Simulation and Numerical Analysis of Gain Model Based on Thulium-Doped Fiber Amplifier

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Abstract: With the advent of the 5G epoch and the gradual development of information technology, people need broader transmission bandwidth. However, the current devices which can compensate for optical fiber transmission loss, such as the rare-earth-doped fiber amplifiers, can compensate for transmission losses for a narrow range of wavelengths, which cannot satisfy people's needs. Therefore, this paper will further study the gain of 1865nm Thulium-doped fiber amplifier. The Matlab program is used in this experiment to carry out the numerical simulation. Using the numerical simulation method, this article analyzed the gain, pump power, signal power, and amplified spontaneous emission as functions of fiber length. This paper also verified the influence of the overlap factor on the gain of the Thulium-doped fiber amplifier. This experiment can provide a reference for length-choosing of the Thulium-doped fiber amplifier for a maximum gain.

Keywords: Thulium-doped fiber amplifier, Gain, Overlap factor.

1. Introduction

With the rapid demands for communication systems, there is a blank on how to compensate for a wider range of transmission loss. This paper focus on what kind of optical fiber amplifier to compensate for optical signals and how to prevent signal distortion during transmission due to its factors and the influence of external noise. For example, it is mentioned in Wang Feng's article on the development and application of doped fiber amplifier[2]. The Thulium-doped fiber amplifier (TDFA) in the rare-earth-doped fiber amplifier (RDFA) current studied only compensates for the wavelength in the range of 1530~1570nm effectively.

RDFA refers to the use of special means to blend rare earth elements in extremely small concentrations such as erbium, praseodymium, Thulium, and other elements, into the deposition of the core layer of optical fiber, so as to produce the corresponding fiber[3]. RDFA, compared with other optical fiber amplifiers, can directly dope the fiber core with rare earth elements that can generate photons. Therefore, the light energy emitted by the laser diode LD pump through rare earth elements is converted to signal light by rare earth elements. The advent of RDFA replaced traditional photoelectric light, i.e., photoelectric conversion, elector-optical conversion, and regenerative relaying, which can directly amplify the optical signal. Subsequently, it can greatly simplify the structure of fiber optic communication systems and reduce their production costs and difficulties.[4,5,6]

Since people started to study fiber optic amplifiers， and in that period, human beings, through a large number of experiments and analyses, there was a deeper knowledge of the spectral properties of Thulium ions (Tm^{3+}) in crystals and glasses. In the early 1980s, B.M. Antipenko et al. obtained the first 1470nm band leap in Tm^{3+}, Yb^{3+} co-doped crystals that could be used for fiber optic communication [7], thus opening the door to TDFA. Thereafter, T. Komukai et al. used a 1064nm titanium sapphire laser single-wavelength pumping of a fluorine-based Thulium-doped fiber in the 1470nm band to achieve a small signal gain greater than 25 dB[8]. Following that, Yano et al. in the 2000s achieved a small signal gain greater than 25 dB in the 1475-1502nm wavelength range using 1400nm/1560nm dual-wavelength pumping [9].

As time goes on, a variety of optical fiber amplifiers have been extensively researched by scientists. However, these only optical fiber amplifiers cannot fulfill the widely growing needs of global communication services. Therefore, one of the effective solutions to such a problem is to expand the

communication systems to a wider range of fields. And since the energy level structure of Tm^{3+} ions happen to meet the leap of amplification in ultra-long wavelength bands to a greater extent. And it can also achieve effective amplification of optical signals. Therefore, Thulium-doped fiber amplifiers can further compensate for the signal light in a wider wavelength range and prevent its distortion. According to this, the deeper research of Thulium-doped fiber amplifiers has important urgency and practical significance. In recent years, according to Kazemi S.H. [10], Yucel Murat [11] and others, most of the research on Thulium-doped fiber amplifier is in S-band and C-band, and all of these have been more mature. Therefore, this paper will study the Thulium-doped fiber amplifier in the 1800-2000nm band to enrich the research content in this field.

In order to effectively compensate for the larger range of signal light and prevent its distortion during transmission, this paper combines the numerical model of rate equation and power propagation equation. In the simulation environment, Matlab is programmed as the experimental process. Finally, the curves of TDFA gain, pump power, optical signal power, and amplified spontaneous radiated power with fiber length are obtained. Moreover, it also verifies the overlap factor's effect on TDFA gain. It provides some reference for the subsequent fabrication of Thulium-doped fiber amplifiers with compensation for the signal light at 1800~2000nm.

