

Simulation Study of Motion Processes Based on Isometric Solenoidal Equation and Differential Evolutionary Algorithm

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Abstract. This paper mainly focuses on the position and speed changes during the traveling process of the bench dragon, collision detection, optimization of the turning path and the maximum traveling speed of the dragon head. Using the equidistant solenoidal equation and differential evolution algorithm, the key position coordinate model of each part of the bench dragon team is established and collision detection is carried out. Firstly, this paper deduces the movement path and speed of the dragon head through the solenoidal equation; secondly, based on the collision detection model, it analyzes the collision moments and conditions of the bench dragons in the marching; lastly, through the optimization of the turnaround path and the calculation of the speed, it determines the optimal turnaround path. The study shows that the model can effectively solve the optimization problem in the marching process of the complex dancing dragon team.

Keywords: Isometric screws, collision detection, turnaround paths, bench dragons.

1. Introduction

As a local traditional folklore activity, the bench dragon not only has profound cultural value, but also shows complex geometric marching characteristics in kinematics. In order to ensure the smoothness and safety of the dragon dance, avoid collision between benches, and successfully complete the turnaround in the limited space, we scientifically calculate and analyze the traveling path, speed change and collision of the dragon dance team. Firstly, we establish the mathematical model of the traveling process of the bench dragon and describe the traveling path of the dancing dragon team by combining the equidistant solenoidal equation [1-2]. Then, the position and speed of each bench in the traveling process are determined by the collision detection model, and we conduct real-time collision tests [3-4]. Finally, in order to achieve the condition of the maximum speed of the dragon's head to keep the dragon dance smooth, we also determine the optimal turning path and traveling speed of the bench dragon through the optimization of the turning path [5-6].

2. Traveling path analysis based on equidistant solenoidal equations

We first collected data on a certain bench dragon through field research. The bench dragon consists of 223 sections, of which the first section is the dragon's head, the next 221 sections are the dragon's body, and the last section is the dragon's tail. The length of the dragon's head is 341 cm, the length of the dragon's body and tail is 220 cm, and the width of all the benches is 30 cm. each bench has two holes, the diameter of which is 5.5 cm, and the center of the holes is 27.5 cm away from the head of the nearest bench. two adjacent benches are connected by handles, and a front view of the dragon is shown in Fig. 1.

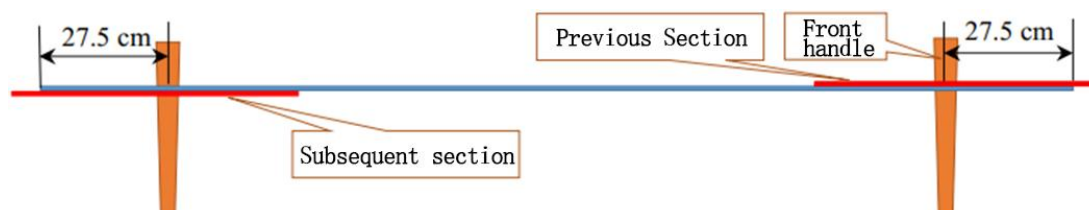


Figure 1. Front view of bench dragon

According to the research, the dragon dance team was coiled clockwise along an isometric screw thread with a pitch of 55cm, and the center of each handle was located on the screw thread. The traveling speed of the handles in front of the dragon's head was always 1 m/s. Initially, the dragon's head was located at the tangent point on the right side of the 16th circle of the thread. Disregarding the width of the bench, the bench dragon is regarded as several line segments, according to which the traveling route of the dragon dance team can be simulated by the equation of the spiral line. Establishing the coordinate axes, the equidistant solenoidal polar coordinate equation can be determined as:

$$r(\theta) = a + b\theta \quad (1)$$

Where, $r(\theta)$ for the helix pole diameter; a for the initial radius of the screw thread, here $a = 0$; b for the relationship between the pitch and angle of the screw thread, the pitch p is 0.55 m, so $b = \frac{0.55}{2\pi}$; θ for the pole angle, that is, the point on the screw thread rotated around the center of the angle; faucet position, that is, the angle at the point A is $\theta = 16\pi$.

