Kinematic Analysis and Trajectory Planning of a 6-DoF Palletizing Robot

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Abstract. With the rise of the intelligent handling industry, palletizing robots are becoming more and more widely used in industry. How to correctly and accurately analyze the kinematics and trajectory planning of the palletizing process has gradually become the research focus of the industry. However, in the research process, there are still problems of non-correspondence between the kinematic positive and inverse solution results, discontinuous robot motion curves, and unstable motion. Therefore, in this paper, kinematic modelling analysis and path planning of Fanuc industrial robot m20id/25 are carried out using the Devanit-Hartenberg (D-H) parametric method and fifth-degree polynomial. The robot is also required to pass through different working points in different work stages to ensure stable variations of motion profile. The motion trajectory animation of the robot is successfully simulated using the Robotics Toolbox of MATLAB. Smooth and continuous motion curves are also obtained. The research of this paper has certain reference significance for the applications of palletizing robots in actual production environments. It also provides a theoretical foundation for the subsequent further research.

Keywords: 6-DoF robot; D-H model; Quintic polynomial; Motion trajectory planning.

1. Introduction

Under the advancement of intelligent manufacturing technology, Industry 4.0 has been rapidly developing and the modern intelligent manufacturing model has put forward more sensitive and precise performance requirements for industrial robots [1] But there is no denying the fact that, with the arrival of an intelligent and unmanned era, industrial robots have demonstrated through their applications in the production field that they do play a significant role in raising the level of production automation, enhancing labor productivity, improving product quality, and so on [2]. In this paper, the kinematics analysis and trajectory planning are the main areas of interest by using a six-axis industrial robot as the object of the study.

Robot kinematics analysis is the foundation of a robot's ability to perform practical tasks smoothly, accurately, and efficiently [3]. On the other hand, the efficiency, motion smoothness, and energy consumption of industrial robots are greatly influenced by the performance of trajectory planning, which forms the foundation of motion control for these machines [4]. For the kinematic analysis of the robot, Wu et al. used higher-order logic for the kinematic analysis of the robotics arm, using screw theory for the forward analysis of the manipulator [5]. Zhang et al. used the traditional D-H parametric method to establish positive and inverse kinematic models for an $R-\theta$ type silicon wafer handling robot [6]. The motion control of the robot was successfully achieved. Ma et al. considered the Lie group concept while combining the D-H model with the Hayati and Mirmirani models [7]. The kinematic analysis of the robotics arm using this improved method optimized the algorithm by avoiding the trigonometric and inverse trigonometric function solutions. For trajectory planning of robot movement, Chen et al. performed cubic polynomial, composite pendulum, and quintic polynomial end-of-foot trajectory trajectories for the bionic trajectories of a quadrupedal robot, and compared the performance of the trajectories in terms of velocity, acceleration and other dimensions [8]. In the end, the conclusion was that the quintic polynomial fit was optimal in a comprehensive way. Long et al. proposed the trajectory planning of the robot using fifth-degree polynomials [2]. Although the computational effort was large, the acceleration constraint introduced after function derivation ensures the continuity of velocity change. Therefore, the movement trajectory is smooth and less shock. The fifth-degree polynomial trajectory planning can obtain better motion performance.

Although all of the mentioned methods can solve the problem from different theoretical perspectives, there are still some shortcomings in analyzing the kinematics and trajectory planning of robots. In the inverse equations of motion of the kinematic model, multiple sets of solutions are sometimes found because of the trigonometric properties. It is also necessary to determine the actual solutions by corresponding joint motion limitations. When planning the trajectory of the robot, there are some scenarios where the robot needs to pass through specific working points required. If there is still a need to maintain the smoothness of the motion profile, then some adjustments are made for polynomial fitting. It is ensured that the curve does not have sudden changes or discontinuities on account of the presence of working points.

Therefore, this paper adopts the method based on D-H parameters to establish the kinematic model for the robot and solve the positive and inverse kinematic equations based on matrix calculations. Combining the advantages and disadvantages of the planning algorithms for various types of motion trajectories, the fifth-degree polynomial is chosen to plan the trajectory of the robot for the palletizing process. Meanwhile, to improve the smooth continuity of the motion curves, the segmented planning method is used in this study to set different points that the robot must pass during the palletizing process so that the robot can complete the whole work by reaching these points. It improves the accuracy, efficiency, and orderliness of the complete working process.

