

Azimuth-only passive positioning technology of UAV based on triangulation and iteration

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Abstract. In recent years, the strategic position of bearing-only passive positioning technology has been continuously improved with the increasing application of UAV cluster flight in military and civil fields. This technology uses as few UAV signals as possible to adjust the position of the UAV formation, so as to achieve the purpose of improving the stealth of the UAV cluster and reducing the external interference. In this paper, the theory and application of the method are described in detail. There are four combinations of UAVs that abstract the demand for de-transmitting signals, and one of them is discussed in detail.

Keywords: symmetry, triangulation, Law of cosines, iterative passive localization

1. Introduction

In recent years, UAV cluster flight plays an extremely important role in military and civil fields, with broad development and application prospects, and has attracted more and more attention. In order to improve the stealth of UAV cluster and avoid external interference, electromagnetic silence should be kept as much as possible to reduce the frequency of transmitting electromagnetic signals when UAV cluster flies in formation. In this regard, the strategic position of azimuth-only passive positioning technology has been continuously improved. It can adjust the position of UAVs on the premise of ensuring the safe flight of the UAV cluster, so as to achieve the purpose of improving the stealth of the UAV cluster and reducing the external interference. So far, the application prospect of bearing-only passive positioning technology is very broad. In this paper, the model is abstracted and discussed in detail for the UAV localization problem. In the research background, 10 UAVs are kept flying on the same altitude plane to form a circular formation. One UAV is located in the center of the circle, and the remaining 9 UAVs are evenly distributed on the circumference. In this paper, we mainly discuss the signals sent by the UAVs located in the center of the circle and the other two UAVs (three UAVs with no deviation and known number), and the other UAVs with slightly deviation can only receive signals passively. Then we solve the positioning model of the UAVs with passive receiving signals[1].

2. Determination of the general position of the UAV to be adjusted

In the problem, it is clear that the transmitted signal of UAV is only composed of the electromagnetic signals of FY00 and the other two UAVs in the formation, and the UAV position of the transmitted signal has no deviation. According to the question meaning, the other UAVs with slightly different positions can be positioned by passively receiving signals. Since there are three UAVs transmitting signals and the FY00 position is known, it can be deduced from the circular

symmetry that there are four combinations of the relative position relations of the other two UAVs transmitting signals. Figure 1 and Table 1[2].

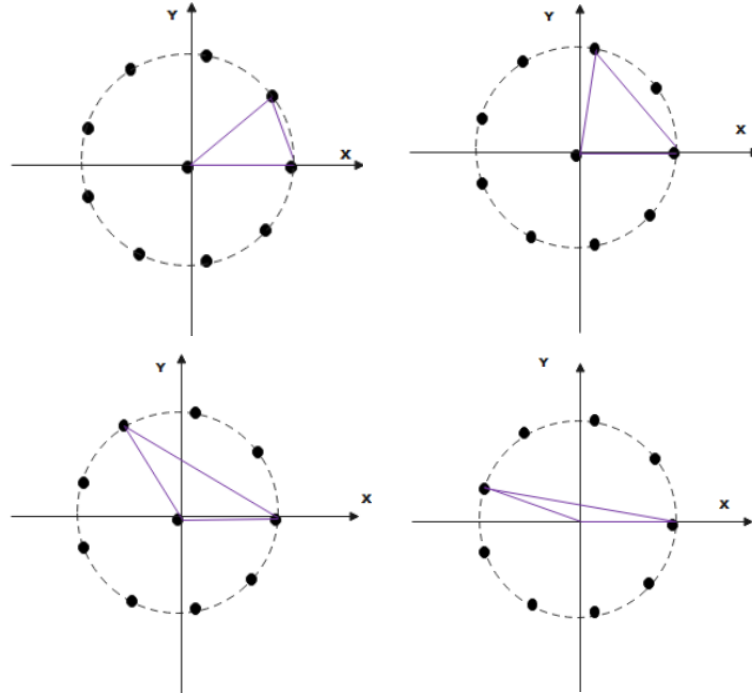


Figure 1. The center of the circle and the position relationship of the remaining two UAVs

