Research on Production Process Decision Making Based on Dynamic Programming and Simulated Annealing Algorithm

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Abstract. In this paper, a comprehensive decision model based on dynamic programming and simulated annealing algorithm is constructed to solve the decision optimization problem of an enterprise in the production of popular electronic products. By analyzing each stage of the production process in detail, including parts purchase, inspection, assembly and finished product inspection, and combining the defective rate, inspection cost, market price and return loss and other key parameters of each stage, a scheme for optimizing production decision is formed. In this paper, the dynamic programming method is used to optimize the production path, and the simulated annealing algorithm is introduced to solve the larger scale production planning problem, and the detection and treatment strategy in the production process of enterprises is optimized. Finally, it provides an effective decision-making basis for enterprises to obtain the maximum production profit under uncertain conditions.

Keywords: Dynamic programming, simulated annealing, Monte Carlo method.

1. Introduction

In the field of production and manufacturing, enterprises are faced with the challenge of how to make optimal decisions in the multi-stage production process in order to maximize profits. Especially in the manufacturing of electronic products, the decision-making process is complicated and many influencing factors are involved due to the assembly and testing of multiple parts. This paper is devoted to studying how to use advanced optimization algorithms, such as dynamic programming [1] and simulated annealing algorithm [2], to solve this practical problem. Through a detailed analysis of the production process of an enterprise's electronic products, this paper aims to build a comprehensive decision model [3] to help the enterprise make the optimal detection and processing decisions in different production stages, so as to improve product quality, reduce production costs, and ultimately enhance the market competitiveness of the enterprise.

2. Construction and Solution of Dynamic Programming Model

This paper designs a decision-making model of dynamic programming for production enterprises, in order to optimize the decision-making of enterprises in each stage of production and testing, so that enterprises can get the maximum expected income. The model considers the parts and finished products in different stages of testing, discarding, disassembly and other decisions, combined with the cost and defective rate of each link, and finally gives the optimal production strategy.

2.1. Establishment of the Model

Objective function: the enterprise's goal is to maximize the expected revenue, which includes the sales revenue of the product minus the costs of testing, disassembling and discarding during production.

Determine the model variables and parameter definitions

di: i whether to test (parts i=1; i=2, parts 2; i=3, finished product; i=4, whether the unqualified product is disassembled) di=1 indicates detection, di=0 indicates no detection.

The relevant parameters of the model include:

$k1, k2$: The defective rate of parts 1 and 2, both set to 0.1.

$C1, C2, C3$: The inspection cost of parts 1, parts 2 and finished products is 4, 18 and 6, respectively.

$det1, det2, det3$: The inspection time of parts 1, parts 2 and finished products is 2, 3 and 3 respectively.

$t1, t2$: The defective rate of finished products when parts are tested and parts are not tested.

$k3$: When both parts are tested, the defective rate of the finished product is set to 0.1.

p: The selling price of each qualified finished product is set to 56.

bc: Cost of dismantling finished goods, set to 5.

ex: Cost of discarding defective products, set to 6.

Monte Carlo method combined with dynamic programming: Dynamic programming is used to decompose the decision problem and find the optimal decision path for each state by recursion. However, due to the uncertainty of certain parameters, the expected returns of each state can be estimated more accurately by simulating each decision point based on dynamic programming through Monte Carlo method [4]. In particular, for the finished product defective rates $t1$ and $t2$, the Monte Carlo method is used to simulate the fluctuation of defective rates in the case of part inspection or non-inspection, thus helping to determine the optimal combination of overall gains.

State space: Set $S = \{s1, s2, ., sn\}$, each state corresponds to a combination of values for a set of decision variables, i.e. whether to inspect parts 1, whether to inspect parts 2, whether to inspect finished products, and whether to disassemble nonconforming finished products. These state Spaces are finite and ordered, reflecting the different stages of the production process.

Phased decision making: At each stage k, the costs and benefits of the possible state transitions from sk to $Sk + 1$ need to be calculated, and the decision of the next stage should be optimized based on the choices made at this stage.

