

Optimization Design of Heliostat Field Based on Particle Swarm Optimization

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Abstract. This paper discusses the optimal design model of circular heliostat field, aiming at improving the efficiency of tower solar power station. By calculating the output efficiency of heliostat field, a multi-objective programming model is constructed and solved by particle swarm optimization algorithm to optimize the parameters of the tower solar power station. The research focuses on solving the shadow occlusion loss efficiency, cosine efficiency and truncation efficiency, as well as using MATLAB and Excel to process a large number of data. Finally, the annual average optical efficiency and output thermal power are obtained, and the parameters of the absorption tower and heliostat are designed under the condition of 60MW, so as to maximize the annual average output thermal power per unit mirror area.

Keywords: Solid geometry model, multi-objective programming particle, swarm optimization algorithm.

1. Introduction

With the formulation of the national "14th Five-Year Plan" outline and the proposal of carbon peak and carbon neutrality, the research of new clean energy technology has been paid more and more attention, and the construction of a new power system with renewable energy as the main body is the focus topic. In this paper, by calculating the output efficiency of heliostat field [1], a multi-objective programming model [2] is constructed, and particle swarm optimization is used as the main method to solve [3], so as to set various parameters of the tower solar power station, so as to make the annual average output thermal power per unit mirror area as large as possible.

2. Parameter Analysis of Heliostat

2.1. Optical Efficiency of Heliostat

The optical efficiency of heliostat can be calculated according to the given heliostat size and installation height, referring to the relevant formula to calculate the corresponding parameters. This paper introduces the definition and calculation formula of optical efficiency, as well as the calculation methods of shadow blocking efficiency, cosine efficiency and truncation efficiency.

2.1.1 Solar declination Angle and solar hour Angle

To calculate the solar altitude Angle of the sun near 39 degrees north latitude on the 21st of each month and other relevant parameters, first determine the direct position of the sun at this point in time, which can be expressed by the solar declination Angle δ , which refers to the tilt Angle of the sun relative to the Earth's equator.

In the cycle of a year, the change range of the sun's direct point is the Tropic of Capricorn, that is $23^{\circ}26'$, the southern latitude to the Tropic of Cancer, that is $23^{\circ}26'$, the northern latitude, the change range of the sun's declination Angle can be approximated -23.5° to $+23.5^{\circ}$, and the value of the sun's declination Angle in a day is very small, and can be approximated as unchanged.

The solar hour Angle is the hour Angle of the center of the helion plane, that is, the angular distance from the observation point to the celestial meridian along the celestial equator to the time circle of the sun. The solar hour Angle is denoted ω , and this Angle can be calculated by longitude and local time.

Based on the local hours, minutes, we can use the following formula to calculate the solar hour Angle:

$$\omega = \frac{\pi}{12}(ST - 12) \quad (1)$$

2.1.2 Solar altitude Angle and solar azimuth Angle

The solar altitude Angle refers to the Angle between the incidence direction of sunlight and the ground plane, while the solar azimuth refers to the Angle between the projection of the sun's rays on the ground plane and the local meridian. We use it α_s to represent the sun's altitude Angle and γ_s to represent the sun's azimuth Angle, respectively. These two angles can be calculated by the sun declination Angle, solar hour Angle and latitude, at this time the latitude φ is represented by symbols, you can use the following formula to calculate the sun altitude Angle and solar azimuth Angle:

Sun altitude Angle:

$$\sin \alpha_s = \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \quad (2)$$

Sun azimuth:

$$\cos \gamma_s = \frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi} \quad (3)$$

2.1.3 Normal direct radiation irradiance

Normal direct radiation irradiance can be expressed in DNI (unit: kW/m^2), which refers to the solar radiation energy received per unit area and per unit time on the plane perpendicular to the sun's rays on the earth. The value DNI depends on factors such as atmospheric transparency, solar radiation intensity and the amount of ozone in the atmosphere [4]. The value of DNI is approximated by the following formula:

$$DNI = G_0 \left[a + b \exp \left(- \frac{c}{\sin \alpha_s} \right) \right] \quad (4)$$

$$a = 0.4237 - 0.00821(6 - H)^2$$

$$b = 0.5055 + 0.00595(6.5 - H)^2$$

$$c = 0.2711 + 0.01858(2.5 - H)^2$$

Where G_0 is the solar constant and its value is 1.366 kW/m^2 , H taken as the altitude (unit: km).

