Spatial Vector-based Shape Adjustment Strategy for "FAST" Active Reflecting Surface

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Abstract. In this paper, we study the shape adjustment strategy of the active reflecting surface of "FAST", which requires the determination of the ideal paraboloid and the adjustment of the active reflecting surface under the constraint of the reflecting panel adjustment so that the feed module can receive as many reflected signals as possible. This paper establishes the ideal paraboloid model, the coordinate change equation model and the reflective panel adjustment model. The algorithm mainly adopts the idea of variable step size. Finally, this paper analyzes and calculates the signal reception rate of the working paraboloid and the reference reflecting sphere according to the three different states of the reflecting panel.

Keywords: "FAST"; Mechanism Analysis; Variable Step Size; Coordinate Transformation; Space Vector; Approximation Processing.

1. Problem Restatement

1.1 Background Knowledge

Chinas eye -the 500-meter spherical radio telescope- is the largest single-aperture radio telescope with independent intellectual property rights in China and has the highest sensitivity worldwide. The FAST consists of an active reflecting surface , a signal-receiving system (feeder module) and related control, measurement and support systems. The active reflecting surface system is an adjustable sphere composed of the main cable network, reflecting panels, lower cables, actuators and support structures.

1.2 Problem Overview

Depending on the location of the observed objects, different ideal parabolic surfaces and adjustment strategies need to be defined for FAST. In this paper, the following problems need to be studied and completed.

Problem 1: Assuming that the object to be observed by the radio telescope is located directly above the reference reflecting sphere, the ideal paraboloid is determined, taking into account the constraints of the reflecting panel.

Problem 2: When the object to be observed is located at azimuth $\alpha = 36.795^{\circ}$ and elevation $\beta = 78.169^{\circ}$, the ideal paraboloid is determined, and the reflector adjustment model is built. The working paraboloid is adjusted by means of an actuator.

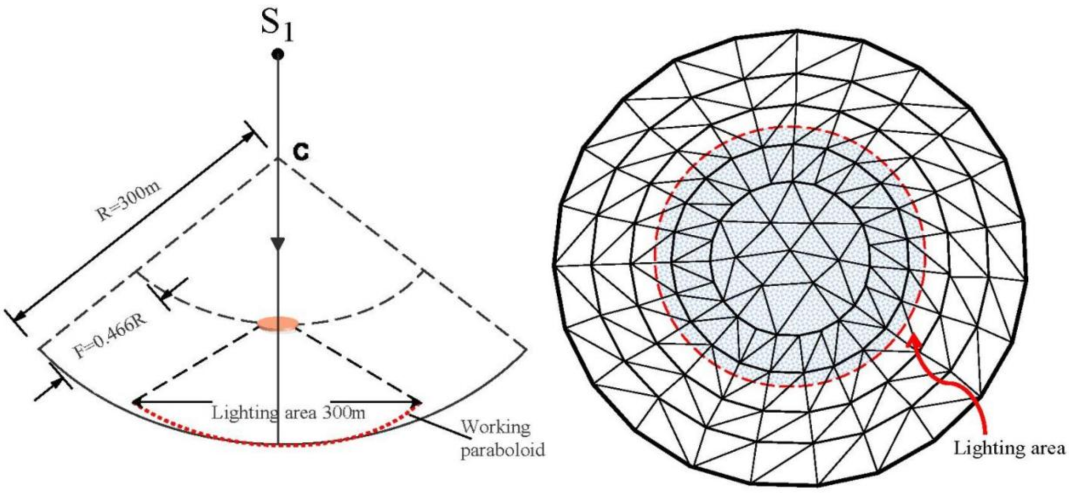
Problem 3: Based on the reflective surface adjustment scheme in Problem 2, the reception ratio of the feeder compartment after adjustment is calculated.

2. Model Assumptions

1. Consider only the radial adjustment.
2. Only the effect of the direct reflection of the signal is considered.
3. In the reference state, the radial expansion of the actuator tip is 0.

2.1 Problem 1: Optimal Strategy for Working Parabolic Adjustment When the Observed Object is Located Directly Above

When $\alpha = 0^{\circ}$ and $\beta = 90^{\circ}$, the active reflecting surface is shown in Figure 1.