Bellman equation:

$$\mathrm{V}\left(\mathrm{sk}\right) = \max(\mathrm{R}\left(\mathrm{sk}, \mathrm{dk}\right) + \sum \mathrm{P}(\mathrm{sk}+1 | \mathrm{sk}, \mathrm{dk}) \mathrm{V}\left(\mathrm{sk}+1\right)) \tag{1}$$

Where $V(sk)$ represents the optimal value in state, $R(sk, dk)$ represents the immediate payoff at the time of the selection decision dk, and $\mathrm{P}(\mathrm{sk} + 1|\mathrm{sk}, \mathrm{dk})$ represents the probability of moving from state $sk + 1$ to sk.

Simulated annealing algorithm: Simulated annealing algorithm can efficiently search large scale solution space when determining sensitivity analysis of two key variables in enterprise production, return loss cost ex and product price p [5].

2.2. Establishment of Model

Based on the above description, the expected returns of the model are calculated as follows.

2.2.1 Defective rate of finished products

Calculate the defective rate $t1, t2$ of finished products under different inspection decisions by Monte Carlo method, the defective rate of parts 1 $k1 = 0.1$, the defective rate of parts 2 $k2 = 0.1$, the number of simulated finished products: set a large number of simulated samples.

For each finished product, the state of two parts is randomly generated. Assuming that part 1 and part 2 have the probability of defective $k1$ and $k2$ respectively, two numbers between 0 and 1 can be randomly selected. If a number is less than the probability of defective parts, then the zero match is considered as defective.

Let N be the number of simulated finished products and the number of sub-products be deno 0ted as N_0, then the formula for calculating the defective product rate t is as follows:

$$t = \frac{N_0}{N} \tag{2}$$

According to the defective rate and cost of each state, we can define the state transition equation. Through the state transition equation, the optimal strategy corresponding to the maximum return can be solved recursively.

2.2.2 Parts detection decision

According to whether parts 1 and 2 are tested, the cost of the enterprise in the test stage is:

$$R_1 = -c_1 - \det_1 \cdot d_1 \tag{3}$$

$$R_2 = R_1 - c_2 - \det_2 \cdot d_2 \tag{4}$$

2.2.3 Decision of finished product inspection

According to the inspection situation of spare parts, different combinations will have different effects on the defective rate of finished products:

If both parts are tested, the defective rate of the finished product is $k3$, and the income of the finished product is:

$$\mathrm{R3} = \mathrm{R2} - \left(\mathrm{c3} + \mathrm{det3.d3}\right).\left(1 - \max\left(\mathrm{k1}, \mathrm{k2}\right)\right) + \mathrm{p.}\left(1 - \mathrm{k3.d3}\right).\left(1 - \max\left(\mathrm{k1}, \mathrm{k2}\right)\right) \tag{5}$$

If only one part is inspected, the defective rate of the finished product is $t1$, and the income is:

$$\mathrm{R3} = \mathrm{R2} - \left(\mathrm{c3} + \mathrm{det3.d3}\right).\left(1 - \mathrm{k1.d1} - \mathrm{k2.d2}\right)\mathrm{R3} + \mathrm{p.}\left(1 - \mathrm{t1.d3}\right).\left(1 - \mathrm{k1.d1} - \mathrm{k2.d2}\right) \tag{6}$$

If neither of the two parts is tested, then the defective product of the finished product is $t2$ and the revenue is:

$$R3 = \mathrm{R2} - \mathrm{c3} - \mathrm{det3.d3} + \mathrm{p.}\left(1 - \mathrm{t2.d3}\right) \tag{7}$$

2.2.4 Decision of disassembly of finished product

If the finished product that fails the test is selected for dismantling, the profit after dismantling is:

$$\mathrm{R4} = \mathrm{R3} - ?\mathrm{d3.ds.k3.}(1 - \max\left(\mathrm{k1}, \mathrm{k2}\right).\left(\mathrm{bc} - \mathrm{c1c2}\right) \tag{8}$$