Table 1. Position combination of the center of the circle and the remaining two UAVs

Set no.	Unmanned aerial vehicle (uav) 1	No. 2	Unmanned aerial vehicle (uav) 3
1	FY00	FY01	FY02
2	FY00	FY01	FY03
3	FY00	FY01	FY04
4	FY00	FY01	FY05

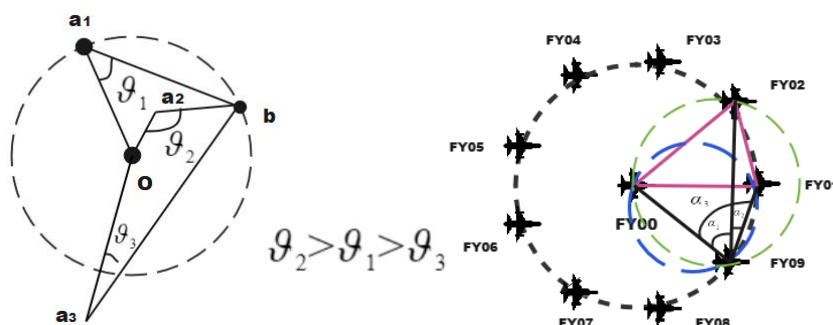


Figure 2. Relationship between the corresponding angles inside the circle, outside the circle and on the circle Figure 3 Position relationship between group No. 1 and FY09 UAV

As shown in Figure 2, a point b on the center and circumference of the circle O is selected. There are three situations in the selection of the point. If a point a_1 is selected on the circle, its corresponding Angle is ϑ_1 ; If a point a_2 is chosen on the circle, its corresponding Angle is ϑ_2 ; If you take a point a_3 on the circle, the corresponding Angle is ϑ_3 . The following relationship can be proved as [3]:

$$\vartheta_2 > \vartheta_1 > \vartheta_3 \quad (1)$$

Select group number 1 and FY09 UAV position combination to analyze and solve as shown in Figure 3.

(1) Select FY00, FY01 and FY09 to be arranged on the same green circle D. The theoretical Angle between FY00-FY09-FY01 can be uniquely determined. α_1 By measuring the actual Angle between FY00-FY09-FY01. θ_1 Compare the relationship between the theoretical Angle and the actual Angle [4].

- ① when $\alpha_1 > \theta_1$, the receiving signal UAV is located outside circle D;
- ② when $\alpha_1 = \theta_1$, the receiving signal UAV is located on circle D;
- ③ when $\alpha_1 < \theta_1$, the receiving signal UAV is located in circle D.

(2) Select FY00, FY02 and FY09 to be aligned on the same black circle E. The theoretical Angle between FY00-FY09-FY02 can be uniquely determined. α_2 By measuring the actual Angle between FY00-FY09-FY02. θ_2 The relationship between the theoretical Angle and the actual Angle A02 was compared [5].

- ① when $\alpha_2 > \theta_2$, the receiving signal UAV is located outside circle E;
- ② when $\alpha_2 = \theta_2$, the receiving signal UAV is located on circle E;
- ③ when $\alpha_2 < \theta_2$, the receiving signal UAV is located in circle E.

(3) Select FY01, FY02 and FY09 to be aligned on the same blue circle F. The theoretical Angle between FY01-FY09-FY02 can be uniquely determined [6]. By measuring the actual Angle between FY01-FY09-FY02. The relationship between the theoretical Angle and the actual Angle A03 was compared.

- ① when $\alpha_3 > \theta_3$, the receiving signal UAV is located outside circle F;
- ② when $\alpha_3 = \theta_3$, the receiving signal UAV is located on circle F;
- ③ when $\alpha_3 < \theta_3$, the receiving signal UAV is located in circle F.

After integrating and analyzing the above position relations, it can be found that there are $27(3^3)$ possible positions. According to the question, the position of the UAV is slightly deviated. The list finds that there are 6 cases with great deviation, which can be directly excluded. Therefore, there are a total of 21 possible situations, and there is no deviation if and only if the drone is on three circles at the same time. By observation, the second, third, and fourth position combinations have the same conclusion as the first one in determining the position, as shown in Figure 3, which will not be discussed here [7].