2. Numerical model

So as to explore the effect of fiber length on the gain of Thulium-doped fiber amplifier with wavelength at 1865 nm signal light, this paper uses Matlab to solve the rate equation and conducts model simulation and numerical analysis of the gain based on the rate equation and power propagation equation. The numerical model and simulation procedure are shown below.

2.1 Energy level structure of Thulium ions

Thulium ion (Tm^{3+}) in Thulium-doped fiber amplifier is a multi-energy level structure system [12]. This paper uses 1575nm pump light for pumping 1865nm signal light, and a quasi-three-energy level structure system realizes the process. Nonetheless, the relaxation time from the pump energy level to the $^3F_4$ energy level is very short, the number of particles on the pump energy level is approximated as 0 in this paper; the three-energy level system is simplified to a two-energy level ($^3F_4$ energy level and $^3H_6$ energy level) system, as shown in Figure 1.

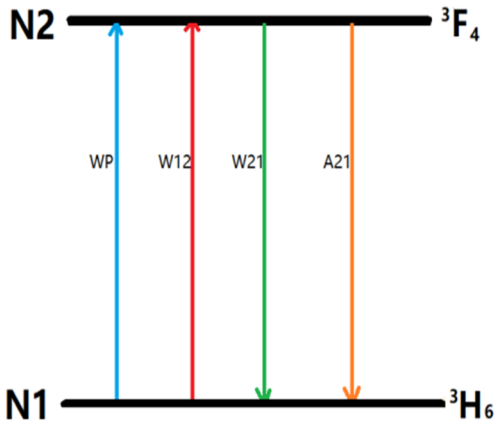


Figure 1. Energy level structure diagram of Thulium ion

Figure 1 shows the changes of each energy level particle during the whole process of amplification of 1865nm signal light by the Thulium-doped fiber amplifier. Firstly, the pump light enters the Thulium-doped fiber amplifier to stimulate the Thulium ion in the ground state $^3H_6$ energy level, which excites it to the $^3F_4$ energy level, and the process is called the pump light absorption rate W_p. After that, the signal light stimulates Thulium ion and makes it produce excited emission from the

$^3F_4$ energy level to the $^3H_6$ energy level, and the process W_{12} is signal light excited emission rate. In this process, the Thulium ion will also spontaneously jump from the $^3F_4$ energy level to the $^3H_6$ energy level, which isspontaneous emission, whereas A_{21} isradiation jumping rate.

2.2 Rate equation model

The variation of the ion bulk density with time for the electrons located at each energy level is called the rate equation. In this paper, the experiments used are Thulium ion is a two-energy level structure system, and its rate equation can be written as:

$$\frac{\partial N_1}{\partial t} = -[W_p + W_{12}]N_1 + A_{21}N_2 + W_{21}N_2 \quad (1)$$

$$\frac{\partial N_2}{\partial t} = [W_p + W_{12}]N_1 - W_{21}N_2 - A_{21}N_2 \quad (2)$$

$$N = N_1 + N_2 \quad (3)$$

Where: t is time. N_1 and N_2 correspond to the particle number density of Thulium ions in Thulium-doped fiber involved in the laser process at energy levels N_1 and N_2 , respectively. N is the total particle number density of the gain medium. W_p is the pump light absorption rate. W_{12} is the signal light absorption rate. W_{21} is the signal light excited emission rate. A_{21} is the chance of spontaneous radiation jump of particles at the N_2 energy level to the N_1 energy level.

In addition, where W_p indicates how many electrons can be excited from the ground state to the excited state for every pump light. It can use parameter $\sigma_p P_p$ divide $h\nu_p A_{eff}$.Where:σ_p is the absorption cross section of the pump light. P_p is the pump light power. ν_p is the pump light frequency. h is Planck's constant. A_{eff} is the core area of the fiber. And A_{eff} can use π mulitiply the square of r, where r is the core radius of the light. π is the circumference of the circle. W_{12} indicates how many electrons are absorbed and excited from the $^3H_6$ energy level to the $^3F_4$ energy level on the ground state for every second signal photons are absorbed. It can use $\sigma_p P_p$ divide $h\nu_{12} A_{eff}$. And σ_{12} is the absorption cross section of the signal light [13]. ν_{12} is the signal light absorption frequency. W_{21} indicates how many photons are generated because of the excited emission for every second. It can use $\sigma_{21} P_s$ divide $h\nu_{21} A_{eff}$. Where σ_{21} is the emission cross section of the signal light [14]. P_s is the signal light power. ν_{21} is the signal light emission frequency. The expressions can be described as follows, respectively.