(a) Side view (b) Top view

Figure 1. Schematic diagram of Question 1

2.2 Definition of Paraboloid

2.2.1 Proof that Parallel Signals Shot into a Parabola Converge at the Focus

Let the coordinates of the focus F of the parabola be (0,a) and the collinear L be y = -a, then the equation of the parabola has $\sqrt{(x-0)^2+(y-a)^2} = |y+a|$. The simplification yields: $y = \frac{x^2}{4a}$. As shown in Figure 2, let the coordinates of the point A on the parabola be $(X, \frac{X^2}{4a})$. Make a vertical line through point A with the intersection of L at point D(X, -a). Connect the points F and D and intersect the x-axis at point C. The equation of the line FD can be obtained as $y = -\frac{2a}{X}x + a$ with slope $k_{FD} = -\frac{2a}{X}$ and the coordinates of point C as $(\frac{X}{2}, 0)$, and the equation of the line AC with slope $k_{AC} = -\frac{X}{2a}$. Since $k_{FD}k_{AC} = -1$, so FD⊥AC.

When the parallel signal is incident on the paraboloid, there is $\angle BAF = \angle CFA, \theta = \angle ADC$. And since AF=AD, △AFC is isosceles, then $\angle CFA = \angle ADC = \theta$. Therefore, the parallel signal incident to the paraboloid converges at the focal point F. The parallel signal incident to the paraboloid can also converge at the focal point F.

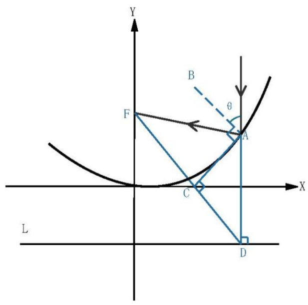


Figure 2. Diagram of the proof that parallel signals shot into a parabola converge at the focus.

2.3 Modeling the Structure of Parabolic Equations

Using the center C of the reference sphere as the coordinate origin, a parabolic profile schematic is created, as shown in Figure 3.

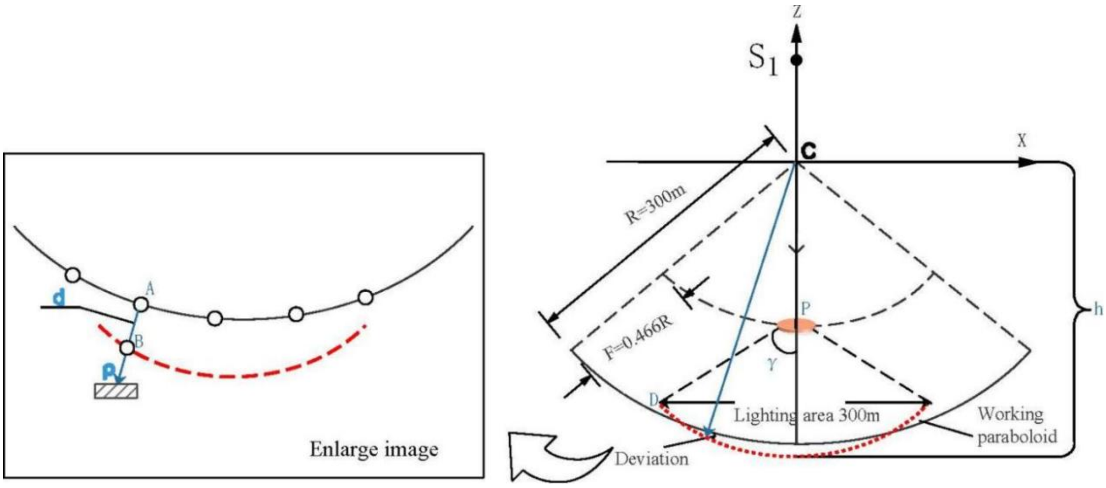


Figure 3. Question 1 Paraboloid Schematic.

In the ZOX coordinate system, based on the standard parabolic equation, the parabolic equation with "h" as the parameter is established by translation and rotation:

$$x^2+y^2 = 4(h-5.34R).(z+h)$$

2.4 Modeling of the Deviation of the Paraboloid from the Main Cable Node of the Reflective Panel

An enlarged view of Figure 3, with the dashed line being the paraboloid and the solid black line being the reflective panel. The n principal rope nodes on the reflective panel with maximum, medium and minimum distances from the z-axis are selected. Assuming the set of n nodes is LAN, if one of the nodes is A and does CA to intersect the paraboloid at B, let the deviation of A be d = AC, then for the deviation of the n nodes on the reflective panel we have: $\Delta LAN = [d_1, d_2, ..., d_n]^T$

