begin{cases} L_1 & S_{4\,\text{Spare part 1}} = 1 \\ (1 - p_1) \times L_1 & S_{4\,\text{Spare part 1}} = 0 \end{cases} \tag{15}$$

$$V_2 = \begin{cases} L_2 & S_{4\,\text{Spare part 2}} = 1 \\ (1 - p_2) \times L_2 & S_{4\,\text{Spare part 2}} = 0 \end{cases} \tag{16}$$

The cost function for this stage is:

$$W_4 = S_4 \times D_2 - (V_1 + V_2) \tag{17}$$

To minimize the final total cost and thus find the optimal decision, the following global objective function is established in this paper to minimize the total cost at each stage:

$$W = \min(W_1 + W_2 + W_3 + W_4) \tag{18}$$

4.4 Establishment of the Dynamic Programming Model

Dynamic programming is a mathematical method used to solve the optimization problem of a multi-stage decision-making process. Its core idea is to decompose a complex question into multiple sub-problems. The optimal solution to the overall question is finally found by finding and storing the optimal solutions of these sub-problems[1]. In this question, this paper calculates the expected cost of each decision by establishing a dynamic programming model to optimize the impact of various decisions on the cost to help the enterprise develop the optimal decision-making scheme.

According to the two properties of the dynamic programming class of algorithms, it is known that dynamic programming is a multi-stage optimal decision-making problem-solving process to decompose a complex problem into several simpler sub-problems[3]. In calculating the expected cost of each decision, starting from the initial state (stage $i = 1$), this paper progresses to the end state (stage $i = 4$) by gradually solving the decision questions in each stage. Combining these decisions constitutes a sequence of decisions and determines the optimal path to complete the whole process. Based on the nature of multi-stage decision-making in dynamic programming, the recursive relation reflects the dependency relationship between each stage. It ensures that the decision of each stage is based on the results of the previous stages. The specific recursive relation of the dynamic programming class algorithm is summarized in the following steps:

Firstly, this paper specifies the stages of the decision-making process and identifies all the possible states of each stage. The stage is a time period in the question-solving process, denoted by variable i; the state represents the situation of the analyzed object in a certain stage, denoted by variable j.

Then, this paper determines the decisions that need to be made in each state, when the state of each stage is determined, this paper can make different decisions to determine the stages and states that are allowed to enter into the current i stage j state corresponds to 0 and 1. For the above decision-making processes can be categorized into four cases: $i = 0$, $j = 0$; $i = 1$， $j = 0$; $i = 0$, $j = 1$; $i = 1$, $j = 1$.

Additionally, this paper can define the set of allowed decisions. Since the values of the decisions are usually restricted to a certain range, this paper makes the selection by comparing the cost values of different scenarios. The set of allowed decisions is obtained by comparing the cost values of each decision scheme and storing the number (0 or 1) representing the optimal decision in an array.

Finally, this paper defines the function to solve the recurrence relation equation. The definition steps are as follows[4]:

• Since the possible values of state j in the current stage i are not allowed to be derived directly by taking the values between the decisions, this paper defines the strategy representation function to find the global optimum as $dp(i,j)$, i.e., the optimal total cost in stage i and the choice of state j, and at the same time, allows to use 0 and 1 in the choice of the different decision-making choices.

• In this paper, by defining the auxiliary function as Q_i, i.e., it is used to represent the size of the cost brought by the selection strategy at each stage, which is used to assist in judging whether the decision meets the optimal solution. Where $Q_i(0)$ denotes the loss when there is no detection or dismantling, and $Q_i(1)$ denotes the cost when there is detection or dismantling.

• This paper defines the optimization function min, which is used to specify the optimization objective and to find the minimum total cost of each decision.

• To measure the quantitative index of the advantages and disadvantages of the selected strategy, this paper defines the optimal index function as $F(i,j)$ at stage i and state j are selected.

Finally, the dynamic programming recursive relation equation obtained in this paper is shown below:

$$dp(i,j) \to F(i,j) = \min\big(dp(i-1,0) + Q_i(0), dp(i-1,1) + Q_i(1)\big) \tag{19}$$

4.5 Solution of the Dynamic Programming Model

The solution of the dynamic programming model mainly includes the following two parts: backward calculation and forward solving.

(1) Backward computation: Starting from the last moment of the discrete post-cyclic decision making, traverse the state variable ψ_{ij} at each moment i, then traverse the cost variable W_i, and compute the total cost at that stage and state through equation (16), and finally choose the strategy with the lowest total cost as the optimal solution F, and so on until the initial moment of the cyclic decision making.

$$\psi(_{W_i}) \to F = W_i - W_{i-1} \tag{20}$$

(2) Forward solving: Starting from the initial moment of cyclic decision-making, according to the results of reverse calculation, through the recursive relationship to derive and calculate the minimum value of each stage, and then through the state transfer equation to reach the next state, and so on until the last moment, to get the global total cost of the lowest path.