$$W_p = \frac{\sigma_p P_p}{h\nu_p A_{eff}} \quad (4)$$

$$W_{12} = \frac{\sigma_p P_p}{h\nu_{12} A_{eff}} \quad (5)$$

$$W_{21} = \frac{\sigma_{21} P_s}{h\nu_{21} A_{eff}} \quad (6)$$

$$A_{eff} = \pi r^2 \quad (7)$$

2.3 Power propagation equation model

Pump power, signal power, and amplified spontaneous radiation (Amplified Spontaneous Emission, ASE) power with the propagation length are called the power propagation equation. The pump power propagation equation can use $(-\sigma_p N_1 - \alpha_a)P_p$ to calculate. Where, α_a is the loss coefficients of the fiber material.After that, the signal power propagation equation can use the similar equation $(-\sigma_{21}N_2 - \sigma_{12}N_1 - \alpha_s)P_s$ to calculate. And, α_s is the loss coefficients of the fiber material. Lastly, when calculate the ASE power propagation equation , can use $(\sigma_{21}N_2 - \sigma_{12}N_1 - \alpha_s)P_{ase} + 2\sigma_{21}N_2 h\nu\Delta\nu$ to solve the problem and ν is the signal optic frequency. P_{ase} is the amplified spontaneous radiated power. $\Delta\nu$ is the frequency half-height full width. And the rest of the

parameters are mentioned above. z is the fiber position and all power propagation equations can be described as:

$$\frac{dP_p}{dz} = \left(-\sigma_p N_1 - \alpha_a\right)P_p \quad (\mathbf{8})$$

$$\frac{dP_s}{dz} = (-\sigma_{21}N_2 - \sigma_{12}N_1 - \alpha_s)P_s \quad (\mathbf{9})$$

$$\frac{dP_{ase}}{dz} = (\sigma_{21}N_2 - \sigma_{12}N_1 - \alpha_s)P_{ase} + 2\sigma_{21}N_2 h\nu\Delta\nu \quad (\mathbf{10})$$

According to the above given model, the equation of Thulium-doped fiber amplifier gain, which can be described as:

$$G = 10 * log10\left[\frac{P_s(z)}{P_s(0)}\right] \quad (\mathbf{11})$$

Where: G is the fiber amplifier gain. $P_s(z)$ is the signal optical power at the output of the fiber. $P_s(0)$ is the signal optical power at the input of the fiber.

Light will interact with Thulium ions after entering the fiber. Still, because the wavelength of pump light and signal light is too long, most of the light will enter the fiber cladding, and only a small portion exists in the fiber's core. So, to make the light better propagated in the fiber's core and prevent its distortion during the propagation. T_s and T_p are the overlap factors of the signal light and pump light, respectively. And their respective roles are to distinguish the signal light from the pump light and reduce the overlap. The introduction of these two parameters will change equation (8-9) to equation (10). And equation (10) can be described as:

$$\frac{dP_p}{dz} = P_p T_p\left(-\sigma_p N_1\right) - \alpha_a P_p \quad (\mathbf{12})$$

$$\frac{dP_s}{dz} = P_s T_s(-\sigma_{21}N_2 - \sigma_{12}N_1) - \alpha_s P_s \quad (\mathbf{13})$$

$$\frac{dP_{ase}}{dz} = P_{ase} T_s(\sigma_{21}N_2 - \sigma_{12}N_1) - \alpha_s P_{ase} + 2\sigma_{21}N_2 h\nu\Delta\nu \quad (\mathbf{14})$$

3. Experimental results

3.1 Experimental scenario

For investigating the effect of fiber length on the gain of Thulium-doped fiber amplifier with wavelength at 1865nm signal light and to verify the accuracy of the numerical model, this paper uses the Matlab modeling method for numerical simulation. Firstly, the expressions of N_1 ,N_2 are obtained for the rate equation using the symbolic solution operation, as shown in equation (15) and (16).

$$N_1 = \frac{[N \times (A_{21} + W_{21})]}{A_{21} + W_{12} + W_{21} + W_p} \quad (\mathbf{15})$$

$$N_2 = \frac{\left[N \times \left(W_{12} + W_p\right)\right]}{A_{12} + W_{12} + W_{21} + W_p} \quad (\mathbf{16})$$

Then the corresponding function is written to solve it. Among them, for solving the power propagation equation as a non-rigid (Nonstiff) ordinary differential equation, this paper calls the ode45 function from the Matlab function library to perform the solution operation. Finally, the resulting results are plotted in a plane right-angle coordinate system.