2. Methodology

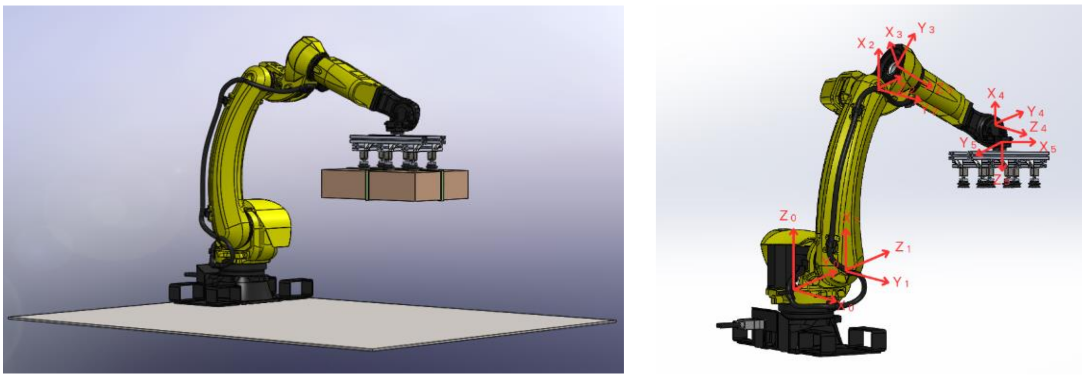
2.1. Modelling of the 6-DoF Palletizing Robot

This paper proposed a study of the kinematics and trajectory planning of a six-axis robot during the palletizing process. Cartons are the palletizing objects and have a range of specifications, the sizes of cartons are shown in Table 1. To ensure that the palletizing process can be applied to all sizes of cartons, the largest value of sizes is used as the parameter of the carton in this study.

Table 1. Parameters of the carton.

physical quantity	range of value(mm)
Length	500-800
Width	300-600
Height	80-150

The Fanuc industrial robot m20id/25, a tandem articulated robot with six degrees of freedom (DoF), is utilized in this work. Fig. 1(a) shows the position of the robot and the cardboard box. As seen in Fig. 1(b), a coordinate system is created based on the robot's basis and a joint coordinate system for each joint is also completed [9].



(a) (b)

Fig 1. 6-DoF robot and carton (Picture credit: Original).

After establishing of each joint's coordinate system, the relationship between the position of the robot's basis and its end-effector can be described [10-11]. According to the previous descriptions, the D-H parameters of this 6-DoF robot can be described, particularly the axis's length, displacement, phase, and rotation angle [12]. Four parameters of the robot are shown in Table 2. Since all six joints of the robot are rotational, the joint angles $\alpha_i(i = 1,2,3,4,5,6)$ are variables.

Table 2. D-H parameters.

Link	$\theta_i(rad)$	$d_i(mm)$	$\alpha_i(rad)$	$a_i(mm)$
1	θ_1	199	$-pi/2$	199
2	θ_2	0	0	840
3	θ_3	-106	$pi/2$	215
4	θ_4	597	$-pi/2$	0
5	θ_5	0	$pi/2$	0
6	θ_6	197	0	0

2.2. Kinematic Analysis

The process of determining the attitude and position of the robot's end-effector is known as kinematic analysis. It is divided into positive kinematic and inverse kinematic analyses, both of which study the matrix transformation relationship between the robot joint angles and the end position.

2.2.1. Positive analysis

Positive kinematics describes the transformation relationship between the spatial coordinates of the robot joints to the base coordinates. Knowing the starting point and the angular values of the individual joint rotations, the position of the robot end-effector in space can be calculated. And it needs to use the transformation matrix obtained from the D-H parameters [13].

In this study, the transformation matrix for rotating around the axis and moving along the axis is required to be used. The conversion matrix between the previous coordinate system and the next coordinate system can be calculated, which is shown in Formula 1.

$$ {}^{i-1}_{i}T = Rot(Z_{i-1}, \theta_i)Trans(Z_{i-1}, d_i)Rot(X_i, \alpha_i)Trans(X_i, a_i) \tag{1} $$

$$ Rot(Z_{i-1}) = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 & 0 \\ sin\theta_i & cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ Rot(X_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha_i & -sin\alpha_i & 0 \\ 0 & sin\alpha_i & cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ Trans(Z_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ Trans(X_i) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

According to the above two types of matrixes, the transformation matrix of each joint can be calculated in Formula 6.