4.6 Decision Method

Combining the above models and solving them using dynamic programming, this paper obtains the global optimal solution as shown in Table 4.:

Table 4. Optimal decision visualization table of Question 2

Situation	Spare prat 1	Spare prat 2	Finished product	Dismantling nonconforming products	Minimum total cost
1	1	1	0	1	19.40
2	1	1	0	1	22.80
3	1	1	0	1	21.80
4	1	1	1	1	20.60
5	0	1	0	1	19.62
6	0	1	0	0	34.51

Where, state 0 means no detection or dismantling and state 1 means detection or dismantling.

To visualize the specific decision to minimize the cost in each case, this paper plots the “optimal decision” curves for six different cases, as shown in Fig. 6, and each curve shows how to decide on the path of choice in these cases. The horizontal coordinates are the four different stages, and the vertical coordinates are the decision states. A vertical coordinate of 1 indicates detection or dismantling, while a vertical coordinate of 0 indicates no detection or discard.

For the above results, this paper gives the following optimal decisions for different cases:

For case 1, for spare part 1 detected, spare part 2 detected, the finished product not detected, and dismantling the unqualified product, the final minimum cost is 19.40;

For case 2, for spare part 1 detected, spare part 2 detected, the finished product not detected, and dismantling the unqualified product, the final minimum cost is 22.80;

For case 3, for spare part 1 detected, spare part 2 detected, the finished product not detected, and dismantling the unqualified product, the final minimum cost is 21.80;

For case 4, for spare part 1 detected, spare part 2 detected, the finished product detected, and dismantling the unqualified product, the final minimum cost is 20.60;

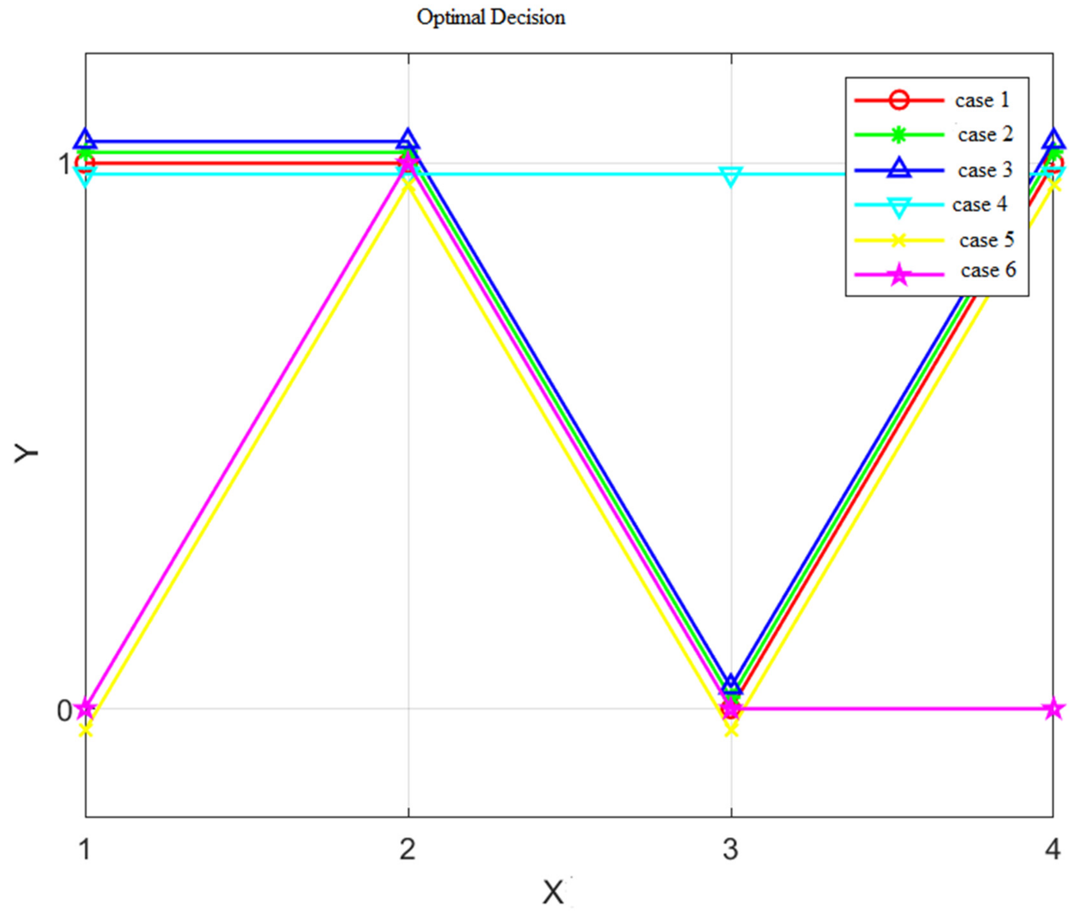

Fig 6. Optimal decision visualization map of Question 2

For Case 5, for spare part 1 not detected, spare part 2 detected, the finished product not detected, and dismantling the unqualified product, the final minimum cost is 19.62;

For case 6, for spare part 1 not detected, spare part 2 detected, the finished product not detected, and not dismantling the unqualified product, the final minimum cost is 34.51.