The parameters for the theoretical simulation are shown in Table 1, where L is the fiber length. P_p0_1, P_p0_2, and P_p0_3 are three different pump powers. N_d is the doping concentration; $\lambda_s$is the signal light wavelength. $\lambda_p$is the pump light wavelength. c is the speed of light.

Table 1 Theoretical simulation parameters

Parameter	Value	Parameter	Value
L / m	15	σ_{21} / m²	3.59×10^{-25}
P_p0_1 / W	1	σ_{12} / m²	1.1×10^{-25}
P_p0_2 / W	2	C / m/s	3.0×10^{8}

P_p0_3 / W	5	h	0.626×10^{-34}
I_{ase} / W/m/m	0	a_a / /m	0.1
N_d / ions/m	2×10^{25}	a_s / /m	0.1
λ_s / m	1.565×10^{-6}	$\Delta\nu$ / Hz	12.5×10^{12}
λ_p / m	1.575×10^{-6}	A_{21} / /s	3000.0
v_p / m	1.0×10^{8}	T_s	0.6
r / m	2.5×10^{-6}	T_p	0.4

3.2 Gain

At any position along the length of the fiber, the amplification of the amplifier for the signal light can be expressed by the following equation:

$$A = \frac{I_{(L)}}{I_{(0)}} \tag{17}$$

Where: A is the amplification and its unit is %. $I_{(L)}$ is the signal light intensity at the output; $I_{(0)}$ is the signal light intensity at the input. In fiber optic communication, the amplification of the fiber optic amplifier is usually referred to as the gain, and equation **(11)** above is the formula for calculating the gain, which unit is decibels (dB). Figure 2 shows a numerical simulation of the gain model of a TDFA using Matlab software, and the experimental data is summarized and plotted.

As seen in Figure 2, the gain does not vary linearly along the length of the fiber. Within a certain length range (0-50m), the fiber gain from the pump power are P_p0_1, P_p0_2 and P_p0_3 all become larger with increasing fiber length, but the rate of increase gradually decreases. When the fiber length approaches to 30m, all three gains will be at their peak. And this peak is the point needs to ensure the gain of TDFA keeps maximum. However, if the length of the fiber continues to increase, too much of the energy of the signal light will be absorbed by the fiber and the gain will be reduced. According to the curves corresponding to the three different pump powers in Figure 2, other factors being equal.

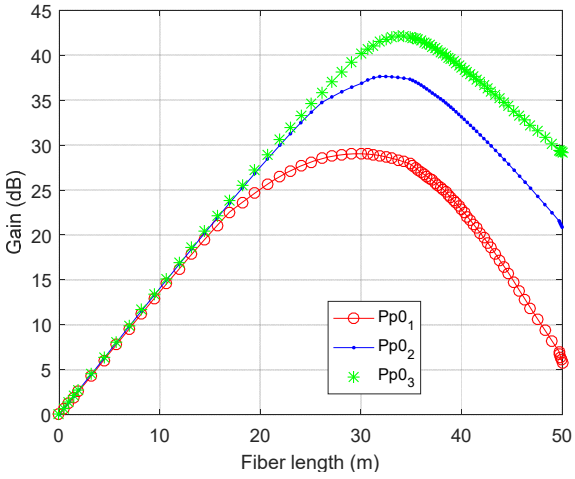


Figure 2. Gain versus fiber length for different pump powers (T_s =0.6, T_p =0.4)

In order to verify the effect of T_s on the gain of TDFA, the value of T_s is changed in Figure 3.1 and Figure 3.2 below, with T_{s3} = 0.65 and T_{s4} = 0.59 respectively. Following that, it cn be concluded that when T_{s3} =0.65 , the gain of three different pump power are approaching to 33dB, 42dB and 46dB respectively. Meanwhile, when T_{s4} =0.59 , there is a decline of 5dB in the gian of three pump power respectively. Lastly, by comparing Figures 2, Figure 3. (a) and Figure 3. (b), it can be concluded that all other factors being equal, the higher the value of T_s the longer the fiber length required for the gain of the TDFA to reach its maximum value; the higher the value of T_s , the higher the gain of the TDFA for the same fiber length. Therefore, it can conclude that among these three different data of T_s , T_{s3} =0.65 is the guarantee to ensure the gain of TDFA keeps the manximum value.