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

The transformation matrices from the base to the end of the robot can be obtained by multiplying the transformation matrix of the two adjacent joints in turn. It is shown in Formula 7.

$$ {}^0_6T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T \cdot {}^3_4T \cdot {}^4_5T \cdot {}^5_6T \tag{7} $$

2.2.2. Inverse analysis

The inverse kinematics of the robot is the process of finding the joint variables of the robot by knowing the position of the end-effector in space [3]. Based on Formulas 7, and 8 can be extended, which is able to represent the end of the robot in all directions in space.

$$ {}^0_6T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T \cdot {}^3_4T \cdot {}^4_5T \cdot {}^5_6T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8} $$

Substitute the known D-H parameters in expanded matrices from Formula 8 (shown in Formula 9), and obtain the angle $\alpha_i (i = 1,2,3,4,5,6)$ of each joint. The inverse kinematics equations can be calculated by getting all ranges of value of six joints [12].

$$ ({}^0_1T)^{-1} \cdot \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_2T \cdot {}^2_3T \cdot {}^3_4T \cdot {}^4_5T \cdot {}^5_6T \tag{9} $$

2.2.3. Polynomial trajectory planning

During the palletizing process, the robot needs to reach different work points and complete the special actions at each point. At the same time, the end of the robot needs to move from the starting point to the target work point. Trajectory planning is the calculation of the changes in velocity and acceleration due to the transition of work points [14]. In this study, a fifth-degree polynomial is used for path planning in order to reduce the abrupt changes in curve fitting. The quintic curve can effectively reduce mechanical vibration and friction [15]. Quintic polynomial and its derivative formula are shown in Formula 10, which represents the angular displacement, angular velocity, and angular acceleration.

$$ \begin{cases} \theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 \\ \dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 \\ \ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3 \end{cases} \tag{10} $$

In a real environment, the palletizing process is described as four steps based on the general working process of robot palletizing operations: positioning, gripping, yarding, and repositioning. Four stages correspond to six sets of robot actions the specific process is shown in Fig. 2.

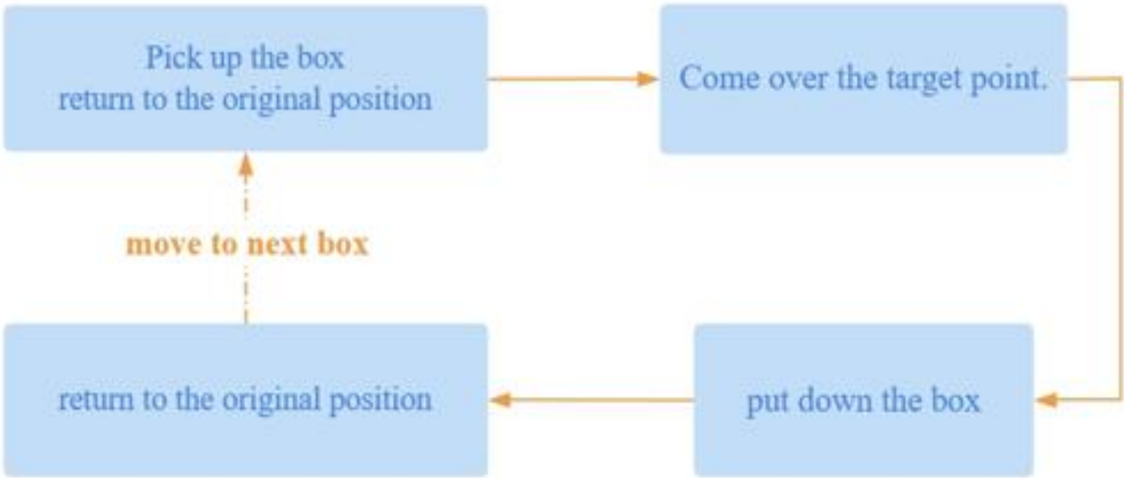


Fig 2. The process of palletizing. (Picture credit: Original).

According to the four steps mentioned above, four different working points are set up in three-dimensional space. Table 3 shows working points and their coordinate values. Through path planning, the end of the robot passes these four points every time when it works. It can ensure that the palletizing process is stable, orderly, and efficient.

Table 3. Four working points.

Number	Working point	Coordinate value
1	Initial point	(550,150,1200)
2	Pick-up point	(600,200,900)
3	Transition point	(250,700,700)
4	Target point	(250,700,500)

When the robot starts to work, some motion conditions are needed to regulate the path planning. The time of each step is limited to 5 seconds, which means that the time relationship between the two working points is the same. And there are some limited conditions of velocity and acceleration shown in Formula 11.

$$\begin{cases} v(T_{t=0}) = v(T_{t=5}) = 0 \\ a(T_{t=0}) = a(T_{t=5}) = 0 \end{cases} \quad (11)$$

After calculating the original value angle of each joint based on the inverse kinematics equations of the robot, the initial angle of the joint and the limitations of the velocity and acceleration can be brought into the formula 10. Solving the equation, the constant parameters of the quintic polynomial can be obtained.