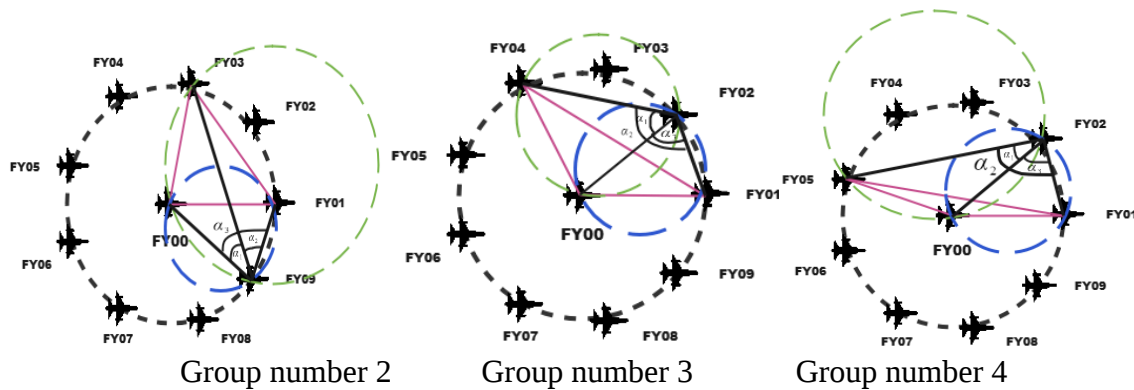


Figure 3. Diagram of position relationship of each UAV under other group numbers

3. Quantification of specific position information of UAVs to be adjusted

3.1. Principle of triangulation method

The principle of triangulation is that a triangle can be formed according to a measured target point and two known reference coordinate points. By calculating the length of the reference side of the triangle and measuring the Angle formed between the two reference coordinate points and the target point, the coordinates and distance of the target point can be found out [8].

3.2. Triangulation with FY00, FY01, FY04 and FY07

The coordinates of the three known points FY04, FY00 and FY01 are respectively, assuming that the coordinates of the unknown point FY06 are $(x_1, y_1), (x_2, y_2), (x_3, y_3)(x, y)$, combined with Figure 4, we can know that[9]:

$$\begin{cases} x_1 = R \cos \theta \\ y_1 = R \sin \theta \\ x_2 = 0 \\ y_2 = 0 \\ x_3 = R \\ y_3 = 0 \end{cases} \quad (2)$$

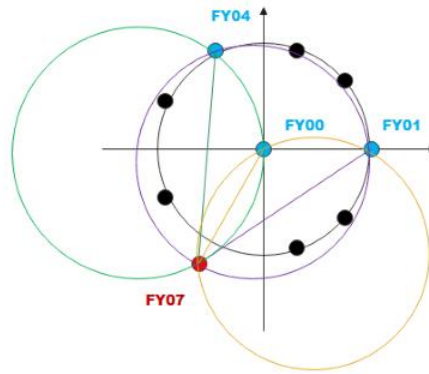


Figure 4. Reference diagram of triangulation method

The angles between unknown UAV FY07 and known UAV FY04, FY00 and FY01 are known $\theta_1, \theta_2, \theta_3$ respectively. Based on the coordinate information and Angle of known points FY04 and FY01, the center of the circle can be uniquely determined as the circle with radius, then the center of the circle is (x_{o1}, y_{o1}) , and let be the Angle between FY04 and FY01. The following formula exists[10]:

$$\begin{cases} (x_{o1} - x_1)^2 + (y_{o1} - y_1)^2 = D_1^2 \\ (x_{o1} - x_3)^2 + (y_{o1} - y_3)^2 = D_1^2 \\ (x_1 - x_3)^2 + (y_1 - y_3)^2 = 2D_1^2 - 2D_1^2 \cos \alpha \end{cases} \quad (3)$$