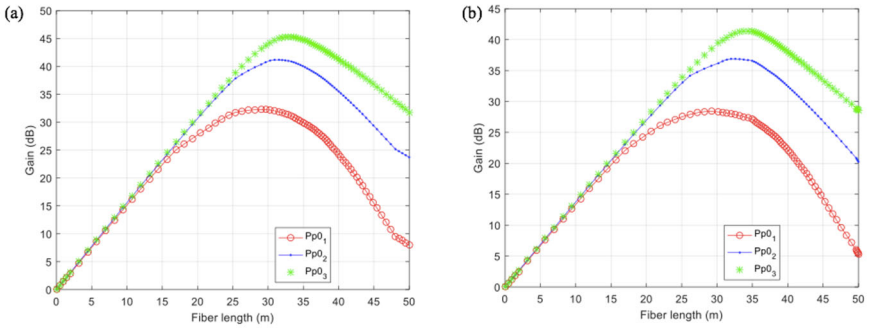


Figure 3. The relationship of gain and fiber length for various pump powers
(a) T_{s3} =0.65 **(b)** T_{s4} =0.59

In order to verify the effect of T_p on the gain of TDFA, this paper changes the value of T_p to produce the following Figure 4.1 and Figure 4.2 for T_{p3} =0.43 and T_{p4} =0.3 respectively. According to these two figures, the result can be got that when T_{p3} =0.43, the peak of three different pump power are approaching to 29dB, 37dB and 42dB respectively. Moreover, when T_{p4} =0.3 , the maximum gain of different pump power are approaching to 32dB, 40dB and 45dB. By comparing these data, can find that there is a climb of 3dB in the gain of every different pump power. Therefore, by comparing Figures 2, Figure 4. (a) and Figure 4. (b), it can be concluded that, with other factors being the same, the larger the value of T_p , the shorter the fiber length required for the gain of TDFA to reach its maximum value; in the same fiber length the smaller the value of T_p , the higher the gain of the TDFA. Hence, it can conclude that T_p is one of the most significant key factors affecting the gian of TDFA and among these three data, it can find that when T_p =0.3, the gain of TDFA is the maximum.

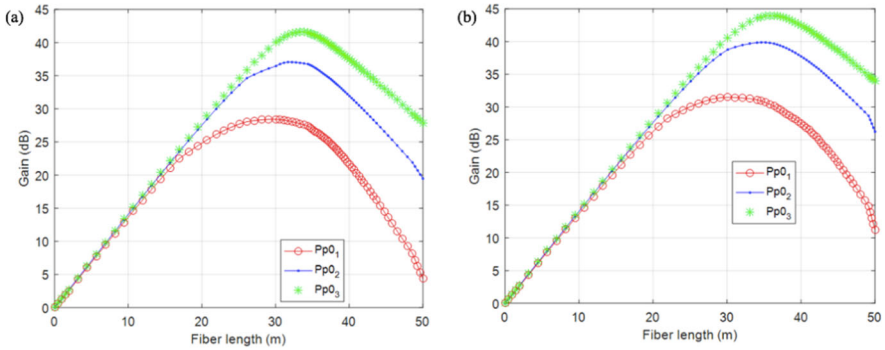


Figure 4. The relationship of gain and fiber length for various pump powers
(a) T_{p3} =0.43. **(b)** T_{p4} =0.3

In a nutshell, in order to ensure that the gain of the TDFA is as large as possible, suitable values of T_s and T_p should be selected so that T_s is as large as possible and T_p is as small as possible, which in turn enables the gain of the same length of TDFA to achieve better results and reduce energy loss in fiber optic communication.

3.3.1 Pump power

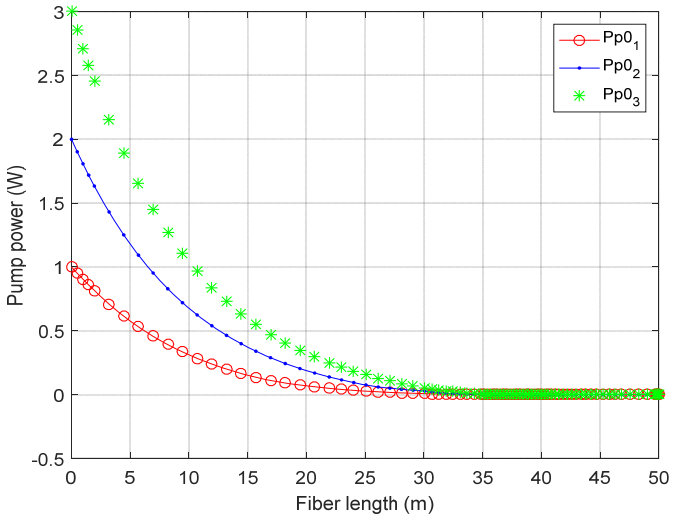


Figure 5. Pump power varies with fiber length.