Let the maximum radial expansion of the actuator be $L_c^{\max}$,In this paper, we stipulate that the deviation of any of its main cable nodes should be less than or equal to the maximum radial expansion of the actuator, and we have: $d_k \le L_c^{\max} (k = 1, 2, ..., n)$

For an ideal paraboloid, this paper expects that the maximum deviation value of the main rope nodes in the LAN set should be as small as possible, with:

$$\min\max(\Delta LAN)$$

In this paper, we consider the tensor angle of the paraboloid, as in Figure 3, and let the tensor angle be O_a , $O_a = 2\gamma = 2.\frac{|D_x|}{|D_z|-0.534R}$ of which, D_x and D_z is the value of the point D on the x-axis and z-axis coordinates.

To sum up, the maximum deviation value of the minimising paraboloid from the main cable node of the reflective panel is modelled as follows.

$$\min\max(\Delta LAN)$$

$$s.t.\begin{cases} d_k \le L_c^{\max} (k = 1, 2, ..., n) \\ \Delta LAN = [d_1, d_2, ..., d_n]^T \end{cases}$$

2.5 Variable-step Multiple Search Algorithm for Seeking the Ideal Parameter "h"

1.Determining the main rope nodes in the datum sphere

A straight line is made through point P with a calibre of 300m to the parabolic boundary at points A and B, intersecting the datum sphere at points C and D, resulting in a datum sphere with point E as the vertex and CD as the tangent line, as in Fig. 4. The data such as the number and coordinates of the main rope nodes in its area are derived.

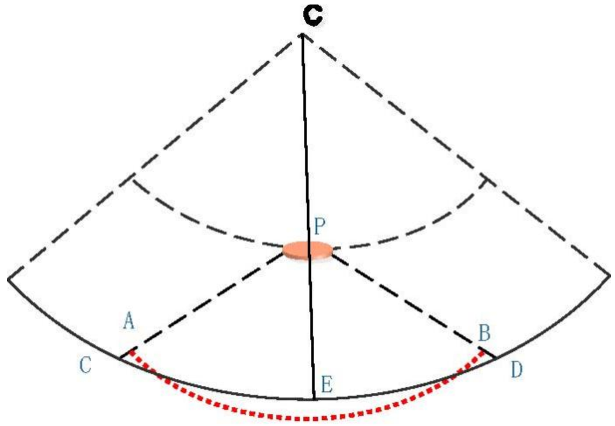


Figure 4. Schematic diagram of the algorithm for determining the master cable node.

2.Multiple search pan parameter h

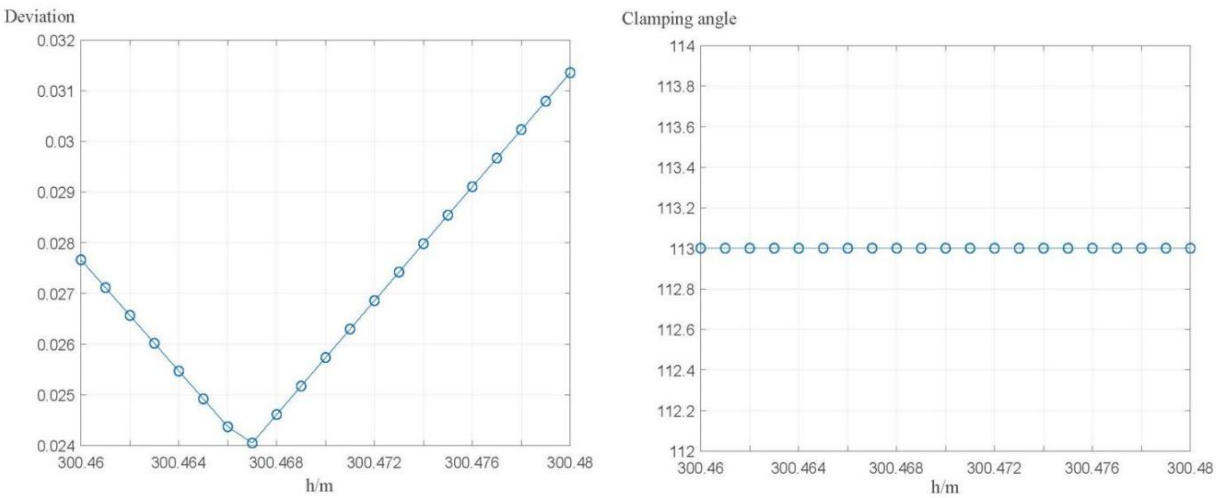
Determine the step size and range of the multiple searches. Use n nodes to calculate the deviation values from the parabola to reduce the amount of computation. If the node deviation is within the maximum adjustable range of the actuator for a given translation h, record the translation h and the maximum deviation value of the main rope node in the database; otherwise change the search step and range search by means of the pinch forcing criterion. When finished, exit and go to 3.