If disassembly is not carried out, then there will be the cost of discarding, and the total income of the enterprise is:

$$\mathrm{R5} = \mathrm{R4} - \left(1 - \mathrm{d3}\right)\mathrm{k3.}\left(1 - \max\left(\mathrm{k1}, \mathrm{k2}\right)\right).\left(\mathrm{ex} + \mathrm{c1} + \mathrm{c2} + \mathrm{c3} + \mathrm{det1.d1} + \mathrm{det2.d2} + \mathrm{det3.d3}\right) \tag{9}$$

2.2.5 Maximum expected value

By setting a certain number of simulation times N, in each simulation, the total benefit under each simulation is calculated according to whether the parts are inspected or not, the defective rate and the related cost, and the benefit is recorded. After all simulations are completed, take the average of these benefits as the approximate expected benefits:

$$E\left[R\right] \approx \frac{1}{N} \sum_{i=1}^{N} R_i \tag{10}$$

Where R_i denotes the return from the first i simulation.

2.3. Solving the Model

2.3.1 Defective rate of finished products

Monte Carlo Method: Based on the first case, it is assumed that the finished product defect rate $t1$ and $t2$ obey some probability distribution (such as normal distribution), respectively representing the finished product defect rate when the part is partially inspected or not inspected.

Monte Carlo method is used to generate multiple samples of defective rate, assuming that the defective rate $[kmin, kmax]$ fluctuates, dynamic programming is performed on each sample to calculate the expected returns under different detection strategies.

By repeating the above simulation process for many times, the probability that the finished product is defective in each simulation is calculated, and the proportion of finished product is calculated. Through recursion, we derive the global optimal solution step by step from the underlying decision. State recursion equation:

$$\mathrm{R}\left(\mathrm{d1}, \mathrm{d2}, \mathrm{d3}, \mathrm{d4}\right) = \max\left\{\mathrm{R}\left(\mathrm{d1}, \mathrm{d2}, \mathrm{d3}, \mathrm{d4}\right)\right\} \tag{11}$$

Where, $R(d1, d2, d3, d4)$ represents the maximum return under different combinations of detection decisions.

Backtracking path: After obtaining the optimal value, the optimal decision of each state can be found through the backtracking algorithm [6]. This can clearly indicate how to choose the detection or not decision at each stage, so as to achieve the global optimal.

$t1 = 0.19$ (that is, when only one part is inspected, the defective rate of the finished product is 19%).

$t2 = 0.271$ (that is, the defective rate of the finished product when no parts are tested 27.1%).

2.3.2 Dynamic programming method

Sensitivity analysis: In order to evaluate the impact of return cost ex and market price p on the expected revenue of the enterprise, the simulated annealing algorithm can be solved under multiple different ex and p values. By gradually adjusting the initial values of ex and p, and recording the expected return under each decision combination, the change trend is analyzed.

Cost of returns ex: When the cost of returns increases, the expected revenue of the business decreases significantly. Therefore, enterprises should reduce the return rate and returns-related costs as much as possible to improve the overall revenue.

Market price p: As the market price increases, the expected return increases. However, too high a price may lead to a loss of market competitiveness, so the firm needs to balance the price increase with the rate of product defects.

The optimal decision path under dynamic programming is to test parts 1, not parts 2, to test finished products, not to disassemble unqualified products, and the maximum expected value is 22.58 yuan.

2.3.3 Simulated annealing algorithm

Model initialization: We first define the key parameters in the model: Initial temperature To: The initial temperature of simulated annealing, which is used to control the degree of exploration of the solution space. Temperature drop rate a: The temperature update rule during annealing. Usually selected as $0 < a < 1$.

Termination temperature $Tmin$: The algorithm stops when the temperature drops to a certain threshold. Number of iterations N: The number of iterations in each temperature state determines the intensity of the search for each round.

Status update: During each round of temperature drop, a new solution is generated, and it is decided whether to accept the solution according to the following rules:

If the return of the new solution $Rnew$ is better than the return of the current solution $Rcurrent,$ the new solution is accepted; If the return of the new solution is poor, the solution is accepted with a certain probability, the acceptance probability is determined by the formula $exp((Rnew - Rcurrent)/T)$, and the probability gradually decreases as the temperature drops.

Temperature update: At the end of each iteration, the temperature $T = aT$ is updated and the search continues in the next temperature state.

