Different Methods of Linear Phase IIR Filer Realization

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Abstract: Signal processing has become one of the most popular research topics. Researchers have designed a variety of digital filters to exclude unwanted random noises during the transmission or extract part of the signal in the desired range. There is a prevalent trend for digital filters to replace analog ones since they do not require hardwires. All the performances are operated on a single processor and are free from the effect of external factors. Finite impulse response (FIR) and infinite impulse response (IIR) filters in digital filters respectively provide infinite and finite impulse responses. In the application, it is preferred to have a linear phase digital filter, and FIR filters are naturally linear. However, FIR filters have higher orders and group delay than IIR filters, so researchers found various ways to implement linear phase IIR filters for improvement. This paper introduces and compares Powell and Chau Linear Phase IIR Filter, Kwan Linear Phase IIR Filter, and Xiao, Oliver, and Agathoklis Linear Phase IIR Filter.

Keywords: Digital Filter, Linear Phase, IIR Filter.

1. Introduction

Under the increasing reliance on information exchange in multiple forms: audio, video, image, radar, sonar, etc., signals have become an indispensable element in these processes. To process these signals during transmission, filters come into use to suppress noises and remove interfering components [1]. Analog filters and digital filters are the two basic types into which filters can be divided. Analog filters use resistors, capacitors, and op amps to achieve the filtering effect. Thus, their performance depends on the temperature, the hardware could limit the frequency of the signal, and the design gets complicated when combining multiple filters in parallel. However, digital filters make up for these disadvantages. They employ a digital signal processor to process numerical calculations. Digital filters are stable, easily designed, and programmable, therefore replacing analog filters [2]. Digital filters can be either infinite impulse response (IIR) or finite impulse response (FIR), and these two variants are the most common. While IIR filters are recursive and deal with finite impulse responses, FIR filters are non-recursive and deal with infinite impulse responses [3]. Recursion in this context refers to the feedback circuitry, which combines previous output with current input. Furthermore, FIR filters can operate on linear phase filtering, while IIR filters cannot. The following article will introduce and compare three implementation techniques of IIR filters which are linear-phase from the perspectives of functionalities and properties.

2. Powell and Chau Linear Phase IIR Filter

In 1991, Powell and Chau published their design of a linear phase IIR filter with the best combination of performance and delay compared to other realization methods at that time. To be an exact linear phase, the transfer function should fulfil

$$\mathrm{H}(\mathrm{z}) = \mathrm{H}(\mathrm{z}^{-1}) \tag{1}$$

so that there are mirror image poles and zeros [4]. The efficiency of this implementation technique is illustrated in the fewer multiplications operated than traditional FIR filters and less memory space required than other implementation methods.

The noncausal filter enables the input to be a continuous and infinite sequence, which would be divided into limited sections by using block processing. Powell and Chau's technique depends on the modification of the famous time-reversing technique, capable of reconstructing a source event and generating the best carrier signal for transmission [5]. Based on the traditional two-pass filter, this technique has the block processing operated after the first pass [4]. Thus, the noncausal transfer function is produced in the first pass, and the causal transfer function is produced in the second pass [4, 6].

As shown in Figure 1, the Powell-Chau IIR filter divides the input signal x(n) into segments of length L samples and stores the segments into a(n).

$$a(n) = \{\big(x(0), \dots, x(L-1)\big), \big(x(L), \dots, x(2L-1), \dots \dots\}\tag{2}$$

The intermediate operation could be implemented on a LIFO stack of the capacity of L samples to process the real-time data stream [4]. To keep the streamlined, the time reversal block continuously loads the current input, unloads it in the reverse order, and loads the following input. The output a(n) then goes into an overlap-add block in figure 2. This scheme produces output for each section based on two successive segments from the input convolved with the h(-n), which is the summation of the trailing response and leading response, as shown in figure 2 [4]. As a result, as this scheme is applied in the overall block diagram, a noncausal transfer filter $H(z-1)$ is built, as shown in (3).

$$b(n) = h(-n) * x(n)\tag{3}$$

With the IIR filter $H_2(z)$, which satisfies (4), for which $H(z)$ is the original transfer function, the final equivalent transfer function is shown in (5), which is linear-phase with magnitude equals $|H(z)|^2$.