2.1.4 Optical efficiency of heliostat

The optical efficiency of a heliostat refers to the proportion of sunlight reflected by the heliostat as sunlight collected by the heliostat [5]. This efficiency depends on factors such as the design of the heliostat and the material of manufacture. It can be represented by symbols η .

The formula for optical efficiency of a heliostat is as follows:

$$\eta = \eta_{sb} \eta_{\cos} \eta_{at} \eta_{trunc} \eta_{ref} \quad (5)$$

Where: η_{sb} represents shadow blocking efficiency, and $\eta_{sb} = 1 - \text{shadow blocking loss}$; $\eta_{\cos}$ represents cosine efficiency, and $\eta_{\cos} = 1 - \text{cosine loss}$; η_{at} represents atmospheric transmission, and $\eta_{at} = 0.99321 - 0.0001176d_{HR} + 1.97 \times 10^{-8} \times d_{HR}^2 (d_{HR} \leq 1000)$; η_{trunc} represents the collector truncation efficiency; η_{ref} represents the reflectivity of the mirror, which can be a constant.

2.2. Total Output Power of Mirror Field

2.2.1 Calculation of shadow occlusion efficiency

The shadow occlusion loss mainly includes three parts: (1) the loss caused by the shadow of the absorber to the mirror field; (2) the loss caused by the back heliostat being blocked by the front heliostat when receiving sunlight; (3) the back heliostat is blocked by the front heliostat when reflecting sunlight and does not reach the blocking loss on the collector.

Because this paper assumes that there is no mutual occlusion between the adjacent heliostat, the shadow occlusion loss between the heliostat is not considered. The total height of the absorber and collector is mainly considered because of the shadow generated by the sun.

First of all, according to the latitude of the direct solar point to speculate the north latitude of the sun height Angle 39.4° , according to the calculation formula of the sun height Angle:

$$\alpha_i = 90^\circ - |39.4^\circ - \delta| \quad (6)$$

Which α_i is the sun height Angle of the 21st northern latitude of each month, δ is the sun declination Angle, the range is -23.5 to 23.5 , take negative value when directly in the southern hemisphere, take positive value when directly in the northern hemisphere.

Since the absorber is a known height of 84 meters, so the shadow length can be calculated by the tangent value of a right triangle to calculate the shadow length of the absorber tower, assuming that the shadow length Y is represented by a symbol, then:

$$Y = \frac{84}{\tan \alpha_i} \quad (7)$$

The shadow length of the absorber at 12 noon can be obtained by calculation.

According to the verified data, north latitude winter solstice sunrise 7:30 sunset 16:30; Summer solstice sunrise 4:30 sunset 19:30; Equinoxes sunrise 6:00 sunset 18:00. Therefore, continue to use the symmetry of time and the symmetry of the sun moving back and forth between the Tropic of Cancer, you can find the shadow length of the absorption tower at 9 am and 10:30 am.

According to the symmetry of time, we can get the same number of heliostats blocked at 9 am and 15 PM; The number of heliostats blocked at 10:30 am and 13:30 PM is the same. And there is no heliostat within 100 meters, and the distance between the center of the adjacent heliostat base is more than 5 m more than the width of the mirror, and because the heliostat is a square with a side length of 6 meters, more than 100 meters according to the adjacent 11 meters of each heliostat. The average shielding efficiency and the average annual shielding efficiency of the 21st day of each month can be calculated.

2.2.2 Calculation of cosine efficiency

In theory, the magnitude of the cosine efficiency is related to the Angle of incidence. From the model, the magnitude of the cosine efficiency is related to the position of the heliostat.

As shown in Fig. 1, x , y , and z represent the three-dimensional coordinate system, with the X-axis being due east and the Y-axis being due north; Hn point is the center of the heat absorber; An , Bn , Cn and Dn represent the 4 apex angles of the heliostat at any position respectively, and $0n$ is the center point of the heliostat reflector; $0nTn$ is the incident direction of light, $0nHn$ is the reflection direction of light; $TnEn$ and $Hn0$ are perpendicular lines of the plane $x-y$, En and 0 are perpendicular lines.