Termination condition: When the temperature $Tmin$ drops or the yield is no longer significantly improved, the algorithm terminates and outputs the current optimal solution.

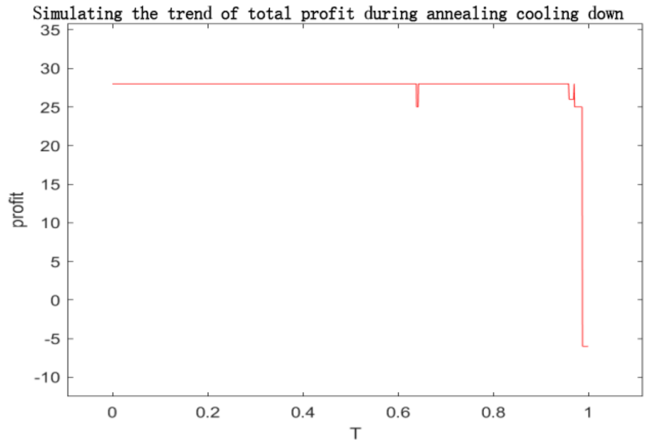

Figure 1. Total profit change during simulated annealing

As shown in Fig. 1, as the temperature decreases, the profit gradually converges from a wild fluctuation to 28, with a maximum expected value of 28.

This chapter constructs a dynamic programming optimization model to optimize the decision path. Based on the production process of the product, the purchase price of spare parts, the cost of testing, the market price and the loss of return, the model determines the decision-making process of whether to carry out testing at each stage and whether the unqualified product needs to be disassembled. The model traverses all possible combinations of inspection and treatment, and finally, finds the optimal strategy to maximize the profits of the enterprise.

3. Application and Optimization of Simulated Annealing Algorithm

In view of the complex production environment with many processes and many parts, this paper puts forward a decision scheme to optimize the production flow. Based on the defective rate of parts, semi-finished products and finished products, the model takes into account the inspection cost, market price and return loss of each stage, and uses dynamic programming and simulated annealing algorithm to optimize. The decision scheme includes: whether to detect the spare parts and semi-finished products, whether to detect and disassemble the finished products, and through sensitivity analysis, to find the optimal path to maximize profits, to provide efficient production decision-making basis for enterprises.

3.1. Establishment of the Model

3.1.1 Parameter setting

Set the parameters involved in the production process, including the related costs of spare parts, semi-finished products and finished products, defective rate and market price, etc. We use the following symbols to indicate

1) Parts stage:

k_i: The defective rate of the i parts, a total of 8 parts, that is $i = 1,2,\dots,8$.

c_i: The purchase cost of part i, where in $c = [2,8,12,2,8,12,8,12]$.

det_i nspection cost of part i, where in $det = [1,1,2,1,1,2,1,2]$.

2) Semi-finished product stage:

K_j: The defective rate of the j semi-finished product $K_j = 0.1$, a total of 3 semi-finished products, that is $j = 1,2,3$.

C_j: the assembly cost of the JTH semi-finished product, where in $C = [8,8,8]$.

Det_j: Inspection cost of the JTH semi-finished product, among which $Det = [4,4,4]$.

bc_j: The dismantling cost of the J-th semi-finished product, where in $bc = [6,6,6]$.

3) Finished product stage:

KK: The defective rate of finished products $KK = 0.1$.

CC: assembly cost of finished products, $CC = 8$.
$DDet$: The cost of testing the finished product, $DDet = 6$.
BBc: disassembly cost of finished product, $BBc = 10$.
p: the market price of the finished product, $p = 200$.
ex: Loss on replacement of finished products, $ex = 40$.
These parameters define the underlying economics and factors of production of the model.

3.1.2 Objective function: Calculation of expected profit

The objective is to maximize the expected profit. The expected profit function can be expressed as:

$$\text{Profit} = \sum_{i=1}^{8}\left(\left(1-k_i\right)\cdot c_i - \det_i\right) + \sum_{j=1}^{3}\left(\left(1-K_j\right)\cdot C_j - \text{Det}_j\right) + \left(1-KK\right)\cdot p - CC - \text{DDet} - \text{BBc} - ex \quad (12)$$

3.2. Establishment of the Model

When facing the production process optimization problem of multiple processes and multiple parts, it is necessary to comprehensively consider the defective rate of spare parts, semi-finished products and finished products, assembly cost, inspection cost, and dismantling cost. Therefore, this paper establishes a dynamic programming model to describe each stage of the entire production process, and combines the simulated annealing algorithm to optimize the solution, and finally determines a reasonable detection and production decision, in order to maximize the profits of enterprises.