Then the coordinates of the position of the point on the solenoid in the plane rectangular coordinate system are When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question.

$$\begin{cases} x = (a + b\theta) \cos\theta \\ y = (a + b\theta) \sin\theta \end{cases} \quad (2)$$

The position of the center of each handle of the bench dragon can be determined from the above equation. The velocity of the corresponding node can be calculated by numerical differentiation as follows:

$$v(t) = \frac{d}{dt} \sqrt{x(t)^2 + y(t)^2} \quad (3)$$

Where $v(t)$ is the velocity of the bench at the time. $(x(t), y(t))$ can be expressed as the coordinates of the position of the handle center of a bench at the time t .

The movement of each section of the bench are based on the trajectory of the previous section of the bench, the bench dragon led by the dragon's head to march, and thus can be calculated by the different moments of the position of the dragon's head of the bench to introduce the position of the center of the handle of each subsequent bench. To the i -th dragon body of the first bench, for example, known as the location of the center of the front handle coordinates $(x_{i,front}, y_{i,front})$, the request of the bench after the handle location coordinates $(x_{i,rear}, y_{i,rear})$, the two meet the relationship between the following formula:

$$\sqrt{(x_{i,front} - x_{i,rear})^2 + (y_{i,front} - y_{i,rear})^2} = 2.2 - 0.275 * 2 \quad (4)$$

The center of the front and rear handles $(x_{i,rear}, y_{i,rear})$ are located on the screw line, combined with the equations of the right-angle coordinate system of equidistant screw lines, i.e., Equation (2) and the above equation, the positional coordinates of the center of the rear handle can be obtained.

According to the initial position and velocity of the dragon head, using the above solenoidal equations, we dynamically simulate the position and velocity changes of the dragon head and each section of the dragon body and tail.

Assuming that the dancing dragon team has been in a disc shape before the beginning of the disc entry, Fig. 2 shows the simulation of the traveling route of the dancing dragon team at each moment, the red-blue overlap indicates the route that the dancing dragon team has already traveled on the spiral line, and the blue part of the inner circle where no overlap occurs indicates the route that the dancing dragon team has not yet passed through.