3. Results and Discussion

3.1. Palletizing Curve Equation

The trajectory simulation of the robot was carried out using MATLAB. A polynomial curve function was created with 50 sampling points; the coordinates of the start and end positions were entered, and the values of the motion parameters were obtained by solving the joint angles according to the inverse kinematics equations and bringing them into the curve function.

The whole process of palletizing was divided into four stages, each stage had different polynomial path curves for each joint. And there were a lot of equations involved. As a result, only one stage was discussed in this paper. The polynomial functions for each joint angle of step 2 are listed in Formula 12.

$$\begin{cases} \theta_1(t) = 5.59 - 13.97t - 9.31t^2 + 0.1534t^5 \\ \theta_2(t) = -2.02 + 5.06t - 3.37t^2 - 0.23t^5 \\ \theta_3(t) = -2 + 5.01t - 3.34t^2 - 0.144t^5 \\ \theta_4(t) = 0 \\ \theta_5(t) = 4.03 - 10.07t + 6.72t^2 + 1.94t^5 \\ \theta_6(t) = -5.59 + 13.96t - 9.31t^2 + 1.42t^5 \end{cases}, 10 \leq t \leq 15 \tag{12}$$

Table 4 shows the joint angles corresponding to the four points in the whole process by using inverse kinematics equations.

Table 4. Coordinate and angle of each point.

Point	Angle Group (°)
Initial point (550,150,1200)	(4.5394, -8.5697, -13.7472, -0, 112.3169, 85.4606)
Pick-up point (600,200,900)	(8.7866, -18.1048, -29.4319, -0, 137.5367, 81.2134)
Transition point (250,700,700)	(62.1475, -32.2562, -27.4584, 0, 149.7146, 27.8525)
Target point (250,700,500)	(62.1475, -42.4355, -31.8720, 0, 164.3075, 27.8525)

3.2. Trajectory Simulation

3.2.1. Robot position display

The Fanuc m20id/25 robot was modelled using the Robotic toolbox function in MATLAB and the palletizing process was animated in four steps. In order to ensure the correctness of the simulation, *fkine* and *ikine* functions in MATLAB were used in the research to solve the joint angle and position of the robot. Fig. 3 shows the different positions of the robot in each step of palletizing.