3.Determine the translation h of the ideal paraboloid

The equation of the ideal paraboloid is determined by selecting the translation h of the minimum deviation value as the final parameter in the database.

2.6 Problem 1 Model Solving Results

The literature [3] suggests that,The amount of translation is h=300.4732m. In this paper, multiple searches are performed in the interval (299m,301m) in steps of 0.1m, 0.01m and 0.001m respectively. The resulting translation h is related to the ideal paraboloid as shown in Figure 5



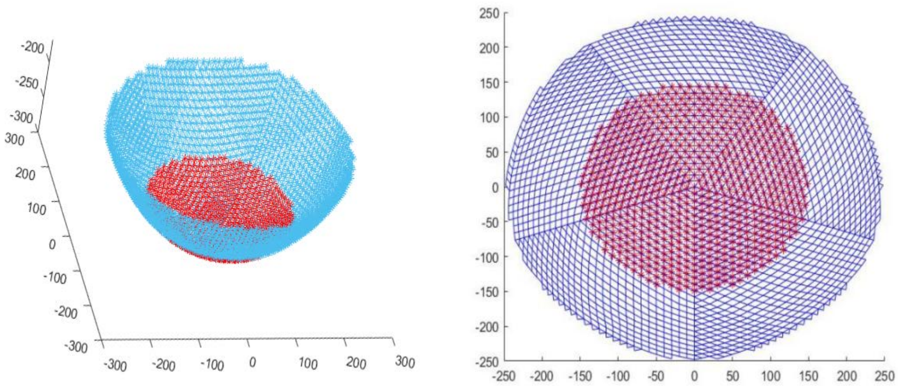
(a) Deviation-h (b) Clamping angle-h

Figure 5. Question 1 Plot of translation h versus ideal paraboloid

As can be seen from Figure 5, the deviation is smallest when the translation amount h=300.4670m. Therefore, this paper selects the translation amount h=300.4670m, At this point the maximum deviation of the main rope node is 0.0240m and the ideal parabolic tension angle $O_a = 112.5327^\circ$.

$$x^2 + y^2 = 561.068\ (z + 300.467)$$

As shown in Figure 6. The blue nodes indicate the main rope nodes outside the illuminated area and the red nodes indicate the main rope nodes inside the illuminated area.



(a) Lighting area 3D map (b) Top view of lighting area

Figure 6. Question 1 active reflective surface diagram.

3. Problem 2: Optimal Strategy for Working Parabolic Adjustment When the Observed Object is Located in an Arbitrary Orientation

This problem builds on question 1, where the global coordinate system OXYZ and the Z axis point to the celestial body in the accompanying coordinate system OXYZ do not coincide , requiring a coordinate transformation.

3.1 Coordinate Transformed Structural Equation Model

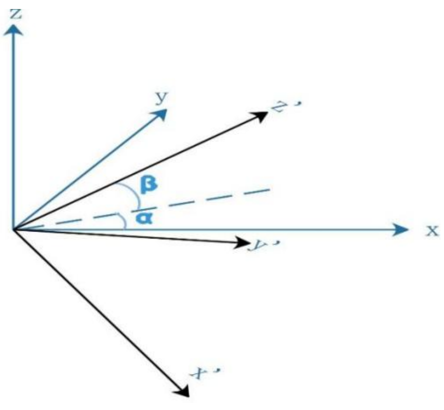


Figure 7. Coordinate transformation diagram.

Adopting the idea of rotation matrix, the forward rotation direction is specified to satisfy the right-hand rule with the coordinate axis pointing.