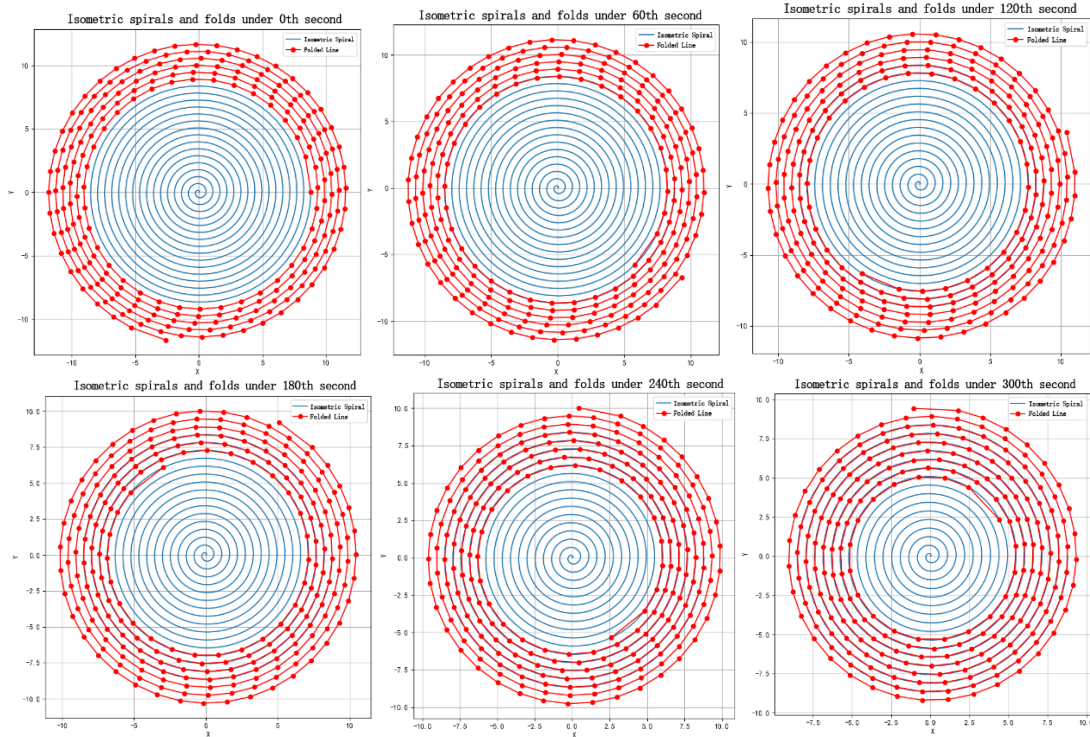


Figure 2. The marching route of the dragon dance teams at each moment

3. Collision Determination and Detection Model Based on Traveling Route

During the process of coiling the bench dragon along the isometric spiral, the radius of the path of the dragon head may be too small, resulting in the collision of the benches of different sections. In order to avoid collision, it is necessary to detect the position coordinates of the four corners of the benches to determine whether the benches are in contact or not. In this paper, we simulate the disk-in process of the dragon dance team, update the position of the benches in real time, and conduct the collision test. Starting the detection in a specific range, gradually reducing the step size, the bench for the first time to meet the discriminatory conditions of collision occurrence, determine this time for the termination of the marching moment. In addition, when the dragon cannot travel along a given spiral path, the disk-in is also terminated.

Considering the i -th bench as a rectangle, the coordinates of the four corners of the first bench are (x_{i1}, y_{i1}) , (x_{i2}, y_{i2}) , (x_{i3}, y_{i3}) , (x_{i4}, y_{i4}) , $i \in [0, 222]$ determined from the known length and width data of the bench and the position of the center of the handle on the solenoid, which is $i = 1$ for the dragon's head bench, $i = 222$ for the first section of the dragon's body bench, and so on for the dragon's tail bench.

Since the first i -th bench is head to tail with neighboring benches, they do not collide. The benches after the dragon's head are traveling along the path of the dragon's head, and as the radius of the spiral gradually decreases, the first collision is between the dragon's head and one of the benches after the dragon's head. The horizontal and vertical coordinates of each of the four corners of the bench A_i , B_i are represented as 2×2 matrices:

$$A_i = \begin{bmatrix} x_{i1} & x_{i2} \\ x_{i3} & x_{i4} \end{bmatrix}, B_i = \begin{bmatrix} y_{i1} & y_{i2} \\ y_{i3} & y_{i4} \end{bmatrix}, i \in [0, 222] \quad (5)$$

Where A_i denotes a matrix consisting of horizontal coordinates and B_i denotes a matrix consisting of vertical coordinates. When $i = 0$, that is to say, the leading bench four corners of the horizontal coordinates, vertical coordinates of the respective composition of the 2×2 matrix.

When the rectangles specified in the four corners overlap each other (except for the front and rear rectangles), it is considered that the bench collides with other benches except for the head-to-tail benches, and the corresponding discriminative conditions are as follows:

$$\begin{cases} \max x_{A_i} < \min x_{A_j} \\ or \\ \max x_{A_i} < \min x_{A_i} \end{cases} \quad or \quad \begin{cases} \max y_{B_i} < \min y_{B_j} \\ or \\ \max y_{B_i} < \min y_{B_i} \end{cases}, \quad j - i > 1 \quad (6)$$

Where $\max x_{A_i}$, $\max x_{A_j}$, $\max y_{B_i}$, $\max y_{B_j}$ denote the maximum values of the horizontal and vertical coordinates of the coordinates of the four corners of the first i -th bench and the j -th first bench respectively; and $\min x_{A_i}$, $\min x_{A_j}$, $\min y_{B_i}$, denote the minimum values of the horizontal and vertical coordinates of the corresponding points.