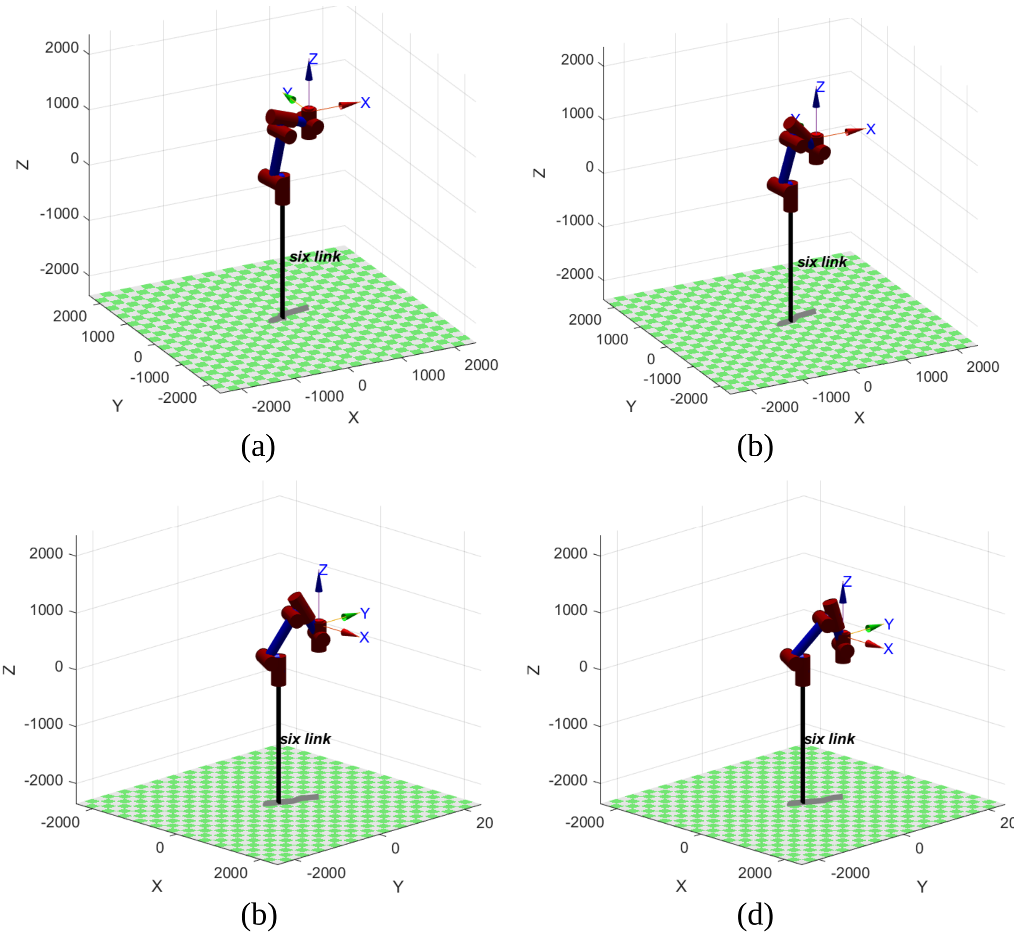


Fig 3. Simulation models of different positions. (a) Initial point
(b) pick-up point (c) transition point (d) target point. (Picture credit: Original).

3.2.2. Joint motion analysis

The variation of angular displacement, angular velocity, and angular acceleration of each joint of the palletizing robot could also be plotted using MATLAB. The *jtraj* function was used to determine the motion parameters and trajectories of each joint given the coordinates of the start and finish positions of the robot end [16].

The variation curves of the angle of each joint are shown in Fig. 4. It is clear to see that every curve is smooth, which means that fifth-degree polynomial curves were effective in resolving continuous changes in joint position. Whether there was a significant rise or fall in the curve, the whole process of change was smooth and without abrupt points [17]. The variations in joints 1 and 6 were larger. Because when the robot was palletizing, the two joints had the greatest impact on the control of the robot's end position. It's worth mentioning that joint 4 didn't change its position during the whole process to ensure the stable transportation of cartons.