3.2.1 Decision Variables

Parts testing decision variable: For each part whether to test, set binary variable di.
If the test is $di = 1$, otherwise $di = 0$ a total of 8 parts.
Semi-finished product testing decision variables: For each semi-finished product whether to test, set binary variables d_{i+8}, a total of 3 semi-finished products.
Final product inspection decision variables: For whether the finished product is tested, set binary variables.
A total of 16 decision variables, respectively corresponding to parts, semi-finished products and finished products in different production stages of the test decision.

3.2.2 Decision Variables

Based on the decision of each stage, the total cost of the entire production process consists of the following parts: Purchase cost of spare parts: Assuming that the defective rate of each spare part is k_i and the purchase cost is c_i, then in the absence of inspection, the assembly may contain defective products, increasing the subsequent disposal cost of defective products.
Inspection cost: Each stage has a corresponding inspection cost, and if the decision is made to conduct inspection, the corresponding inspection cost will be generated.
Disassembly cost: the unqualified parts, semi-finished products or finished products need to be disassembled after the test, and the disassembly will generate certain costs.
Market income: Qualified finished products can be sold in the market, the price is p, but if the finished products are unqualified, it will lead to replacement or loss.

3.2.3 Constraints

The optimization model in this chapter should meet the following constraints:
1) The detection decisions at each stage should be binary variables (i.e.).
2) The total testing cost and dismantling cost cannot exceed a certain budget (can be set according to the actual production environment).

3.3. Solving the Model

The simulated annealing algorithm jumps out of the local optimal solution to find the global optimal solution [7].

Initialization temperature and number of iterations: Set the initial temperature $Tinitial$=1, Final temperature $Tfinal = 0.130$, cooling rate $\alpha = 0.999$, and maximum number of iterations $max\_iterations = 70000$.

Neighborhood solution generation: A new solution is obtained by perturbing the current solution, expressed by the formula:

$$\text{new\_solution} = \text{perturb}(\text{current\_solution}) \tag{13}$$

The new solution represents the new detection strategy.

Calculation of the objective function: For the current solution and the new solution, the corresponding profit is calculated respectively:

$$current\_profit = f(current\_solution) \tag{14}$$

$$new\_profit = f(new\_solution) \tag{15}$$

Where f () is the objective function calculated according to the profit formula.

Acceptance criteria: Use the following criteria to decide whether to accept a new solution:

$$P = \exp\left(\frac{\text{new\_profit} - \text{current\_profit}}{T}\right) \tag{16}$$

If $new\_profit >$ $current\_profit$, accept the new solution unconditionally.

If $new\_profit \leq current\_profit$, accept the new solution with probability.

Temperature update: After each iteration, the temperature gradually decreases according to the following formula:

$$T_{k+1} = \alpha \cdot T_k \tag{17}$$

Until the temperature drops to $Tfinal$ or reaches the maximum number of iterations.

Iteration termination conditions: There are two kinds of iteration termination conditions for the simulated annealing algorithm:

1) Reach the set maximum number of iterations

$$\text{max\_iterations} = 70000 \tag{18}$$

2) The temperature is reduced to the set value $Tfinal$, indicating that the search process has fully converged.

The final optimal detection strategy and maximum profit are obtained through the solving process. According to the cooling process of simulated annealing, the local optimal can be avoided to a certain extent [8], and the cooling strategy will eventually converge to the global optimal solution. In order to analyze the change of profit with temperature, the current temperature and corresponding profit value are recorded for each iteration in the simulated annealing process, and a graph is drawn.