Through dynamic programming, this paper can be divided into multiple stages of the production process decision-making problem into sub-problems and sequential solutions, to find the global optimal solution. The result is to help companies reduce the cost of testing, dismantling, and return processing while improving production efficiency and maximizing corporate profits.

5. Modeling and Solving Question 3

5.1 Question Analysis

For question 3, the decision scheme for the production process in question 2 is repeated with known defective rates for each type of spare part, semi-finished product, and finished product, where the enterprise has to face different stages of decision-making to increase productivity, reduce defective rates and minimize the associated costs. The topic required for the enterprise production stage contains two processes and eight spare parts of the situation to make decisions, and the given situation to give specific decision-making basis and indicator results.

This paper first preprocesses and fills in the missing values for the data in the table. Like question 2, this paper can still build a cost-benefit model to calculate the total cost of each decision and establish a global objective function to minimize the total cost of each stage. Since this problem still needs to optimize the multi-stage decision-making process, this paper can still use the establishment of a dynamic programming model to solve the model established above. After deriving the recursive relationship equation of dynamic programming, the final global optimal solution can be calculated, thus providing a reasonable decision basis for each situation and realizing the lowest cost.

5.2 Data Analysis

To solve another situation encountered by enterprises in production, this paper uses Python for data preprocessing, and the specific data preprocessing results are shown in the following table:

Table 5. Situations encountered by enterprises in production (Question 3)

Project	Defective rate	Unit cost of purchase	Testing cost	Assembly cost	Market price
Spare part 1	10%	2	1		
Spare part 2	10%	8	1		
Spare part 3	10%	12	2		
Spare part 4	10%	2	1		
Spare part 5	10%	8	1		
Spare part 6	10%	12	2		
Spare part 7	10%	8	1		
Spare part 8	10%	12	2		
Semi-finished product	10%		4	8	
Semi-finished product	10%		4	8	
Semi-finished product	10%		4	8	
Finished product	10%		6	8	200

5.3 Establishment of the Cost-benefit Model

This problem involves optimizing the assembly and inspection decisions for multiple spare parts, semi-finished products, and finished products in multiple production stages to minimize the total cost of production, inspection, and dismantling. Again, each stage requires a decision on whether to inspect or dismantle, and all incur certain costs and benefits. This paper still constructs a cost-benefit model to calculate the total cost of each decision:

$$\text{Total cost} = \sum_{i=1}^{n} \begin{pmatrix} \text{Cost of spare parts } + \text{ Loss on exchange } + \text{ Cost of parts} \\ + \text{ Cost of whether semi} - \text{finished product is tested } + \\ \text{Cost of dismantling semi} - \text{finished product } + \\ \text{Cost of whether finished product is tested } + \\ \text{Cost of dismantling finished product } - \\ \text{Recovery value of spare parts } - \\ \text{Recovery value of semi} - \text{finished product} \end{pmatrix} \tag{21}$$

For the different variables affected by different decisions at each stage of the production process, this paper defines the defective rate of eight spare parts as p_1 to p_8 the defective rate of semi-finished products as p_9 to p_{11}, the defective rate of finished products as p_{12}, the defective rate of finished products after testing of the spare parts as $p_1^{'}$ to $p_8^{'}$, the assembly cost of the semi-finished product is N_1, the assembly cost of the finished product is N_2, the testing cost of the spare part is C_1 to C_8, the testing cost of the semi-finished product is C_9 to C_{10}, the purchasing cost of spare part 1, 2 and 3 is $L_{(1,9)}$, $L_{(2,9)}$ and $L_{(2,9)}$, and the purchasing cost of spare part 4, 5, and 6 is $L_{(4,10)}$, $L_{(5,10)}$ and $L_{(6,10)}$, respectively, the purchasing cost of spare part 7 and spare part 8 are $L_{(7,11)}$ and $L_{(8,11)}$, the dismantling cost of detected unqualified products of finished products are D_1, D_2, D_3, the dismantling cost of detected unqualified products of finished products are D_4, the dismantling cost of returned unqualified products are D_5, the exchange loss is M, the recycling value of 8 spare parts is V_1 to V_8, the recycling value of 3 semi-finished products is V_9', V_{10}', V_{11}', and the cost of the six phases is W_1, W_2, W_3, W_4, W_5, W_6, respectively.

For this decision analysis of the situation encountered by the enterprise in production, this paper still adopts the 0-1 variable state S_i to define whether the state of each stage is detected or dismantled. As mentioned above, state $S_i = 0$ means no detection or dismantle, and state $S_i = 1$ means detection or dismantle.