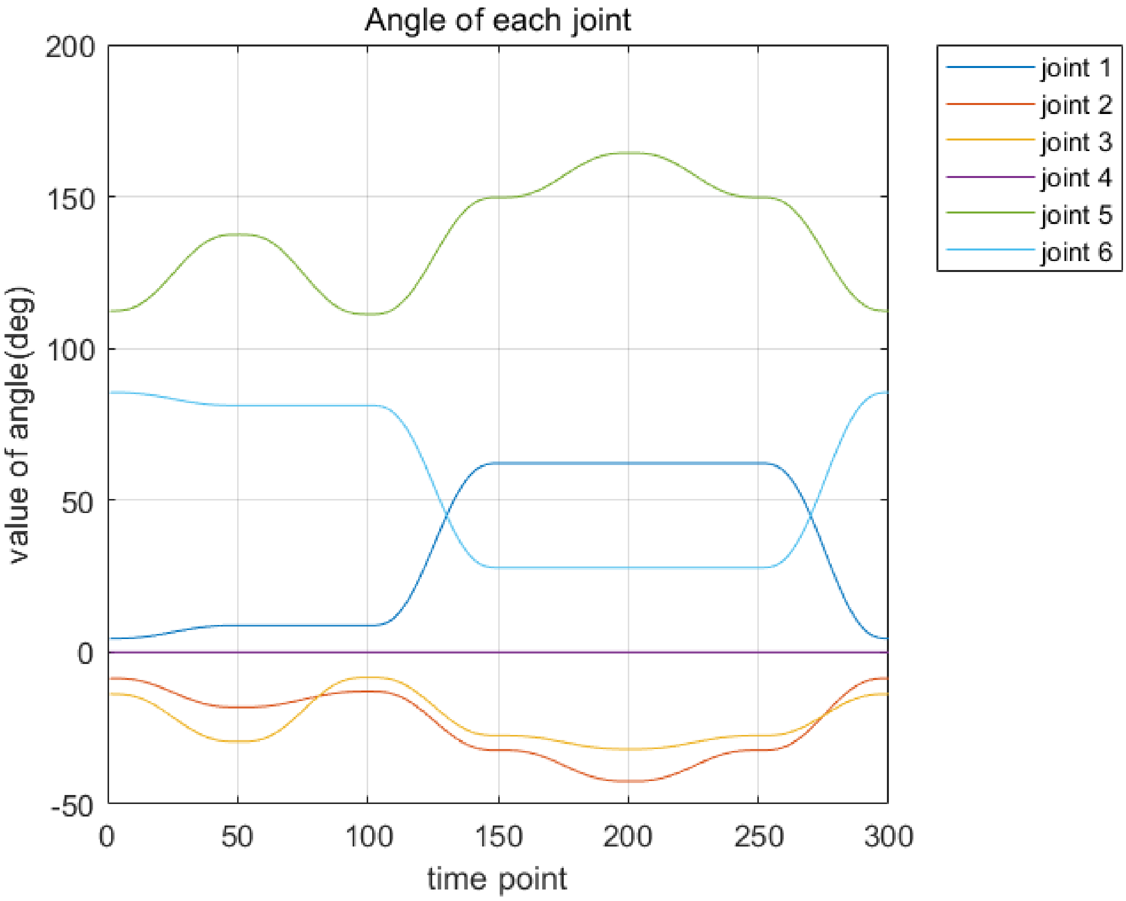
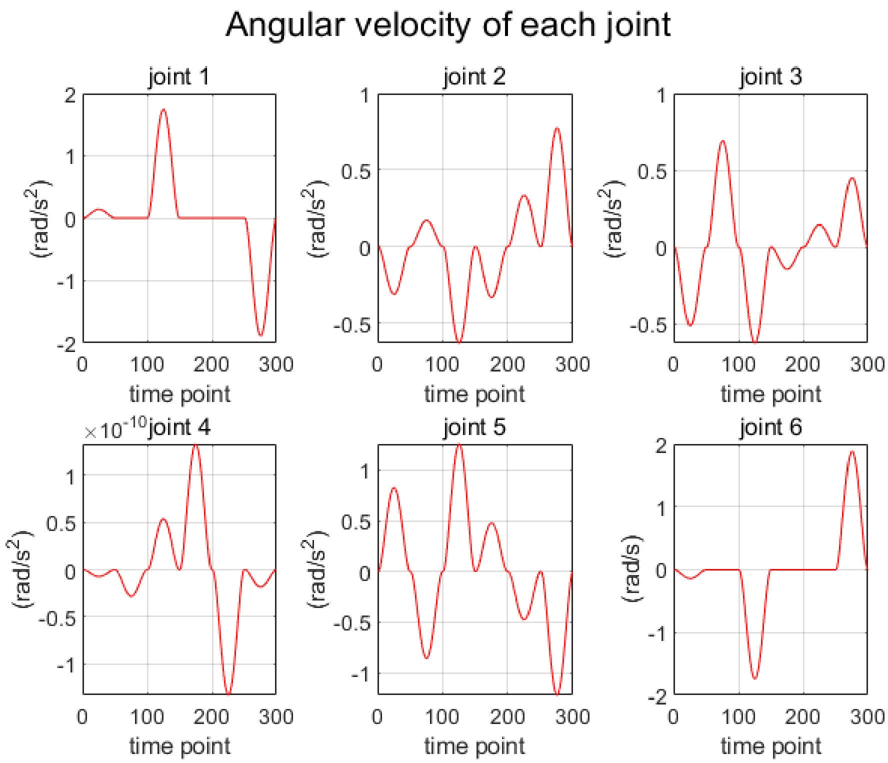
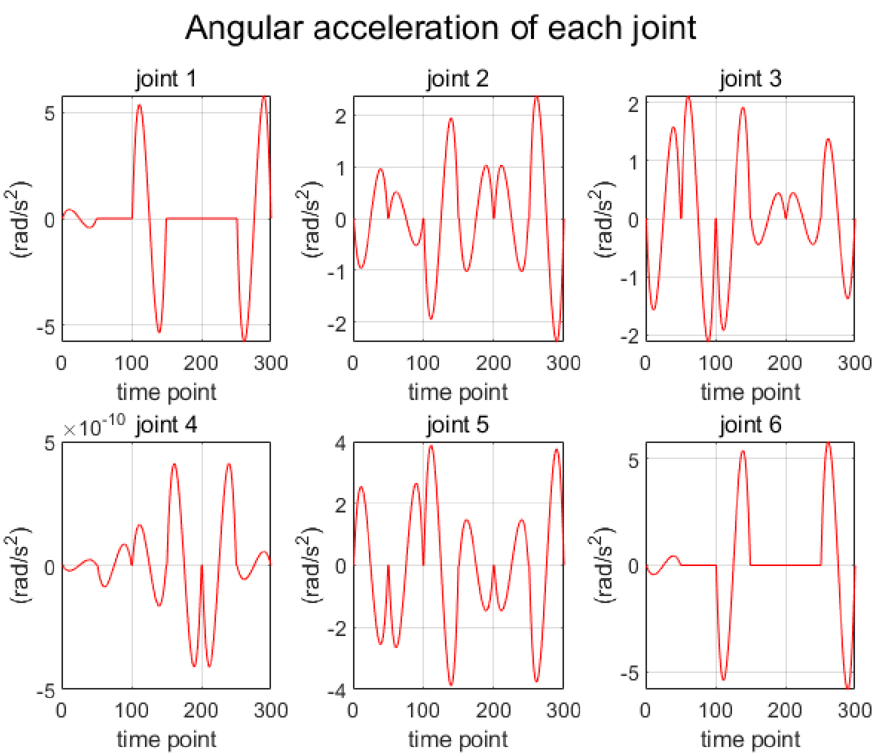


Fig 4. Angle curves of each joint. (Picture credit: Original).