The radius of the center of the circle O_1 can be determined from the above three equations. The radius is:

$$D_1^2 = \frac{(x_1 - x_3)^2 + (y_1 - y_3)^2}{2(1 - \cos \alpha_1)} \quad (4)$$

Similarly, the corresponding radii of FY04, FY00, FY07 and FY00, FY01, FY07 are obtained. The radii are:

$$\begin{aligned} D_2^2 &= \frac{(x_2 - x_3)^2 + (y_2 - y_3)^2}{2(1 - \cos \alpha_2)} \\ D_3^2 &= \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2(1 - \cos \alpha_3)} \end{aligned} \quad (5)$$

Finally, three sides are used to determine the coordinates of the FY07 UAV. By simultaneous simplification of equations (2), (3), (4) and (5), the relationship between the ordinate y and the abscissa x can be obtained as follows:

$$Y = AX \quad (6)$$

At this point, solving the coordinates of the unknown UAV is transformed into solving the least squares problem:

$$\min_x \|Y - AX\|_2^2 \quad (7)$$

The corresponding analytical solution is solved to establish the positioning model of the passive receiving signal UAV.Z

$$Z = (A^T A)^{-1} A^T Y \quad (8)$$

3.3. Minimum number of UAVs required to transmit signals other than FL00 and FL01

The core of this paper is to determine the number information of other signal transmitting UAVs except FL00 and FL01. When the position information of the signal transmitting UAVs is determined, other UAVs will be determined accordingly. Therefore, the problem is transformed into determining the position of the signal transmitting UAVs. In order to determine the position of the transmitting signal UAV, multiple UAVs transmit signals in order to determine the respective position of each other. The law of cosines is used to quantitatively analyze the relationship between the distance and Angle between the transmitting UAV and the receiving UAV. Multiple equations are constructed, and the number of unknowns is less than or equal to the number of equations, so as to calculate the number of UAVs that need to transmit signals, and further obtain the position of any UAV.

(1) The central UAV and any other UAV are selected as the signal transmission source, as shown in Figure 5.

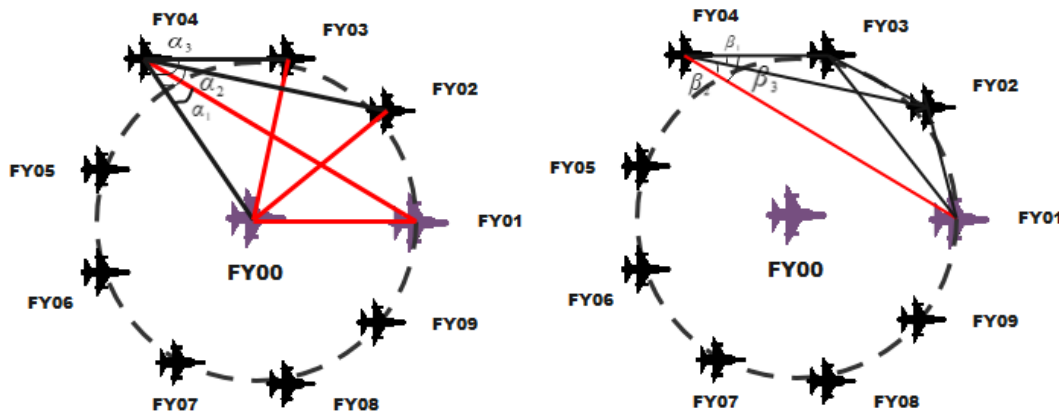


Figure 5. Location diagram of FL00, FL01 and any UAV **Figure 6** Location diagram of any two UAVs on the circumference

(1) Determine the unknowns:

Choose FY04 as the UAV receiving the signal, connect FY04, FY00 and FY01, assume that the distance between FY04 and FY01 is known is, so the triangle composed of FY00, FY01 and FY04 only the distance L between FY04 and FY00 is unknown. L_1 is the number of UAVs transmitting signals except FY00, and the UAVs transmitting signals, such as FY02, are selected to connect FY00, FY02 and FY04, L is the common edge, and the distance between FY00 and FY02 is the radius R of circle C , so only one unknown quantity is added: the distance between FY02 and FY04. L_2 In the same way, an unknown quantity is also added to the UAVs with the i th transmitting signal, and the total number of unknowns in this case can be known $n_1 = 1 + C_{i-1}^1$.