$$H_1(z) = H_2(z) = H(z)\tag{4}$$

$$H_{eq}(z) = H_2(z)H_1(z^{-1})e^{-j\omega(L-1)/2}\tag{5}$$

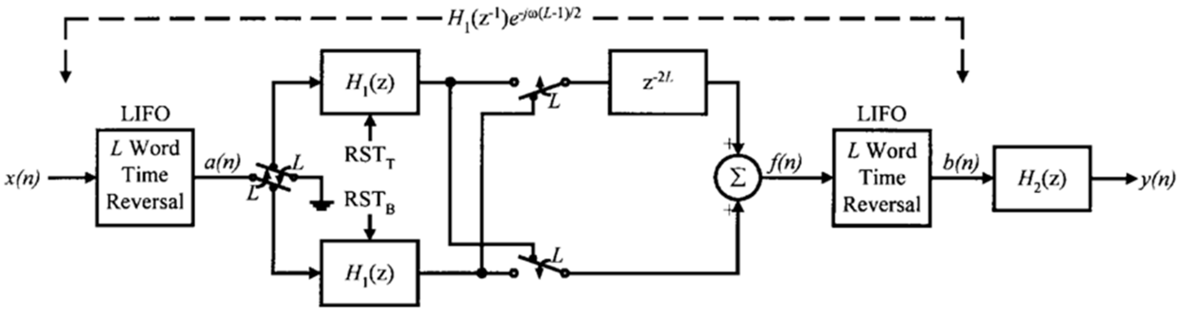

Figure 1. Powell and Chau implementation block diagram [7]

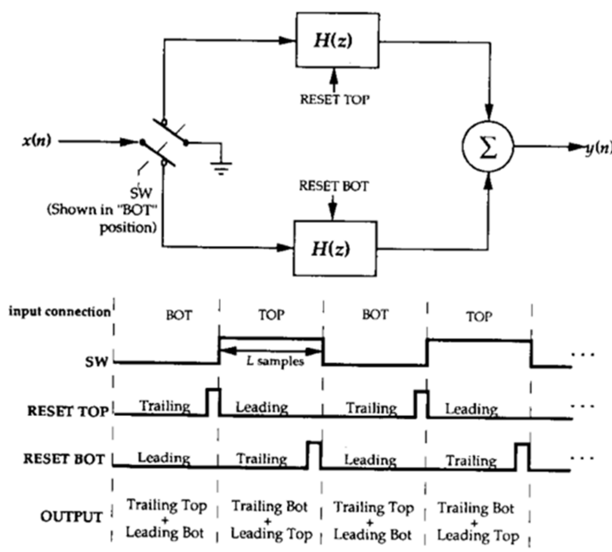

Figure 2. Overlap-add truncate block diagram and timing [4]

3. Kwan Linear Phase IIR Filter

In 2002, Kwan published an equiripple linear phase IIR filter design consisting of ripples of the same magnitude. This design produces a constant group delay and approximately linear phase and takes in both mirror image numerator polynomial and all-pole transfer functions [8]. With this combination, the transfer function could be expressed in the form (6).

$$H(z^{-1}) = \frac{N(z^{-1})}{D(z^{-1})} \tag{6}$$

The denominator D(z-1) is Thiran's all-pole function, which provides a constant group delay, and the numerator N(z-1) is the mirror image polynomial, which produces an equiripple magnitude response [8]. To be compatible with the properties of this filter, these two transfer functions could be further expressed in the form of (7) and (8), where T is the filter order and τ is the constant group delay [8].

$$\frac{1}{D(z^{-1})} = \frac{\frac{2T!}{T!}\frac{1}{\prod_{i=T+1}^{2T}(2\tau+i)}}{\sum_{k=0}^{T}\left[(-1)^k \prod_{i=0}^{T}\frac{2\tau+i}{2\tau+k+i}\right]z^{-k}} \tag{7}$$

$$N(z^{-1}) = z^{-q}N_m(z^{-1}) \tag{8}$$

By plugging the $e^{j\omega T}$ for z, the mirror response could be written as shown in (8), and the real transfer function could be expressed in (9) [8].

$$N_m\left(e^{j\omega T}\right) = b_0 + b_1\cos\omega T + b_2\cos 2\omega T + \cdots + b_q\cos q\omega T \tag{9}$$

By further implementing Remez exchange algorithm, which is an efficient way to compute the minmax polynomial, the coefficients in (10) could be solved [9]. Furthermore, this algorithm could also solve the magnitude of the peak ripples in passband and stopband filters.

$$H_r\left(e^{j\omega T}\right)\left|D\left(e^{j\omega T}\right)\right| = b_0 + b_1\cos\omega T + b_2\cos 2\omega T + \cdots + b_q\cos q\omega T \tag{10}$$

4. Introduction of Xiao, Oliver, and Agathoklis Linear Phase IIR Filter

In 2001 Xiao, Oliver and Agathoklis published a novel way to design a linear phase IIR filter using weighted least square approximation [10]. The approximation process is based on Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm, which solves non-linear equations iteratively.