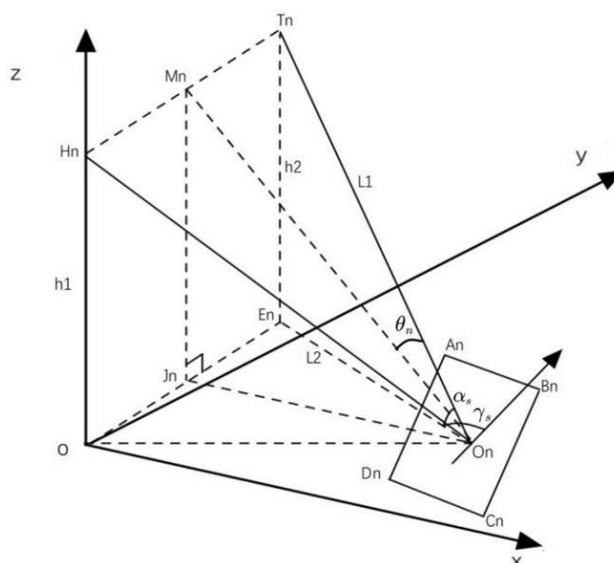


Figure 1. Schematic illustration of the incidence and reflection angles of a heliostat

Let $|0nHn| = |OnTn| = L1$, $TnEn = h2$, $0nEn = L2$, the coordinates $(x_n, y_n, 0)$ of the point be En , the coordinates $(x'_n, y'_n, 0)$ of the point $0n$, the triangle $Tn0nHn$ is the isosceles triangle, and $OnMn$ bisects the $TnHn$ vertically.

$$\begin{aligned} h_2 &= L_1 \sin \alpha_s \\ (y_n - y_n') / L_2 &= \cos \gamma_s \end{aligned} \quad (8)$$

Where: α_s, γ_s respectively are the sun altitude Angle and the azimuth Angle; θ_n is the human Angle of the nth heliostat.

The solar azimuth and altitude angles are related to geographical latitude, the sun's hour Angle, and declination.

$$\sin \alpha_{\varsigma} = \sin \delta \sin \varphi + \cos \delta \cos \varphi \cos w \quad (9)$$

Cosine efficiency of simultaneous available heliostat:

$$\eta_{cn} = \cos \theta_n = \frac{(L1 + \sin \alpha_s h1 + x_n \cos \alpha_s \sin \gamma_s + y_n \cos \alpha_s \sin \gamma_s)^{1/2}}{(2dn)^{1/2}} \quad (10)$$

Where: $L1 = (h1^2 + x_n^2 + y_n^2)^{1/2}$.

2.2.3 Calculation of truncation efficiency

Due to the mirror's tracking error, light scattering, sunlight cone propagation, sun shape error and other factors, the sunlight reflected by the heliostat can not be all received by the collector, resulting in some energy loss. In this paper, the HFLCAL model is used to calculate the truncation efficiency. The advantage of this model is that the calculation speed is faster and the simulation accuracy is more guaranteed. The circular Gaussian distribution is used to represent the energy flux density function of the surface of the heat absorber. The calculation formula is as follows:

$$\eta_{t \text{ trunc}} = \frac{1}{2\pi\sigma_{\text{tot}}^2} \int_x \int_y \exp\left(-\frac{x^2 + y^2}{2\sigma_{\text{tot}}^2}\right) dx dy \quad (11)$$

Where σ_{tot}^2 is the total standard deviation of the spot on the surface of the heat absorber, and its influencing factors include reflection distance d , heliostat focal length f , sun shape error standard deviation σ_{sun} , beam quality error standard deviation σ_{bq} , astigmatism error standard deviation σ_{ost}

and tracking error standard deviation σ_{track} . The specific calculation formula of each parameter is as follows:

$$\begin{aligned}\sigma_{\text{tot}} &= \sqrt{d(\sigma_{\text{sun}}^2 + \sigma_{\text{bq}}^2 + \sigma_{\text{ast}}^2 + \sigma_{\text{track}}^2)} \\ \sigma_{\text{ast}}^2 &= \sqrt{0.5(H_t^2 + W_s^2) / 4d} \\ H_t &= \sqrt{LW \times LH} \left| \frac{d}{f} - \cos\omega \right| \\ W_2 &= \sqrt{LW \times LH} \left| \frac{d}{f} \cos\omega - 1 \right|\end{aligned}\quad (12)$$

σ_s is the standard deviation of the slope error; H_t and W_s are the dimensions of the light spot on the heat absorber in meridian direction and sagittal direction; LH and LW are the height and width of the heliostat; f is the axial focal length of the heliofix, and in this case, if the heliofix is a flat mirror, f tends to infinity; ω is the incidence Angle of light rays.