4. U-turn path optimization based on geometric analysis

The Dragon Dance Team will switch from disk-in to disk-out, and from clockwise disk-in to counterclockwise disk-out, which requires a certain amount of headroom. After investigation, the pitch of the coil-in screw thread is 1.7m, the head-turning space is a circular area with the center of the screw thread as the center and the diameter of 9m, and the head-turning path is an S-shaped curve connected by the tangency of two circular arcs. In this chapter, we discuss the shape of the turnaround curve and adjust the circular arcs to make the turnaround curve shorter.

We first discussed the shape of the turnaround head curve, the turnaround path is an S-shaped curve formed by tangentially connecting two circular arcs, the ratio of the radii of the two arcs (the adjustment of the shape) does not affect the length of the turnaround curve, and the length of the turnaround curve is a constant value in this case. The following is an illustration of the conclusion of the argument.

Any two parallel lines A 、 B , respectively, take the point, so that AB perpendicular to the two lines, the vertical foot are points A 、 B . Set $AB = d$. By all parts are tangent, easy to prove that the length of the curve is πd .

Any two parallel lines, any take two points L 、 J 、 LJ on the line, not perpendicular to the two lines. Make two straight lines of the perpendicular line, the perpendicular foot I 、 K for the point, respectively, and intersect another straight line at the point I , K . The shape of the curve of the head is shown in Fig. 3, easy to prove that the trajectory of the M two tangent points of the circle is LJ , and $\triangle LNM \sim \triangle JOM$. At this time, the radius r and the sum of intercepts d is $LM + MJ = LJ$, intercepts proportion are known.

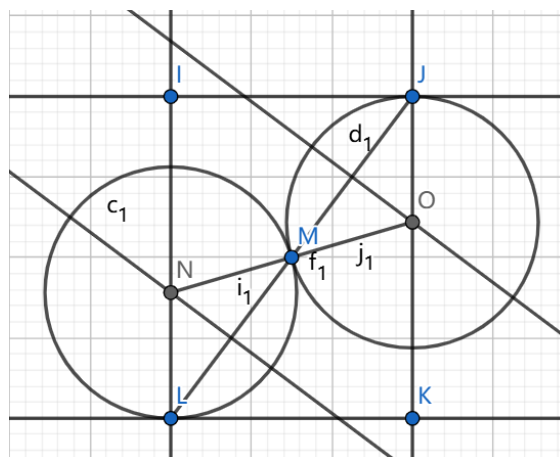


Figure 3. Shape of turnaround curve

We calculate the perpendicular distance from the center of the circle to the chord h . The formula is $h = \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$; Calculate the angle of the center of the circle α . Using the general cosine value of the central angle of a circle $\cos\left(\frac{\alpha}{2}\right) = \frac{r-h}{r}$, we introduce the central angle of the circle $\alpha = 2 \cdot \cos^{-1}\left(\frac{r-h}{r}\right)$; Calculate the length of the arc of the circle C . $C = r \cdot \alpha$. The length of the arc is calculated. It is easy to know that the length of the turnaround curve at this time is a constant value.

Let the angle of the two rays for 2α , point O for the intersection of the two rays. To the origin O , one of the rays for the x axis to establish the plane coordinate system. Let the radius of the circle D is r_1 , the radius of the circle E is r_2 , the tangent point of the two circles is $P(x, y)$. According to the tangent nature of the circle, can be obtained:

$$OD = \frac{r_1}{\sin \alpha}, \quad OE = \frac{r_2}{\sin \alpha} \quad (7)$$

Because the two circles are tangent to each other, so $DP = r_1 + r_2$, $EP = r_2 + r_1$, that is $DP = EP$. According to the formula for the distance between two points, we have $DP^2 = \left(x - \frac{r_1}{\sin \alpha}\right)^2 + y^2$, $EP^2 = \left(x - \frac{r_2}{\sin \alpha}\right)^2 + y^2$. From the cause $DP = EP$, the following relationship is deduced:

$$\begin{aligned} \left(x - \frac{r_1}{\sin \alpha}\right)^2 + y^2 &= \left(x - \frac{r_2}{\sin \alpha}\right)^2 + y^2 \\ x^2 - 2x \frac{r_1}{\sin \alpha} + \frac{r_1^2}{\sin^2 \alpha} + y^2 &= x^2 - 2x \frac{r_2}{\sin \alpha} + \frac{r_2^2}{\sin^2 \alpha} + y^2 \\ 2x \frac{r_2 - r_1}{\sin \alpha} &= \frac{r_2^2 - r_1^2}{\sin^2 \alpha} \\ x &= \frac{r_2 + r_1}{2 \sin \alpha} \end{aligned} \quad (8)$$

Because $DP = r_1 + r_2$, $OP = DP - OD = r_1 + r_2 - \frac{r_1}{\sin \alpha} = r_2 \left(1 + \frac{1}{\sin \alpha}\right)$. According to the collinear theorem, the following relations can be deduced:

$$\begin{aligned} OP^2 &= x^2 + y^2 \\ r_2^2 \left(1 + \frac{1}{\sin \alpha}\right)^2 &= \frac{(r_2 + r_1)^2}{4 \sin^2 \alpha} + y^2 \\ y^2 &= r_2^2 \left(1 + \frac{1}{\sin \alpha}\right)^2 - \frac{(r_2 + r_1)^2}{4 \sin^2 \alpha} \\ y^2 &= r_2^2 \left(\frac{\sin \alpha + 1}{\sin \alpha}\right)^2 - \frac{(r_2 + r_1)^2}{4 \sin^2 \alpha} \end{aligned} \quad (9)$$

Therefore, the equation of the trajectory of the point of tangency of the two circles is $y^2 = r_2^2 \left(\frac{\sin \alpha + 1}{\sin \alpha}\right)^2 - \frac{(r_2 + r_1)^2}{4 \sin^2 \alpha}$. It is easy to prove that the turnaround path curve is constant at this point. Therefore, other conditions remain unchanged, adjusting the ratio of the radii of the two arcs can not make the turnaround curve shorter.

It is known that the head-turning space is a circle with a radius of 4.5 m, and the inward and outward threads are centrosymmetric, so the pitch of both threads is 1.7 m. It is determined that the point where the inward thread intersects with the head-turning space is the starting point of head-turning. For the aesthetic value and ornamental considerations of the plate dragon, it is assumed that the starting point and ending point of the turning path of the bench dragon are also centrosymmetric

about the center of the screw line, so that the location of the ending point in the plate out of the screw line can be determined.

Initial radius $a = 0$, by the center of symmetry, it is easy to know that the disk into the disk out of the screw line equation is:

$$r_{in} = b\theta, \quad r_{out} = -b\theta \quad (10)$$

Where $b = \frac{1.7}{2\pi}$, the former is the disc-in solenoidal polar equation and the latter is the disc-out solenoidal polar equation.

First, calculate the tangent slopes of the two solenoids k :

$$k = \frac{dy}{dx} \quad (11)$$

Through the tangent line combined with the solenoid's positional equation in the plane rectangular coordinate system, the tangent equation of the solenoid can be found to be:

$$p = k(q - b\theta \cos \theta) + b\theta \sin \theta \quad (12)$$

Where p, q are respectively the horizontal and vertical coordinates of the point on the tangent line of the spiral. Through the tangent point of the spiral tangent line for the tangent line, get the equation of the vertical line is:

$$m = -\frac{1}{k}(n - b\theta \cos \theta) + b\theta \sin \theta \quad (13)$$

Where m, n are respectively the horizontal and vertical coordinates of the points on the vertical line. As the center of the two arcs of the turnaround path are located in the tangent line of the spiral tangent line, and the two radii of the circle to meet the multiplier relationship:

$$r_1 : r_2 = 2 : 1 \quad (14)$$

Where r_1 denotes the radius of the circle where the arc before the turnaround path is located, and r_2 denotes the radius of the circle where the arc after the turnaround path is located. The condition is satisfied when the two circles are tangent:

$$\sqrt{(n_1 - n_2)^2 + (m_1 - m_2)^2} = r_1 + r_2 \quad (15)$$

Where $(n_1, m_1), (n_2, m_2)$ respectively denote the positions of the centers of the two circles on the corresponding vertical lines.