The variation curves of angular velocity and angular acceleration of each joint are shown in Fig. 5. Overall, the change of the curves for these two parameters is more complex. The curves of joints 1 and 6 always had symmetry. According to the analysis of the kinematic properties, they were always moving in opposite directions and with the same value. Moreover, the curve itself was symmetric in 50 sampling points in each step. It becomes confusing when the curves of each stage are combined together. It also can be observed that there were some sharp points in the curves of angular acceleration. Each point of this kind was located at the junction of two steps. Each stage was independent of the other and had different path planning. Therefore, the existence of these sharp points was reasonable. These shark points represent the fact that at the end of each stage of the robot's movement, the acceleration of the motion can always be increased or decreased to zero. Therefore, there is no significant impact or inertia on the robot itself.



(a)



(b)

Fig 5. The curves of motion parameters of six joints

(a) Angular velocity curves (b) Angular acceleration curves. (Picture credit: Original).

Fig. 6 shows the trajectories of the end of the robot during the palletizing process. The six-axis robot's trajectories included sudden stops and step acceleration and deceleration. However, the robot's speed and acceleration were constantly changing, which improved the simulation of palletizing. There was also very little vibration in this case, and the wear and tear on the robot was slight.

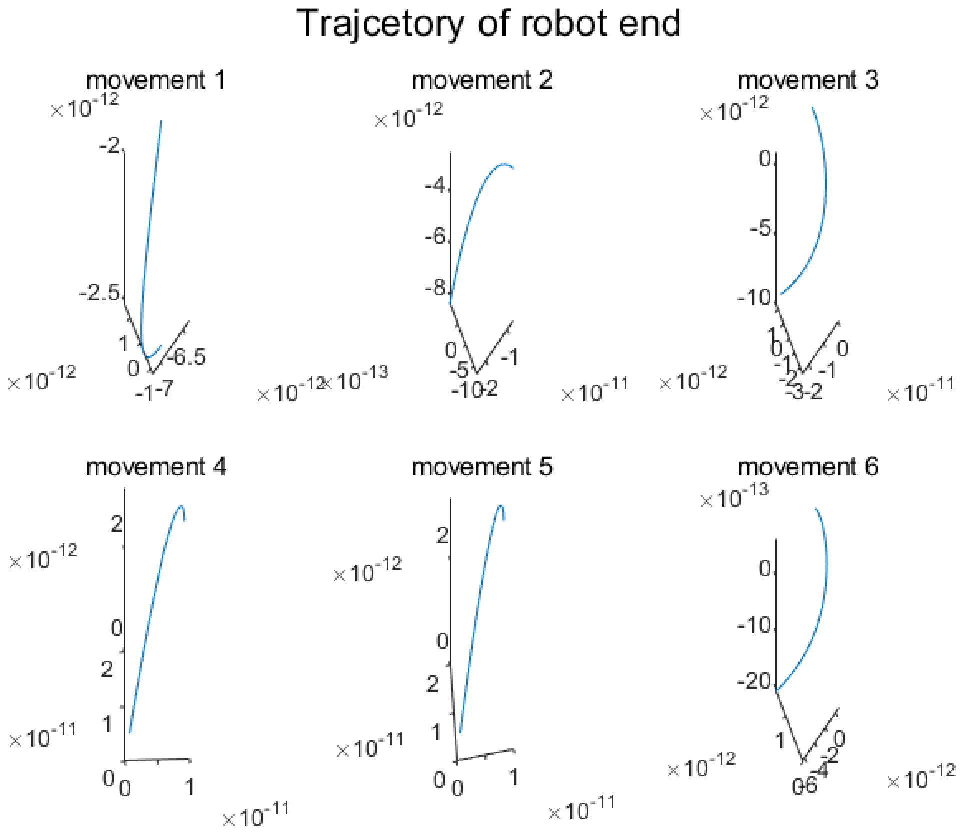


Fig 6. Trajectory of end of robot in six movements. (Picture credit: Original).