For an m-th order digital system that is stable and linear time-invariant, it could be written into a state-space form:

$$x(i+1) = Ax(i) + bu(i) \tag{11a}$$

$$y(i) = cx(i) + du(i) \tag{11b}$$

$$H(z) = c(zI - A)^{-1}b + d \tag{12}$$

$$J_{ew} = \|(H_r(z) - H(z))H_w(z)\| \tag{13}$$

for which $A \in R^{m*m}$, $b \in R^{m*1}$, $c \in R^{1*m}$ and $d \in R$ [10]. Thus, the transfer function of this system could be represented by (12). To acquire the best performance, it is essential to minimize the cost function, as shown in (13), for which $H_r(z)$ is the reduced order system and $H_w(z)$ is the weighted transfer function [10].

This technique starts with a linear phase FIR filter as in (14), sthen change it into the state-space form shown in (11) and gets the cost function. After getting the partial derivative of the cost function with respect to Ar, br , and cr, BFGC algorithm could be utilized to find an optimal solution of Ar, br, and cr and d, which constructs the desired IIR filter, which performs linear phase in the passband.

$$H(z) = \sum_{i=0}^{n} h_i z^{-1} \tag{14}$$

5. Comparison

First, the compositions of these three filters are different. While Kwan Filter starts with an IIR filter and realizes the linear phase property by arithmetic operations on the target filter, Xiao-Oliver-Agathoklis Filter starts with an FIR filter and reduces it to an IIR filter by the weighted-least square method. On the other hand, Powell-Chau Filter is constructed with three copies of the original filter H(z), an adder, 4L LIFO stack space, and a shift memory register, as shown in figure 1. Thus, Powell-Chau Filter has the most complicated composition and most elements in the circuitry. Also, the mirror image numerator polynomial in Kwan Filter adopts an approximation approach, and the Xiao-Oliver -Agathoklis Filter used the BFGS algorithm iteratively for approximation. However, Powell-Chau Filter is designed to be precise. As a result, Powell-Chau Filter has the most precise magnitude. The choice among different types of implementations depends on the functionalities required, input data size and the order of original transfer function.

There are all trade-offs when designing filters and assigning indices. For Powell and Chau Filter, as the segment L increases, the group delay variation and transfer function error decrease, and the memory space needed increases. When L is lower than a critical value, harmonic distortions appear, leading to an invalid linear phase function, and the group delay is not constant. The implementation of the Kwan Filter, however, is based on the specified value of group delay, and the group delay is constant in all circumstances. In this instance, the calculations get more complex and need more processing time the smaller the group delay given. Xiao-Oliver -Agathoklis Filter is the most computationally heavy while computing the reduced order transfer function, and the difficulty and performance alter with different frequency weighted function input.

There are also other implementations of linear phase IIR filters. Maximally flat group delay filters realize linear phases by a direct approximation approach. This technique depends on approximating the polynomial recursive transfer function with its denominator as a Gauss hypergeometric function. Orthogonal projection is also used to improve performance in reducing FIR filter to IIR filter. This method involves optimal search with a convergent iterative algorithm, further reducing truncate error and improving the low-order transfer function performance.

Powell-Chau Filter, since published relatively early, has much space for improvement. It was designed for all-pass filters, and the efficiency would increase if the elliptic transfer function is adjusted for bandpass and stopband specifications as in Kwan Filter. This improvement results in improved amplitude response and reduced distortions to some extent. Furthermore, [7] proposes an improvement by reordering the polynomials on the numerator, which lowers the phase error and truncate length, with a tradeoff of a greater magnitude of L and more transistors used [7].

6 Conclusion

In the data processing, digital filters with a linear phase response and a constant group delay are preferable. IIR filters have more computational efficiency, easier usage, and less delay than their FIR counterpart. Therefore, to save computational difficulties, researchers have tried various ways to implement a linear phase digital filter with IIR filters, decreased group delay, and more precise magnitude. Powell-Chau Filter, using time reversal block, models noncausal filter, so it preserves the precision and some constraints in delay. Kwan Filter and Xiao-Oliver-Agathoklis Filter realized the linear space by computational approximation methods based on the original transfer function, an IIR for Kwan Filter and an FIR for Xiao-Oliver-Agathoklis Filter.

Since there are tradeoffs between the performance of these filters and other indices, there is no absolute best filter with undue advantages under specific parameters. From the comparison, Powell-Chau Filter gives out the best magnitude and the most complicated circuitry design, while the other two consist of only one filter. Further improvement focuses on reducing the truncate error, distortion, and group delay error, but with some sacrifice in the circuitry size and the memory space occupied in the processor.

With the improvement of the technology, the processor speed could increase largely in the future. Thus, more feathers and functionalities could be added to the current filters without significantly increasing operating speed.

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