When the two circles are tangent, it is necessary to further judge whether there is a collision between the benches, i.e., whether the dragon dance team can smoothly enter the disk out of the screw line in accordance with the path of turning around. It is easy to know that if the dragon head can successfully leave the smaller arc (i.e., the second section of the arc tangent to the disk out screw line) and enter the disk out screw line all the time, then the dragon head must be able to pass the larger arc (i.e., the first section of the arc tangent to the disk in screw line) smoothly.

Since the dragon's head is the longest section in the dragon dance team, if the dragon's head can successfully turn its head and coil out from the coil-out screw line, the dragon's body and tail will be able to pass through the path smoothly. Therefore, take a point on the plate out of the screw line, with the point as the center of the circle, the distance between the two aperture centers of the dragon's head bench 2.86 m for the radius of the circle, as shown in Fig. 4 below.

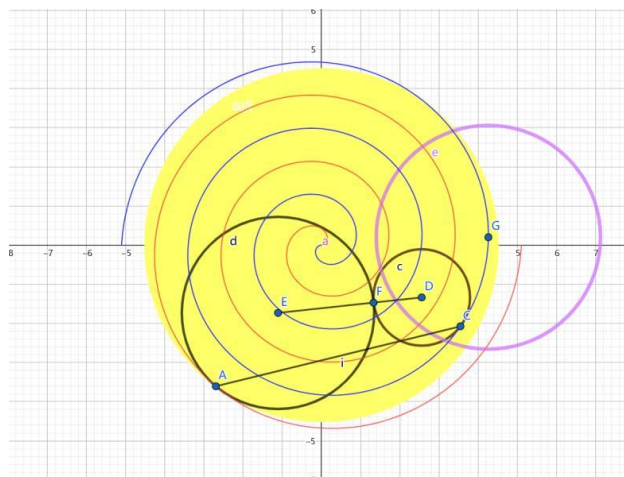


Figure 4. Schematic diagram of the judgment method for detecting collisions

In the above figure, the black circle is the circle where the arcs at the two ends of the turnaround path are located, and the purple circle on the right side is the key circle for detecting whether a collision occurs. If the number of intersections between this purple circle and the turn-around curve is more than one, it means that when the faucet's front handle is located at this point, there are two positions corresponding to the faucet's rear handle. Combined with the actual situation, the faucet is always in motion and cannot stay in place, so this situation means that the faucet cannot pass the turnaround curve smoothly. In all other cases, the faucet can pass through the turnaround curve normally.

The front handle of the faucet travels at a constant speed of 1 m/s, and the circle where the arcs of the two ends of the turn-around path are located is determined by the position model in the previous section. The location of the dragon's head at the beginning of the head-turning moment, i.e., 0 s, is used to determine the positions of the dragon's body and the dragon's tail. According to the collision detection model above to determine whether there is a collision between the benches, the first time the collision between the benches of the head-turning path is the shortest path. We simulate the dynamic adjustment of the dragon dance team's dragon path by going backward and forward to update the position of the dragon's head. Table 1 shows the position of the center of the handle of each bench of the dragon dance team from -100 s to 100 s. The center of the handle of each bench of the dragon dance team is determined by the collision detection model above.