According to the reference data given in the PSA:

$$\sigma_{\text{sun}} = 2.51\text{mrad}, \sigma_s = 0.94\text{mrad}, \sigma_{\text{track}} = 0.63\text{mrad} \quad (13)$$

2.2.4 Calculation of atmospheric transmittance

By the formula of atmospheric transmittance:

$$\eta_{\text{at}} = 0.99321 - 0.0001176d_{\text{HR}} + 1.97 \times 10^{-8} \times d_{\text{HR}}^2 \quad (d_{\text{HR}}, 1000) \quad (14)$$

And:

$$d_{\text{HR}} = \sqrt{\Delta h^2 + x^2 + y^2} \quad (15)$$

Use Excel and MATLAB to batch the data, can be obtained.

2.2.5 Calculate the total output power of the mirror field

The total output power of the mirror field refers to the total amount of solar energy collected and reflected by the entire mirror field. According to the area and optical efficiency of each heliostat, the output power of each heliostat can be calculated, and the total output power of the mirror field can be obtained by summing the output power of each heliostat.

The following formula can be used to calculate the total output power of the mirror field:

$$E_{\text{field}} = DNI \cdot \sum_i^N A_i \eta_i \quad (16)$$

Where DNI is the irradiance of normal direct radiation; N is the total number of heliostats (unit: face); A_i is the daylighting area of the first heliostat (unit: m²); η_i is the optical efficiency of the first mirror.

2.2.6 Calculate the total output power of the mirror field

The average output thermal power per unit area of the mirror refers to the output power emitted per square meter of the mirror area. Divide the power generated by the entire mirror site by the total area of all mirrors in the mirror field to obtain the average output thermal power of the mirror per unit area. As known in Table 1, take the 21st of each month as the average value of the month, and then calculate the average optical efficiency and output power table.

Table 1. Problem annual average optical efficiency and output power meter

Annual Mean Optical efficiency	Annual mean Cosine efficiency	Average annual shadow Occlusion efficiency	Annual average Truncation efficiency	Average annual output Thermal power (MW)	Average annual mirror output per unit area Heat output (kW / m ²)
0.6227	0.7626	0.9888	0.9304	37.8585	0.6027

3. Parameter Optimization Model Based on Particle Swarm Optimization Algorithm

3.1. Establishment and Solution of the Model

The rated average annual output thermal power is 60 MW, and the optimization target is the relevant parameters of the absorber and collector, which belongs to the multi-objective programming problem. Combined with the established models and relevant formulas about various efficiencies, the expression of the average output power of the heliostat field is obtained. After establishing the multi-objective programming model, we solve it by particle swarm optimization algorithm [6]. To get the optimal parameters.

3.1.1 Establishment of the model

The unit output efficiency of heliostat directly affects the efficiency of the whole system, so it is very important to optimize the arrangement of heliostat field. Under normal circumstances, the absorption tower placed in the center of the circular area can reduce the shadow occlusion loss and cosine loss, so as to improve the average annual output of heat power per unit circular mirror area. In this paper, assuming that the ground is flat, and the circular area is not blocked by other obstacles, the absorption tower can be placed in the center of the circular area, and the design and optimization process can be simplified. The independent variable is: heliostat side length a ; Heliostat mounting height h ; The number of heliostats n ; Heliostat coordinates (x_i, y_i) .

1) Objective function

Determine the objective function of the problem: Our goal is to maximize the annual average thermal power output per unit mirror area. Set the objective function as $f(x_1, x_2, x_3, x_4)$, where x respectively represent the design parameters of the heliostat field (heliostat side length a , installation height h , number of heliostat n , heliostat position (x_i, y_i) , etc.