At present, this paper knows eight kinds of spare parts, three kinds of semi-finished products, and the finished product defective rate, so the production process has a total of the following stages needed to make a decision. At the same time, this paper defines the transfer of state equation for each stage:

(1) Spare part detection decision: The specific decision-making scheme and its impact have been explained in the modelling of Question 2 and will not be repeated here. This stage of the state transfer equation can be expressed as:

$$S_1 = \begin{cases} 1 & \text{Detect spare part } N, \text{where } N \in [1,8] \\ 0 & \text{Do not detect spare parts} \end{cases} \tag{22}$$

The cost function for this stage is:

$$\begin{aligned} W_1 &= W_{\text{Spare part detection cost}} + W_{\text{Total assembly cost}} \\ &= S_1 \sum_{i=1}^{8} (p_i \times C_i) + \left(N + \sum_{i=1,j=9}^{i=8,j=11} L_{(i,j)} \right) \end{aligned} \tag{23}$$

(2) Semi-finished product detection decision: This paper needs to decide whether to test the semi-finished product after assembly based on spare part testing. If testing is chosen, it will reduce the number of unqualified products entering the market and reduce the return rate and related losses. However, it may increase the cost of testing semi-finished products as well as the complexity and time of the production process. If testing is not chosen, untested finished products may increase market return rates and losses and may reduce customer satisfaction. This stage of the state transfer equation can be expressed as:

$$S_2 = \begin{cases} 1 & \text{Detect semi} - \text{finished products} \\ 0 & \text{Do not detect semi} - \text{finished products} \end{cases} \tag{24}$$

The expression for the defective rate of semi-finished products after inspection of spare parts is:

$$p_9^{'} = 1 - \left[\prod_{i=1}^{8} (1 - S_1 \times p_{\mathrm{i}}) \right] \times (1 - p_9) \tag{25}$$

$$p_{10}^{'} = 1 - \left[\prod_{i=1}^{8}(1 - S_1 \times p_{\mathrm{i}})\right] \times (1 - p_{10}) \tag{26}$$

$$p_{11}^{'} = 1 - \left[\prod_{i=1}^{8}(1 - S_1 \times p_{\mathrm{i}})\right] \times (1 - p_{11}) \tag{27}$$

The cost function of this stage is:

$$\begin{aligned} W_2 &= W_{\text{Loss of whether to detect semi-finished products}} \\ &= \begin{cases} p_{\mathrm{j}}^{'} \times (N_1 + \sum_{i=1,j=9}^{i=8,j=11} L_{(i,j)}) & S_2 = 1 \\ C_j & S_2 = 0 \end{cases} \end{aligned} \tag{28}$$

(3) Unqualified semi-finished product processing decision: For the unqualified product detected after finished product inspection, this paper needs to choose whether to dismantle it. If dismantling is chosen, the dismantled spare parts can be reused to reduce the waste of resources, but the dismantled spare parts need to be re-tested, which may increase the testing cost. If discarding is chosen, it may cause the loss of spare parts purchase and assembly costs and is not conducive to environmental protection and resource utilization. This stage of the state transfer equation can be expressed as:

$$S_3 = \begin{cases} 1 & \text{Detect unqualified semi} - \text{finished product } N, \text{where } N \in [9,10] \\ 0 & \text{Discard unqualified semi} - \text{finished product} \end{cases} \tag{29}$$

The expression for the recycling value of the semi-finished product is:

$$V_j' = \begin{cases} L_{(i,j)} & S_{3\ \text{Semi-finished product}i} = 1 \\ \sum_{i=1,j=9}^{i=8,j=11} \left[\left(1 - p_j^{'}\right) \times (1 - S_1) \times L_{(i,j)}\right] & S_{3\ \text{Semi-finished product}\ i} = 0 \end{cases} \tag{30}$$

The cost function of this stage is:

$$\begin{aligned} W_3 &= W_{\text{Dismantling cost}} - W_{\text{Recycling value of semi-finished products}} \\ &= \sum_{i=1}^{3}((S_3 \times D_i) - \sum_{j=9}^{j=11} V_j') \end{aligned} \tag{31}$$

(4) Finished product detection decision: The specific decision-making scheme and its impact have been explained in the modelling of Question 2 and will not be repeated here. This stage of the state transfer equation can be expressed as:

$$S_4 = \begin{cases} 1 & \text{Detect finished products} \\ 0 & \text{Do not detect spare parts} \end{cases} \tag{32}$$

The expression for the defective rate of finished products after the inspection of spare parts is:

$$p_{12}^{'} = 1 - \left(1 - S_2 \times p_9^{'}\right) \times \left(1 - S_2 \times p_{10}^{'}\right) \times \left(1 - S_2 \times p_{11}^{'}\right) \times (1 - p_{12}) \tag{33}$$

The cost function of this stage is:

$$W_4 = W_{\text{Loss of whether to detect finished products}} = \begin{cases} p_{12}^{'} \times M & S_3 = 1 \\ C_{12} & S_3 = 0 \end{cases} \tag{34}$$

(5) Unqualified product processing decision: The specific decision-making scheme and its impact have been explained in the modelling of Question 2 and will not be repeated here. This stage of the state transfer equation can be expressed as:

$$S_5 = \begin{cases} 1 & \text{Dismantle unqualified products} \\ 0 & \text{Discard unqualified products} \end{cases} \tag{35}$$