(2) Determine the system of equations:

Since FY04 receives signals from FY00 and FY01, the Angle between FY00-FY04-FY01 is known. In the triangle formed by FY01-FY04-FY01, the law of cosine can be obtained as follows. α_1

$$R^2 = L_1^2 + L^2 - 2LL_1 \cos \alpha_1 \quad (9)$$

Similarly, if the Angle between FY0i, FY00-FY04-FY0i, another UAV transmitting signal on the circle is given, in the triangle formed by FY00-FY04-FY0i, the law of cosine can obtain:

$$R^2 = L_i^2 + L^2 - 2LL_i \cos \alpha_i \quad (10)$$

Since the number of combinations of a UAV transmitting signal on the circle can be selected as $m_1 = C_i^1$, a triangle can be formed $m_1 = C_i^1$. Therefore, a cosine theorem can be listed, that is $m_1 = C_i^1$, there is a system of equations

(2) Any two UAVs on the circle are selected as the transmitting signal sources, as shown in Figure 6.

(1) Determine the unknowns:

FY01 and FY02 are selected as signal transmitting UAVs, and these two aircraft are connected with FY04, so the distance between FY01 and FY04 is known. Because the distance between FY04 and FY02 has been calculated into the total number of unknowns in ①, it is ignored and not included in the total number of unknowns, while the distance between FY01 and FY02 is unknown, so it should be included in the total number of unknowns. Similarly, the distance between the two UAVs transmitting signals is unknown every time. Therefore, there are a total of unknowns in this situation.

(2) Determine the system of equations:

Since FY04 receives signals from FY01 and FY02, the Angle between FY01-FY04-FY02 is known as β_1 . In the triangle formed by FY01-FY04-FY02, the law of cosine can be obtained as follows:

$$L_{12}^2 = L_1^2 + L_2^2 - 2L_1L_2 \cos \beta_{12} \quad (11)$$

Similarly, when FY0i and FY0j are selected as the other two UAVs transmitting signals on the circle, the included Angle between FY0i-FY04-FY0j is known as α_{ij} . In the triangle formed by FY0i-FY04-FY0j, the law of cosine can be obtained as follows:

$$L_{ij}^2 = L_i^2 + L_j^2 - 2L_iL_j \cos \beta_{ij} \quad (12)$$

To sum up, the total number of unknowns is

$$n = n_1 + n_2 = 1 + C_{i-1}^1 + C_i^2 \quad (13)$$

The total number of equations is

$$m = m_1 + m_2 = C_i^1 + C_i^2 \quad (14)$$

To:

$$n \leq m \quad (15)$$

That is:

$$1 + C_{i-1}^1 + C_i^2 \leq C_i^1 + C_i^2 \quad (16)$$

Available

$$i = 2, 3, 4, 5 \dots \dots \quad (17)$$

At that time, the received signals of FY04 were known, and the distance between FY04 and the transmitting signal UAV was obtained by solving the linear equations. However, due to the symmetry axis of circle C, the FY04 direction signal could not be judged by a length, so the value of I needed to be increased. $i = 2$

At that time, all the signal directions at FY04 were known. $i = 3$ The distance between FY04 and the transmitting UAV can be obtained by solving the linear equations, and the distance between the transmitting points can also be obtained, so as to obtain the information direction reference value of FY04. By comparing the reference value with the actual value, the specific orientation of point F can be determined.

To sum up, the minimum number of signal transmitting UAVs in the upper circle is 3. Since the position of FY01 UAV is known, in addition to FY00 and FY01, another i' UAV is needed to transmit signals

$$i' = i - 1 = 2 \quad (18)$$

4. Conclusion

In this paper, three circles of different sizes are drawn, and the general position of the target UAV is estimated by using the nature of the central Angle. Then, the position of the UAV is determined by triangulation, and the position information is quantified by azimuth only passive positioning technology, which has good properties such as high concealment and long action range. It is of great significance to improve the combat ability and survival ability in the complex environment. In modern high-tech warfare, passive positioning technology has been widely used in various services. In addition, bearing-only passive positioning technology can also play an important role in civil and commercial fields, such as communications.

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