Table 1. The position of the center of each bench handle of the dragon dance team at each moment in time

	-100 s	-50 s	0 s	50 s	100 s
Mixer x (m)	7.203885				
Mixer y (m)	7.526460				
Section 1 of the Dragon's Body x (m)	4.930648				
Section 1 of the Dragon's Body y (m)	9.261979				
Section 51 of the Dragon's Body x (m)	-5.148630				
Section 51 of the Dragon's Body y (m)	11.325032				
Section 101 of the Dragon's Body x (m)	-4.948786				
Section 101 of the Dragon's Body y (m)	13.226366				
Section 151 of the Dragon's Body x (m)	5.738366				
Section 151 of the Dragon's Body y (m)	14.531236				
Section 201 of the Dragon's Body x (m)	16.985695	-16.094024			
Section 201 of the Dragon's Body y (m)	-0.477295	-1.641284			
Dragon's Tail (back) x (m)	-9.512595	12.128031			
Dragon's Tail (back) y (m)	14.783649	-11.615808			

It can be determined that the shortest path for the turnaround is 13.481247m, and the traveling route of the dragon dance team is shown in Fig. 5 below.

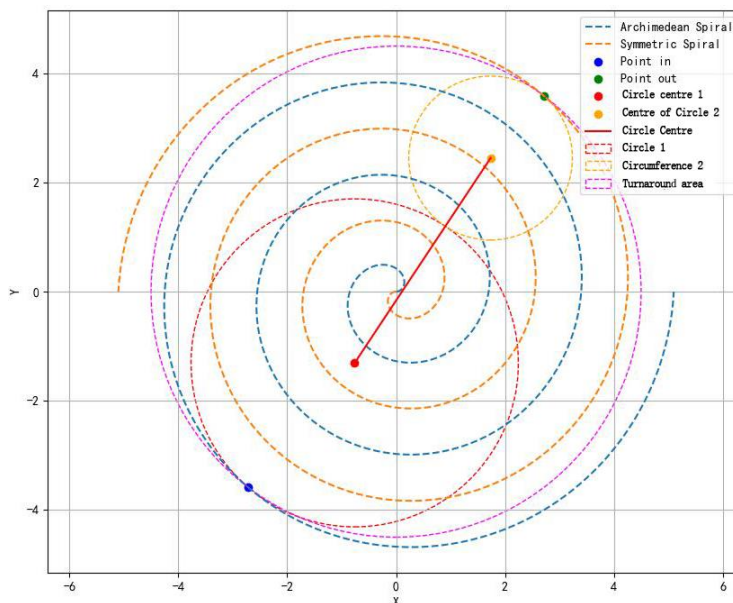


Figure 5. Paths of the Dragon Dance Team during the entry, turn-around and exit of the Dragon Dance Team

5. Summary

In this paper, the position and velocity changes, collision conditions and turnaround path optimisation of the bench dragon in the process of travelling are investigated through the establishment of equidistant solenoidal equations and collision detection models. It is shown that the mathematical model proposed in this paper can effectively solve the collision problem in the complex travelling path of the bench dragon, and provides reliable theoretical support in the optimization of the turning path and speed control. Future research can further extend the model and apply it to more complex motion scenarios.

References

- [1] Chan, K.C. The living bench dragon: shaping China's indigenous tree burial model with ritual design [J]. Landscape Architecture (in Chinese and English), 2022, 10 (04): 96-108.
- [2] Wu Lingping. Aesthetic Review of Folk Sports Programs in Huizhou--Based on the Field Survey of Right Dragon Bench Dragon [J]. Sports Research and Education, 2021, 36 (04): 76-80.
- [3] Principles and calculations of isometric spirals [J]. Practice and understanding of mathematics, 2018, 48 (11): 165-174.
- [4] SHAO Bing, WANG Xunjie, TIAN Yuheng, et al. A study of algebraic Archimedean spiral vortex profiles [J]. Journal of Lanzhou University of Technology, 2004, (05): 64-67.
- [5] TO De-Xin, JIANG Fu-Jin. A preliminary investigation of the mathematical properties of isometric conic spirals [J]. Physics Bulletin, 2018, (12): 75-77+80.
- [6] Shao Yi-Heng. Calculation and analysis of Archimedean solenoidal tangent based on mathematical software [J]. New course (in), 2018, (01): 118.