The cost function of this stage is:

$$W_5 = W_{\text{Dismantling cost}} = S_3 \times D_4 \tag{36}$$

(6) Returned nonconforming products processing decision: The specific decision-making scheme and its impact have been explained in the modelling of Question 2 and will not be repeated here. This stage of the state transfer equation can be expressed as:

$$S_6 = \begin{cases} 1 & \text{Dismantle returned nonconforming products} \\ 0 & \text{Discard returned nonconforming products} \end{cases} \tag{37}$$

The recovery value of spare parts is expressed as:

$$V_i = \begin{cases} L_{(i,j)} & S_{6\text{Spare part } i} = 1 \\ (1 - p_i) \times L_{(i,j)} & S_{6\text{Spare part } i} = 0 \end{cases} \tag{38}$$

The cost function of this stage is:

$$W_6 = S_5 \times D_5 - \left(\sum_{i=1}^{8} V_i \right) \tag{39}$$

For the known defective rates of spare parts, semi-finished products and products, to minimize the final total cost and thus find the optimal decision, this paper establishes the following global objective function to minimize the total cost of each stage:

$$W = \min(W_1 + W_2 + W_3 + W_4 + W_5 + W_6) \tag{40}$$

5.4 Establishment of the Dynamic Programming Model

Dynamic programming is a mathematical method used to solve the optimization problem of a multi-stage decision-making process. Its core idea is to decompose a complex question into multiple sub-problems. The optimal solution to the overall question is finally found by finding and storing the optimal solutions of these sub-problems. In this question, this paper calculates the expected cost of each decision by establishing a dynamic programming model to optimize the impact of various decisions on the cost to help the enterprise develop the optimal decision-making scheme.

5.5 Solution of Question 3 Model - the Dynamic Programming Model

To solve the inspection, dismantle, and disposal decisions of spare parts, semi-finished products and finished products in the production process of the enterprise, this paper still adopts the dynamic planning model to comprehensively consider the costs and benefits of each stage and find the optimal decision-making strategy, i.e., by defining the states and decision variables, recursively solving the optimal value of each stage, to get the globally optimal solution.

Unlike the solution process of Question 2, starting from the initial state (stage $i = 1$), this paper gradually solves the decision-making problem in each stage and advances to the end state (stage $i = 6$). At the same time, this paper defines stage i to denote a time period in the question-solving process and state j to denote the specific situation at a certain stage.

This paper then determines the decisions to be made in each state and the conditions that allow access to current stage i and state j. The decision-making process is then defined as follows. The paper categorizes the different decision-making processes mentioned above into four cases and eight cases.

- Four scenarios: $i = 0$, $j = 0$; $i = 1$, $j = 0$; $i = 0$, $j = 1$; $i = 1$, $j = 1$.
- Eigtht scenarios: $i = 0, j = 0, k = 1$; $i = 1, j = 0, k = 1$; $i = 0, j = 1, k = 1$; $i = 1, j = 1, k = 1$; $i = 0, j = 0, k = 0$; $i = 1$, $j = 0, k = 0$; $i = 0$, $j = 1, k = 0$; $i = 1$, $j = 1, k = 0$.

Finally, the recursive equation of dynamic programming solved by defining the function is shown as follows:

$$dp(i,j) \to F(i,j) = \min\big(dp(i,0) + Q_i(0), dp(i,1) + Q_i(1)\big) \tag{41}$$

$$dp(i,j,k) \to F(i,j,k) = \min\big(dp(i,j,0) + Q_i(0), dp(i,j,1) + Q_i(1)\big) \tag{42}$$

5.6 Decision Method

Combining the above models and solving them using dynamic programming, this paper obtains the global optimal solution shown in Fig. 7:

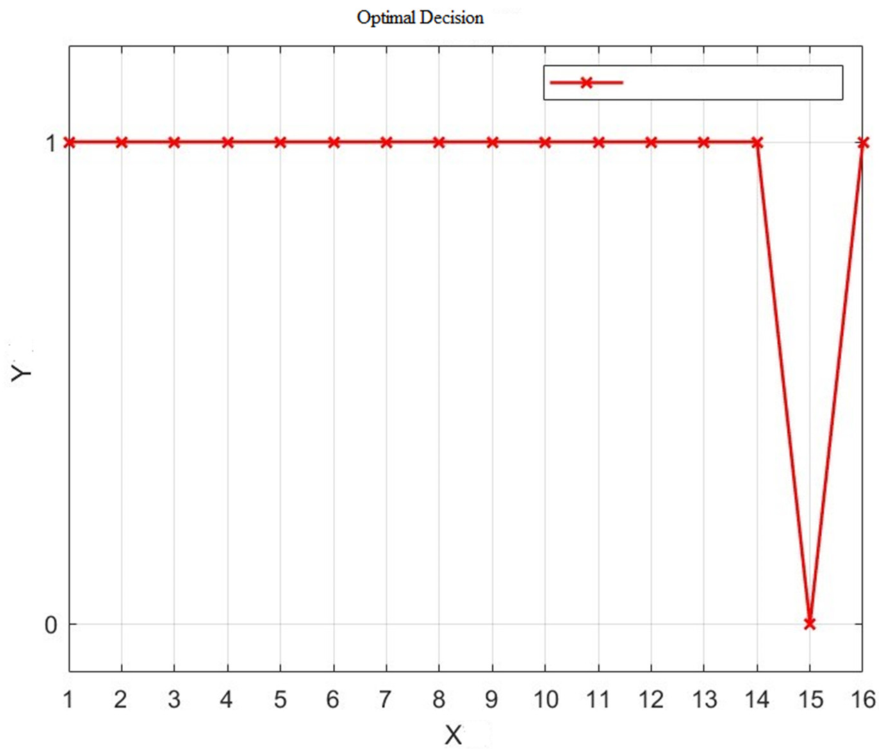

Fig 7. Optimal decision visualization map of Question 3

In conjunction with Fig. 7, it is clear that the state of the "optimal decision" for the scheduling question for two processes and eight spare parts changes. As can be seen from the figure, all other spare parts and semi-finished products are tested and dismantled, and only the finished products are not tested but dismantled. The horizontal coordinate is the number corresponding to spare parts, semi-finished products and finished products, and the vertical coordinate is the decision state. A vertical coordinate of 1 indicates testing or dismantling, while a vertical coordinate of 0 indicates no testing or discarding.

By deriving the recursive relation equation of dynamic programming, this paper can finally use the method of inverse computation and forward solving to obtain the global optimal solution. The specific decision-making method is to detect spare parts 1 to spare parts 8, detect and dismantle semi-finished products, and not detect but dismantle finished products. After this series of optimization steps, the total cost of the enterprise is at least 24.68. This decision-making method can not only help the enterprise to ensure the quality control of each link but also significantly reduce the overall cost, thus providing a reasonable basis for decision-making.

6. Modeling and solving Question 4

6.1 Question Analysis

For question 3, the decision scheme for the production process in question 2 is repeated with known defective rates for each type of spare part, semi-finished product and finished product, where the enterprise has to face different stages of decision-making to increase productivity, reduce defective rates and minimize the associated costs. The topic required for the enterprise production stage contains two processes and eight spare parts of the situation to make decisions, and the given situation to give specific decision-making basis and indicator results.

6.2 Determination of Confidence Intervals for Defective Rates

Since the defective rate of each spare part, semi-finished product and finished product is obtained by sampling method, it can be set that each defective rate is an estimated value $\hat{p}_i$ instead of a fixed value given in question 2 or 3. Since the estimated defective rate has a great uncertainty, in this paper, in the process of dynamic programming and cost-benefit optimization, we need to take into account the change of the confidence interval corresponding to the defective rate of the spare parts through sampling and testing, and the specific formulas are solved as follows:

$$\hat{p}_i \pm Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_i (1 - \hat{p}_i)}{n}} \tag{43}$$

Where, $\hat{p}_i$ is the sample defective rate of spare parts, and n is the sample size of sampling detection.

To determine the confidence level in the production decision, this paper is based on the sampling results to calculate the sample defective rate:

$$\hat{p} = \frac{k}{N} \tag{44}$$

Where, k is the number of rejections detected and N is the total sample size.

Suppose the sample size N is large to make the defective rate estimation more accurate. In that case, this paper needs to narrow the confidence interval to reduce the standard error. The standard error of defective rate SE can be calculated according to the following formula:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{N}} \tag{45}$$

Finally, this paper uses the sample defective rate and standard error SE to calculate the confidence interval:

$$\text{The confidence interval} = \hat{p} \pm Z_{\alpha/2} \cdot SE$$

Meanwhile, $Z_{\alpha/2}$ is set as the critical value of standard normal distribution at a 95% confidence level.

6.3 Establishment and Solution of the Robust Optimization Model

In questions 2 and 3, the decision-making is mainly based on each stage's defective rate and inspection cost. However, due to the uncertainty of defective rate in sampling inspection, the decision can be analyzed by a centralized optimization model robust to multiple discrete scenario distributions to analyze the impact of different confidence levels on the scheduling results[5]. Compared to considering 1-paradigm or ∞-paradigm constraints alone, the combined paradigm constraints can reduce conservatism and optimize the scheduling cost, thus avoiding overly conservative decisions. In addition, robust optimization only relies on worst-case scenarios compared to traditional optimization methods, and the framework provides a more balanced and cost-effective design solution by considering a range of possible scenarios[6].

The core objective of robust optimization is to minimize the maximum cost incurred in the production process over all possible defective rates (i.e. when the defective rate varies within its confidence interval) [7]. To ensure that the total cost of the production process is minimized in the worst-case scenario, this paper redesigns the decision scheme for this purpose:

Instead of directly using the estimated defective rates $\hat{p}_i$ in robust optimization, we use the upper bound of their confidence intervals $p_{i,max}$ to ensure that the decision is still optimal when the worst-case defective rate is high.