The rotation matrix of the forward rotation around the X-axis in the YZ-plane is R_x:

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{pmatrix}$$

The rotation matrix of the forward rotation around the Y-axis in the XZ-plane is R_y:

$$R_y(\theta) = \begin{pmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{pmatrix}$$

The rotation matrix of the forward rotation around the Z-axis in the XY-plane is R_z:

$$R_z(\theta) = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The complex rotational motion can be described by combining the basic rotation matrix. For the second question, the global coordinate system is first orthogonally rotated about the z-axis by an angle α, and then orthogonally rotated about the y-axis by an angle $\pi/2 - \beta$. Thus, the total rotation matrix R is

$$R = R_y(\frac{\pi}{2} - \beta) \cdot R_z(\alpha)$$

So

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_y(\frac{\pi}{2} - \beta) \cdot R_z(\alpha) \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3.2 Modeling of Reflective Panel Adjustment

3.2.1 Establish the Objective Function based on Minimizing the Deviation of the Ideal Paraboloid from the Main Cable Node

This question requires the adjustment of the reflective panel with the objective of minimising the error between the working paraboloid and the ideal paraboloid. As shown in Figure 3, let the deviation $d = AB$, The unit vector normal to the sphere at the main cable node A is denoted as: $\boldsymbol{p} = [p_x \quad p_y \quad p_z]^T$. For the set of main rope nodes in the lighting area LAN , The deviation distribution vector is:

$$\triangle LAN = [\boldsymbol{d}_{L1}^T, \boldsymbol{d}_{L2}^T, \ldots, \boldsymbol{d}_{Lm}^T]^T$$

d_{Lm}^T A is the deviation vector of the nth main rope node in the illuminated area, Then the deviation value D for all main rope nodes in the lighting area is

$$D = \sum_{i=1}^{m} \left\| \boldsymbol{d}_{Lm}^T \right\| = \sum_{i=1}^{m} \sqrt{(p_x \cdot d_i)^2 + (p_y \cdot d_i)^2 + (p_z \cdot d_i)^2}$$

Among them d_m is the deviation value of the m-th principal cable node in the illuminated area.

Objective function: Establish the objective function to minimize the deviation D of the ideal paraboloid from the main rope node

$$\min \ \frac{1}{m} \cdot D$$

3.2.2 Constraints based on Reflective Panel Conditioning Requirements

Based on reflective panel adjustment requirements, The constraints are as follows

1. Constraint on the radial expansion range of the actuator

Let the radial expansion of the i-th actuator be L_c^i, the maximum radial expansion is $L_c^{\max,i}$, The minimum radial expansion is $L_c^{\min,i}$, then $L_c^{\min,i} \le L_c^i \le L_c^{\max,i}$.

2. Constraints on the variation of adjacent node spacing

Let the initial and adjusted coordinates of node i be $(x^i_{init}, y^i_{init}, z^i_{init})$ and $(x^i_{fin}, y^i_{fin}, z^i_{fin})$,if nodes i and j are adjacent nodes, the front-to-back spacing between them is

$$\begin{cases} L^{(i,j)}_{init} = \sqrt{(x^i_{init} - x^j_{init})^2 + (y^i_{init} - y^j_{init})^2 + (z^i_{init} - z^j_{init})^2} \\ L^{(i,j)}_{fin} = \sqrt{(x^i_{fin} - x^j_{fin})^2 + (y^i_{fin} - y^j_{fin})^2 + (z^i_{fin} - z^j_{fin})^2} \end{cases}$$

Then the change in the spacing between adjacent nodes and the initial spacing after adjustment is $L^{(i,j)}_f$, then there are: $L^{(i,j)}_f = \frac{L^{(i,j)}_{fin} - L^{(i,j)}_{init}}{L^{(i,j)}_{init}} \cdot 100\% \le 0.07\%$

Combined, the minimized parabolic main cable node deviation model is established as follows:

$$\min \ \frac{1}{m} \cdot D$$

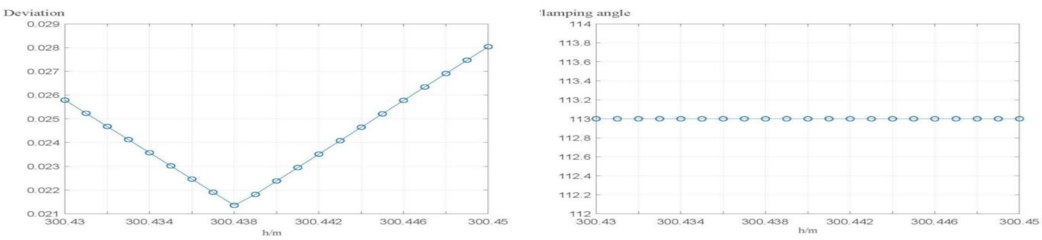
$$s.t. \begin{cases} D = \sum_{i=1}^{m} \left\| \boldsymbol{d}^T_{Lm} \right\| = \sum_{i=1}^{m} \sqrt{(p_x \cdot d_i)^2 + (p_y \cdot d_i)^2 + (p_z \cdot d_i)^2} \\ L^{\min,i}_c \le L^i_c \le L^{\max,i}_c, \quad i = 1,2,\ldots m \\ L^{(i,j)}_f = \frac{L^{(i,j)}_{fin} - L^{(i,j)}_{init}}{L^{(i,j)}_{init}} \cdot 100\% \le 0.07\%, \quad i,j = 1,2,\ldots m \\ L^{(i,j)}_{init} = \sqrt{(x^i_{init} - x^j_{init})^2 + (y^i_{init} - y^j_{init})^2 + (z^i_{init} - z^j_{init})^2} \\ L^{(i,j)}_{fin} = \sqrt{(x^i_{fin} - x^j_{fin})^2 + (y^i_{fin} - y^j_{fin})^2 + (z^i_{fin} - z^j_{fin})^2} \end{cases}$$

3.3 Stepwise Regulation Algorithm to Find the Optimal Reflection Panel Regulation Strategy

After performing coordinate conversion, this problem uses the algorithm of Problem 1 to determine the translation parameter h of the ideal paraboloid. the adjustment is described below.

1. Determine the ideal parabolic equation and the step size of step adjustment.
2. Calculate the deviation values of the n main cable nodes from the ideal paraboloid and rank them from largest to smallest, and update the node number according to the change in the spacing of adjacent nodes or the maximum expansion of actuators.
3. Determine the final reconciliation results.

3.4 Model Solution Results



(a) Deviation-h (b) Clamping angle-h

Figure 8. Question 2 Plot of translation h versus ideal paraboloid.

In this paper, we conduct multiple searches for the translation h in the interval (299,301), and the step size is set as (0.1m,0.01m,0.001m), the relationship between the translation h and the paraboloid is shown in Figure 8. It can be seen that when the translation h=300.4380m, the deviation value is the smallest, and the parabolic tensor angle does not vary significantly with the translation h. Therefore, the translation of the ideal paraboloid h=300.4380m is chosen in this paper, at which time the maximum deviation value of the main cable node is 0.0214m, ideal parabolic tensor angle $O_a = 112.5524^\circ$. The ideal parabolic equation is given by:

$$(0.784x+0.586y-0.205z)^2-68.885y-549.06z-92.108x+(0.599x-0.801y)^2-168531.297=0$$

In this paper, according to the reflective panel regulation model and stepwise regulation algorithm regulation yields. The average deviation of the 688 main cable nodes is 0.0034m, i.e. 3.4mm, and the vertex coordinates of the ideal paraboloid are (-49.3263, -36.8940, -294.0557). The partial results of the main cable node number, position coordinates, and the expansion and contraction of each actuator within 300 m aperture of the reflective surface after adjustment are shown in Table 1.

Table 1. Each parameter after adjustment.

Node	x-axis	y-axis	z-axis	Stretching volume
A0	0	0	-300.0652	0.3348
B1	6.0991	8.395	-299.7933	0.4272
C1	9.8697	-3.2068	-299.8249	0.3956
...	...	...	...	...

4. Problem 3: Solving for the Reception Ratio of the Feeder Compartment

4.1 Modeling of the Signal Reception Ratio under a 300 M Aperture Reflector Panel

Problem 3 requires the calculation of the adjusted reception ratio. Suppose there are q reflector panels, for the i-th reflector panel, if its corresponding three main cable nodes are located in the ideal paraboloid, then the reflector panel is considered to reflect the signal into the feeder compartment, marked as 1; if the main cable node is not on the ideal paraboloid, the panel will be analyzed by discrete processing. The reflector panel with signal reflected into the feeder compartment is marked as 0, and the reflector panel without signal reflected into the feeder compartment is marked as -1. Therefore, the i-th reflector panel judgment quantity is $\mathrm{D}_i \in \{1,0,-1\}$