4. Conclusion

This research is to study the kinematic analysis and trajectory planning of a six-axis robot during palletizing with an industrial robot, Fanuc m20id/25. The kinematic coordinates are established for each joint of the robot, based on which the D-H parameter table of the robot is solved. The positive and inverse kinematic equations of the robot are analyzed according to the coordinate transformation matrix in three-dimensional space. In the paper, the fifth-degree polynomial curve is chosen as the solution function of path planning, which can ensure a more stable and continuous movement process of palletizing. And the coordinate positions of four working points are set for the whole process. The path curve analysis is carried out in stages, and finally, the animated demonstration of palletizing at different stages is successfully simulated. And the robot moves smoothly during the whole process without any influence or sudden change of motion. The variation of curves of the three motion parameters, which are respectively the angle, the angular velocity, and the angular acceleration, are analyzed qualitatively. These curves are smooth without any sudden points. All of the above can show that this palletizing simulation has certain practical reference value, and can well reflect the actual palletizing process in industrial applications. At the same time, it also shows that the fifth-degree polynomial curve improves the smoothness and continuity of the motion curve. However, the experiment of this research is for the simulation of specific types of robots and working points, which may have different effects on different working situations with various requirements. Therefore, in the future, it is necessary to continue in-depth research to further improve the flexibility of analysis methods and the efficiency of planning algorithms. It can apply to different types of palletizing robots as far as possible.

References

[1] Li Tuanjie, Zhang Yan, Zhou Jiaxing. Trajectory planning of high precision collaborative robots. Computer Modeling in Engineering & Sciences, 2019, 118(3): 583-598.

[2] Long Zhang, Li Xiantao, Shuai Tao, et al. Review of research state of trajectory planning for industrial robots. Mechanical Science and Technology for Aerospace Engineering, 2021, 40(06): 853-862.

[3] Wu Ke, Ping Xuecheng. Kinematic analysis and trajectory planning based on six axis robots. Modular Machine Tool & Automatic Manufacturing Technique, 2024, (07): 17-22.

[4] Xiong Gang, Ding Ye, Zhu Limin. A feed-direction stiffness-based trajectory optimization method for a milling robot. 10th International Conference on Intelligent Robotics and Applications, 2017, 10463: 184-195.

[5] Wu A X, Shi Z P, Li Y D, et al. Formal kinematic analysis of a general 6R manipulator using the screw theory. Mathematical Problems in Engineering, 2015, (02): 1-7.

[6] Zhang Yi-mian, Xu Fang, Du Zhen-jun, et al. Kinematic analysis and simulation of silicon wafer handling robot. Machinery Design & Manufacture, 2024: 1-11, https://link.cnki.net/doi/10.19356/ j. cnki.1001-3997.20240617.016.

[7] Ma L H, Zhong Y B, Wang G D, et al. Kinematic analysis of the robot having closed chain mechanisms based on an improved modeling method and lie group theory. Mathematical Problems in Engineering, 2020, (03): 1-16.

[8] Chen Jiupeng, Li Chunlei, San Hongjun, et al. Biomimetic trajectory planning and implementation of quadruped robots based on biological motion characteristics. Advanced Engineering Sciences, 2024: 1-16.

[9] Ren Jun, Wu Zheng-Hu, Cao Qiu-Yu. Trajectory planning and simulation of ER50 manipulator based on MATLAB robotics toolbox. Machinery Design & Manufacture, 2022, (08): 33-36.

[10] Gao R, Zhang W, Wang G F, et al. Experimental research on motion analysis model and trajectory planning of GLT palletizing robot. Buildings, 2023, 13(4): 966.

[11] Okan Duymazlar, Dilsad Engin. Boomerang algorithm based on swarm optimization for inverse kinematics of 6 DOF open chain manipulators. Turkish Journal of Electrical Engineering and Computer Sciences, 2023, 31(2): 342-359.

[12] Jia Lianhui, Liu Shenyao, Cao Chenxu, et al. Kinematics and spatial structure analysis of TBM gunite robot based on D-H parameter method. Scientific Reports, 2024, 14(1): 13649.

[13] Li Darui, Xiao Ping, Sun Yongjiu. Joint trajectory planning and simulation of six axis welding robot. Journal of Dongguan University of Technology, 2023, 30(03): 120-124.

[14] Özge Ekerm, Bekir Aksoy. Trajectory planning for a 6-axis robotic arm with particle swarm optimization algorithm. Engineering Applications of Artificial Intelligence, 2023, 122: 106099.

[15] Wang W J, Tao Q, Cao Y T, et al. Robot time-optimal trajectory planning based on improved cuckoo search algorithm. IEEE Access, 2020, 8: 86923-86933.

[16] Zhou Yang, Yang Qi, Yan Weijia. Kinematics analysis and simalution of six axis robots based on D-H parameters. Mechanical Engineering & Automation, 2024, (03): 56-58.

[17] Haiek D El, Aboulissane B, Bakkali L El, et al. Optimal trajectory planning for spherical robot using evolutionary algorithm. The 12th International Conference Interdisciplinarity in Engineering (INTER-ENG), 2018, 32: 960-968.