For spare part i, semi-finished product and finished product, the following upper bound of the confidence interval of defective rate is used in this paper to compute the result of the worst-case defective rate:

$$p_{i,max} = \hat{p}_i + Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_i (1 - \hat{p}_i)}{n}} \tag{46}$$

The following corresponding confidence intervals can be calculated in this paper:

For question two, the corresponding confidence intervals for the defective rates of spare part 1 and finished product are $[4.12\%, 15.88\%]$, and the corresponding confidence intervals for the defective rates of spare part 2 are $[12.16\%, 27.84\%]$.

For question 3, the confidence intervals for defective rate for spare parts, semi-finished products and finished products are $[4.12\%, 15.88\%]$.

6.4 Defective Rate updating in Dynamic Programming

In addition, the global objective cost function in dynamic planning needs to be recalculated based on the new upper limit of the defective rate, and inspection and recovery decisions in the production process must be designed based on the maximum defective rate value. Robust optimization adds an additional layer of immunity to the dynamic planning model, ensuring it remains stable and performs well during corporate sampling and testing[8]. A higher worst-case defective rate means more workpieces will be rejected or need to be tested, resulting in higher production costs. Therefore, this paper re-optimizes these decisions after updating the defective rate to find the lowest-cost solution.

In each stage, this paper first estimates the defective rate using the results of sampling inspections and updates the defective rate based on the upper limit of the confidence interval. Next, for each artefact, the original deterministic defective rate is replaced by using the solved upper bound defective rate. Finally, this paper recalculates the cost based on the updated maximum defective rate, including inspection cost, swapping loss, dismantling cost, etc., and re-optimizes the decision-making process therein to ensure the lowest cost in the worst-case scenario.

According to the requirements of the question, this paper re-solves the global optimal solution of the dynamic programming model in questions 2 and 3 to determine the optimal decision-making in different cases, and the results are shown below:

The solution regarding the reproduction of Question 2 in Question 4 is as follows, and the specific visualization is shown in Fig. 8:

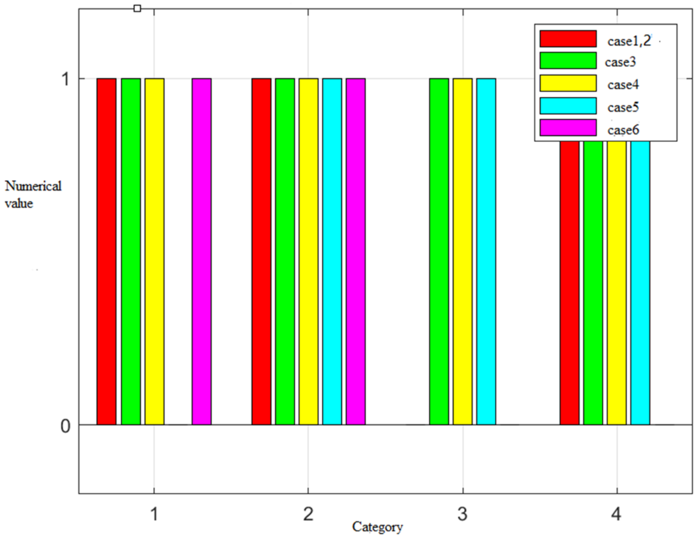

Fig 8. Histogram of optimal decisions for the reproduction of Question 2 in Question 4

For cases one and two, which require testing of spare part 1, testing of spare part 2, non-testing of the finished product, and dismantling of the unqualified product, the minimum total cost is 21.40.

For case three, which requires testing of spare part 1, testing of spare part 2, testing of the finished product, and dismantling of the unqualified product, the minimum total cost is 23.45.

For case four, which requires testing of spare part 1, testing of spare part 2, testing of the finished product, and dismantling of unqualified product, the minimum total cost is 20.45.

For case five, which requires non-testing of spare part 1, testing of spare part 2, testing of the finished product, and dismantling of the unqualified product, the minimum total cost is 22.82.

For case six, which requires testing of spare part 1, testing of spare part 2, non-testing of the finished product, and non-dismantling of the unqualified product, the minimum total cost is 39.03.

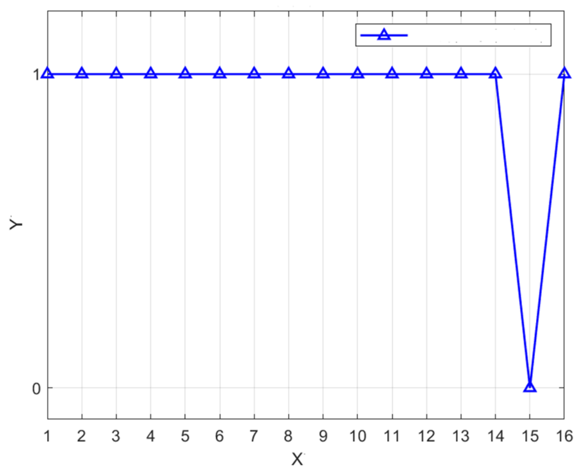

Fig 9. Line graph of optimal decision-making for the reproduction of Question 3 in Question 4

(2) The solution for problem three reproduced in question four is as follows, and the specific visualization is shown in Fig. 9.