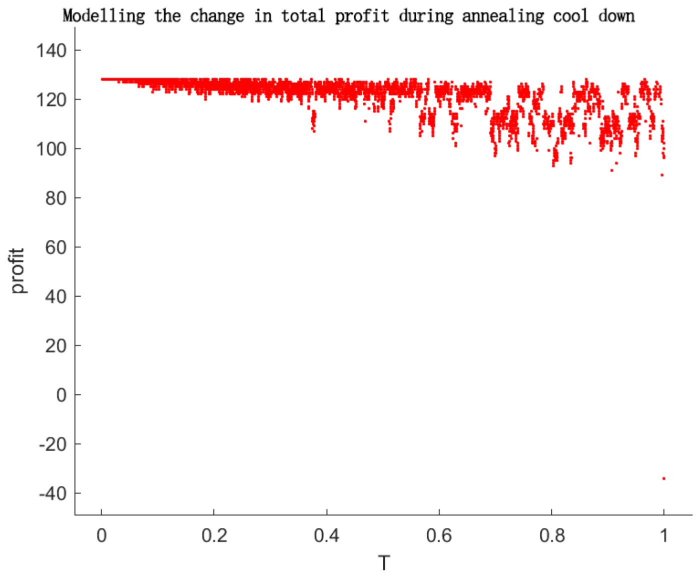

Figure 2. Changes in total profit during simulated annealing after optimization

By drawing the profit curve in the simulated annealing process, as shown in Fig. 2, it can be intuitively seen that the system gradually converges to the global optimal solution during the cooling process. The figure shows how the system profit changes as the temperature drops. At the beginning, when the temperature is high, the system profit fluctuates greatly, but as the temperature gradually decreases, the profit gradually becomes stable and finally reaches the global optimal solution. The optimal detection scheme is that no parts are tested, and all semi-finished products are tested and disassembled after assembly, without testing the finished products. At this time, the total profit to be maximized is 128 yuan.

In this chapter, through the optimization of simulated annealing algorithm, the testing decisions of spare parts, semi-finished products and finished products are gradually adjusted, and finally the best production process decisions are obtained. Through many iterations to generate a new detection scheme, and gradually optimize, the final output of the optimal solution.

4. Conclusion

This paper provides an effective solution to the decision problem of an enterprise in the process of producing popular electronic products by constructing a dynamic programming decision model and optimizing by simulated annealing algorithm. The results show that the expected profit of the enterprise can be improved significantly by optimizing the production decision. At the same time, sensitivity analysis also provides the basis for enterprises to make production decisions in different market environments. Future studies can further explore more efficient optimization algorithms and a wider range of application scenarios to improve the accuracy and applicability of the model. If you follow the “checklist” your paper will conform to the requirements of the publisher and facilitate a problem-free publication process.

References

[1] A. B, J.P. S, J.C. M G, et al. Modified dynamic programming for asteroids belt exploration [J]. Acta Astronautica, 2024, 215142-155.

[2] Alexandru V, Constantin I C, Mihai D C, et al. Simulated annealing algorithms used for microstructural design of composites [J]. Materials Today: Proceedings, 2023 combine (P4): 680-684.

[3] Ahmed E K, Hossan M M C, Pradip R, et al. A decision model for sustainable informal entrepreneurship in cities [J]. International Journal of Climate Change Strategies and Management, 2019, 16 (3): 276-297.

[4] Tartero G, Krauth W. Concepts in Monte Carlo sampling [J]. American Journal of Physics, 2024, 92 (1): 65-77.

[5] Wang Q, Yu D, Zhou J, et al. Data Storage Optimization Model Based on Improved Simulated Annealing Algorithm [J]. Sustainability, 2023, 15 (9).

[6] Claudio C, Romualdo S P. Relevance of backtracking paths in recurrent-state epidemic spreading on networks [J]. Physical Review E, 2018, 98 (5).

[7] Cui S, Zhao J, Shi B. The Fitness Landscapes Exploration Based on the Parallel Simulated Annealing Algorithm [J]. Journal Journal of Big Data and Computing, 2024, 2 (1).

[8] H A C, Ankur G, Guang C, et al. Locally optimal geometry for surface-enhanced diffusion [J]. Physical review. E, 2023, 108 (4-2): 045101-045101.