When $\mathrm{D}_i=0$, it can be regarded as a triangular horizontal reflective surface as shown in Figure 10. Let the point M be the center of gravity of ΔABC,Take the midpoint of the line between the two points, so that the reflective panel is uniformly discrete by m points, noted as set $POINTS_i$. Let the direction vector of the incident signal l be the direction vector $\vec{s}$ not pointing to the reflection point, the normal vector of the triangular horizontal reflective panel is $\vec{n}$,among them:

$$\vec{n}=\begin{cases}\overrightarrow{CB}\times\overrightarrow{CA}, \text{z-coordinate component of } \overrightarrow{CB}\times\overrightarrow{CA}\geq 0\\ -\overrightarrow{CB}\times\overrightarrow{CA}, \text{z-coordinate component of } \overrightarrow{CB}\times\overrightarrow{CA}\leq 0\end{cases}$$

The literature [4] proposed a method to solve the reflected light equation by the vector method, then we have $\vec{s}_1=\frac{2\vec{n}\cdot\vec{s}}{\vec{n}^2}\vec{n}-\vec{s}=(x_0,y_0,z_0)$ .Among them, $\vec{s}_1$ is the direction vector of the reflected ray. As shown in Figure 9, The reflection point (x_1,y_1,z_1) is known, let the reflected signal equation be $\frac{x-x_1}{x_0}=\frac{y-y_1}{y_0}=\frac{z-z_1}{z_0}=t$,then $t=\frac{z-z_1}{z_0}$. i.e. $z=-0.534R$,The values of the x and y-axis coordinates of the point on the reflected signal equation can be solved. If $\sqrt{x^2+y^2}\leq 0.5$,then it is considered that the point can reflect the signal into the feeder compartment, otherwise it is considered that the signal is not reflected into the feeder compartment. In this paper, the area of the i-th reflective panel ΔABC is calculated by using the Helens formula , then we have:

$$S_i=\frac{1}{4}\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}$$

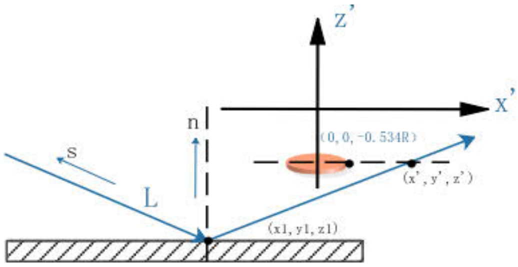


Figure 9. Reflected Signal Judgment.

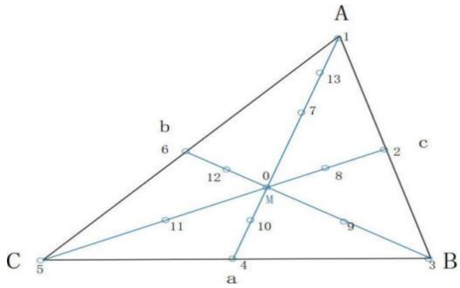


Figure 10. Reflective plate discretization.

Assume that $S_i^{'}$ is the projected area of the feeder compartment plane on the triangular reflective panel of area S_i. Then the reflector panel i can reflect the signal from the object to be observed into the feeder module with an area of S_i^a, there are:

$$S_i^a = \begin{cases} S_i, D_i = 1 \\ S_i^{'}, D_i = 0 \\ 0, D_i = -1 \end{cases}$$

In summary, the feeder compartment signal reception ratio C is modeled as follows:

$$C = \frac{\sum_{i=1}^{q} S_i^a}{\sum_{i=1}^{q} S_i}$$

4.2 Model Solution Results

According to the above model and algorithm, the results are shown in Table 2. It is known that: the signal reception ratio can be significantly improved after the adjustment of the reference reflecting sphere in the range of 300m aperture.

Table 2. Feeder compartment signal reception ratio

District	Reception ratio
Reception ratio of the base reflective sphere (within 300 apertures)	2.90%
Reception ratio of the adjusted reflective panel scheme (within 300 apertures)	69.57%

5. Conclusion

For problem one, this paper uses the variable step multiple search algorithm to determine the ideal paraboloid equation as $x^2 + y^2 = 561.068.(z + 300.467)$. And this paper uses the rotation matrix for problem two to realize the coordinate system transformation and calculates the average radial deviation as 3.4mm based on the reflection panel adjustment strategy. For problem three, the signal reception ratio of the regulated feeder compartment is calculated to be 69.57%. The reflective panel adjustment strategy can significantly improve the signal reception ratio of the feeder compartment.

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