In this paper, the global optimal solution derived after deriving the recurrence relation equation of dynamic programming and solving points to the following specific decision-making scheme for the detection of spare parts 1 to spare parts 8, neither detection nor dismantling of semi-finished products, and detection and dismantling of finished products. Combining the above decision-making steps, the minimum cost of the whole program is 110.88. Therefore, through the sampling and testing method, the optimal decision-making for the two processes and eight spare parts can not only effectively control the various types of costs at each stage but also improve the overall production efficiency and product quality, which will bring more significant economic benefits and competitive advantages for the enterprise.

7. Evaluation of the Model

For question 1, this paper designs a sampling inspection plan with a minimum number of inspections, adopts sequential sampling to inspect spare parts, and puts forward the original hypothesis through hypothesis testing. The defective rate of spare parts is calculated by using normal distribution approximation, and the binomial distribution is approximated to normal distribution through the central limit theorem to determine the critical value of the standard normal distribution and the minimum sample capacity. The results show that at 95% confidence level, the minimum sample capacity is 253; at 90% confidence level, the minimum sample capacity is 188.

For question 2, this paper establishes a cost-benefit model with known spare parts and finished product defective rate and solves it with a dynamic programming model. By defining 0-1 variables and state transfer equations, this paper establishes the cost function of each stage and the global objective function to minimize the total cost. In this paper, after deriving the recursive relationship of dynamic programming, the global optimal solution and the minimum cost are derived using backward computation and forward solving to optimize the multi-stage decision-making process and arrive at the optimal decision.

For problem 3, this paper repeats the cost-benefit model and dynamic programming model of question 2 with known defective rates of various spare parts, semi-finished products and finished products. Since question 3 is more complex than question 2, this paper adds a corresponding step in building the state transfer equation to calculate the global optimal solution. The final decision-making method includes testing spare parts 1 through 8, testing and dismantling semi-finished products, and dismantling finished products without testing but at a minimum total cost to the firm of 24.68.

For question 4, considering that the defective rates of spare parts, semi-finished products and finished products are all obtained by sampling and testing methods, it is necessary to set each defective rate as an estimated value rather than a fixed value. In this paper, after determining the confidence interval, a robust optimization model is used to solve for the worst-case maximum defective rate and update the corresponding defective rate. Subsequently, a dynamic programming model is constructed to find the optimal decision under different scenarios.

7.1 Model Advantages

(1) In this paper, an organized dynamic planning model is constructed by decomposing the complex question into smaller sub-problems. This makes the question-solving more systematic and scientific and improves the rigour and operability of the whole model.

(2) By designing an effective sequential sampling inspection plan, this paper dramatically reduces the time and resources required for inspection while reducing the sample size and the number of inspections under the premise of quality assurance, which reflects the plan's objectivity and efficiency.

(3) This paper avoids repeated calculations by memorizing the results of already calculated subquestions. Compared with the methods relying on the population size and the number of iterations,

the dynamic programming model established in this paper is more advantageous regarding computational efficiency.

(4) To target the sampling inspection process, this paper adopts the robust optimization model to solve the worst-case total cost minimization question. This effectively avoids the loss caused by underestimating the defective rate and ensures the robustness and reliability of decision-making.

7.2 Model Disadvantages

Since the state matrix of dynamic programming is a high-dimensional matrix, it may lead to an explosion of the state space when the number of defined processes m and spare parts n increases, and thus cannot handle large-scale questions.

References

[1] Chulin Ma. Analysis of intelligent detection of electronic components in 5G technology under the background of “Internet+”. Modern Industrial Economy and Informatization, 2023, 13(05): 65-67.

[2] Linfang Wang, Shiqing Xu. Design of sequential sampling test program for the purity of mulberry silkworm varieties. Journal of Biomathematics, 2004, 19(3): 373-378.

[3] Held M, Karp R M. A dynamic programming approach to sequencing problems. Journal of the Society for Industrial and Applied Mathematics, 1962, 10(1): 196-210.

[4] Zhengkang Zuo, Huan Sun, Changjing Wang, et al. Program derivation and mechanical verification of imperative dynamic programming-like algorithms. Journal of Software, 2024, 35(09): 4218-4241.

[5] Trzaskalik T. Multiobjective dynamic programming in bipolar multistage method. Ann Oper Res, 2022, 311: 1259-1279.

[6] Xian Jiang. A robust optimization method for power generation system based on voltage regulation control. Aerospace Precision Manufacturing Technology, 2024, 60(04): 42-45.

[7] Zheng H, Chen W, Lewis K, et al. A multidisciplinary robust optimization framework for UAV conceptual design. The Aeronautical Journal, 2016, 120(1230): 629-657.

[8] Zhao Luo, Bowen Lai, Guihong Bi, et al. Distributed robust optimal scheduling of integrated energy system containing CCUS and P2G. High Voltage Technology, 2024, 50(08): 3486-3499.

[9] Yang X, He HB, Zhong XN. Adaptive Dynamic Programming for Robust Regulation and Its Application to Power Systems. IEEE Transactions on Industrial Electronics, 2018, 65(7): 5722-5732.