Figure 5 depicts the power and fiber length relationship for three distinct pump powers. The figure shows that when the fiber length rises within a specific range, the pump power drops progressively and more slowly (0-50m).And the larger the initial value of the pump power, the more pronounced the change in pump power with fiber length and the faster the decrease. According to the figure, it can find that when the fiber length is about 35m, the three pump powers are apprpaching to 0, and this could attribute to that with the growth of fiber length, there are more and more pump light will be absorbed. Therefore, when choosing the pump power, it should choose the right length of optical fiber to avoid choosing a fiber length that is too long. The pump light is absorbed completely, so the TDFA cannot directly target the signal light for pumping, but instead needs to continue to absorb the signal light. The result can lead to affect the gain and not conducive to TDFA applications.

3.3.2 Signal optical power and ASE power

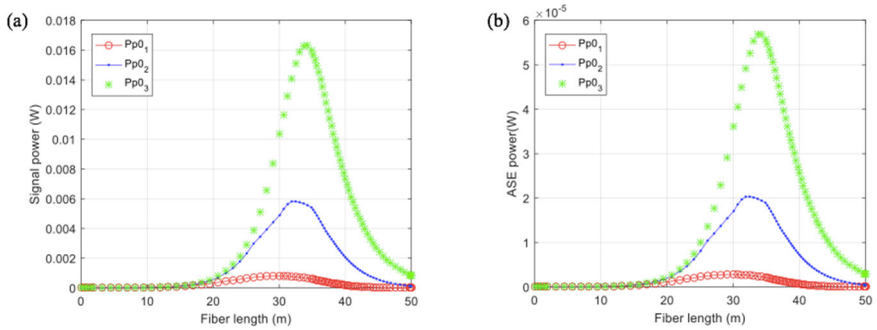


Figure 6. (a) Signal power variation with fiber length
(b) Changes in ASE power as a function of fiber length

Figure 6. (a) shows the variation of optical signal power with fiber length for three different pumping powers. The figure can be seen with the increase in fiber length, in a certain range of fiber length (0 -50m), the optical signal power gradually increases, and the rate increases more and more slowly. The optical signal power is minimal until the fiber length reaches a certain value. After that, the optical signal power will gradually decrease, and the speed line gradually increases, gradually becoming slower. As seen from the line chart, the peak of three different pump powers are close to 0.001w, 0.006w and 0.017w, respectively. It could attribute to different pump powers.

Figure 6. (b) shows the ASE power curve with fiber length for three different pumping powers. As the line chart shows, within a certain fiber length range (0-50m), the ASE power becomes progressively larger as the fiber length increases. When the fiber length reaches a certain value, there is a maximum ASE power—following that, the ASE power continues to decrease as the fiber length increases. According to the figure, it can find that the maximum value of three different ASE powers

are 0.3×10^{-5}w, 2×10^{-5}w and 5.8×10^{-5}w respectively. Therefore, it can conclude that pump power are one of the key factors affecting ASE power.

In summary, it can be concluded that the power of the signal light and the power of the ASE are both related to the length of the fiber, so when choosing the length of the fiber, the energy consumption of the above two powers should be taken into account in order to achieve energy saving and reduce losses.

4. Conclusion

This paper investigates the variation of the gain model of Thulium-doped fiber amplifiers with respect to fiber length, pump power and overlap factor, both from model simulation and experiment. According to the rate equation and power propagation equation , the relationship between length and gain, pump power, optical signal power and ASE power can be calculated by using Matlab program. And this leads to the conclusion that when making a fiber amplifier, it is important to choose the right fiber length according to its needs to avoid meaningless energy loss and reduce energy consumption. The paper also demonstrates the role of the overlap factor: an increase in T_s increases the gain of an equivalent fiber optic amplifier, while an increase in T_p does the opposite. It is therefore important to take T_s and T_p into account when selecting materials for the fabrication of optical fiber, in order to avoid excessive light penetration into the cladding due to the wavelength of the light, resulting in a reduction in the gain of the TDFA and thus an increase in the cost of information dissemination. However, the choice of material for fabricating optical fibers is still at the research stage. Further research will be conducted on the choice of material for optical fibers and the TDFA for the amplification signals in the 1800-2000nm range.

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