2) Optimization objective

$$\max \left(E_{fa} = \frac{E_{\text{field}}}{n \cdot a \cdot b} \right) \quad (17)$$

3) Constraint condition

Determine the range of values of design parameters: Determine the range of values of each design parameter; the distance between heliostats is at least 5 metres more than the length of the sides of the heliostat; the length of the sides of the heliostat is between 5 metres and 8 metres; the height of the heliostat installation is between 2 metres and 6 metres; the average output thermal power of the field of the heliostat is more than 60MW; the coordinates of the heliostats range from 100 metres to 350 metres in radius within the circle.

$$\text{s.t.} \begin{cases} 5 + a \leq \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \\ 5 \leq a \leq 8 \\ 2 \leq h \leq 6 \\ E_{\text{field}} \geq 60 MW \\ 100 \leq \sqrt{x_i^2 + y_i^2} \leq 350 \end{cases} \quad (18)$$

3.1.2 Solution of the model

Particle Swarm Optimization (PSO) is an optimization algorithm based on swarm intelligence, inspired by the rows of animal groups such as swarms and fish schools [7]. Pso is based on searching a population of particles and improves candidate solutions by iteratively trying a given mass measure. Its core new idea is to solve optimization problems by simulating collaboration and information sharing in swarm intelligence [8].

Ideas to solve the above optimization problems:

- 1) Define the objective function: Our goal is to maximize the average output thermal power per unit area of the mirror.
- 2) Determine the constraints: According to the conditions of the problem, determine the constraints, including the distance constraints between the position of the absorber and the heliostat, the distance constraints between the heliostat, the average output thermal power constraints of the heliostat field, etc.
- 3) Initialize the particle swarm: Randomly generate a group of particles, each representing a solution (i.e., the design parameters of the heliostat field), and give an initial velocity and position.
- 4) Calculate fitness function: For each particle, calculate its fitness value, i.e., calculate the value of the objective function.
- 5) Update the speed and position of the particles: Update the speed and position of each particle according to the formula of the particle swarm optimization algorithm to search for a better solution.
- 6) Update the historical optimal solution of the particle swarm: For each particle, compare its current fitness value with its historical optimal fitness value, and update the historical optimal fitness value and location.
- 7) Update the global optimal solution of the particle swarm: Compare the fitness values of all particles to find the optimal solution, and update the global optimal solution.
- 8) Determine the termination condition: Determine whether the termination condition is met, such as reaching the maximum number of iterations or the objective function value converges to a certain threshold.
- 9) Repeat steps.
- 10) Output optimal solution.

3.2. Optimization and Analysis of Model

On the basis of the model establishment, two constraints can be added, that is, the heliostats size can be different, the installation height can be different, still according to the above ideas of modeling and solving. However, in order to simplify the calculation, the heliostat is still set as a square

The constraints are added as follows:

$$\begin{cases} 5 + b \leq \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \\ 5 + b \leq \sqrt{(x_i - x_{i-1})^2} \\ 5 + b \leq \sqrt{(y_i - y_{i-1})^2} \\ 5 \leq a < b \leq 8 \\ 2 \leq h \leq 6 \\ E_{\text{field}} \geq 60 MW \\ 100 \leq \sqrt{x_i^2 + y_i^2} \leq 350 \end{cases} \quad (19)$$

The above solution process, the optimal solution is obtained, as shown in Table 2.

Table 2. Optimized heliostat size, installation height, position coordinates

Serial Number	Heliostat width	Heliostat height	Heliostat x coordinates	Heliostat y coordinates	Heliostat z coordinates
1	6	6	103	0	4
2	6	6	102.396	11.136	4
3	6	6	100.591	22.141	4
4	6	6	97.608	43.248	4
:	:	:	:	:	:
:	:	:	:	:	:
:	:	:	:	:	:
1295	7.96	7.96	253.146	-38.781	4
1296	7.96	7.96	254.786	-25.909	4
1297	7.96	7.96	255.771	-12.971	4

4. Summary

In this paper, the optimization design of circular heliostatic mirror field is deeply discussed, and the multi-objective programming model is successfully constructed. The model comprehensively considers many factors such as shadow occlusion loss, cosine efficiency and truncation efficiency, aiming to maximize the annual average output thermal power per unit mirror area by optimizing parameters such as the size, installation height, number and position of the heliostatic mirror. With the help of particle swarm optimization algorithm, we get the optimal design parameter configuration of heliostat field under the constraint condition of rated power of 60MW. This research result is of great significance for improving the energy conversion efficiency of tower solar power station and promoting the development and application of clean energy technology.

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