Research Methods for Reliability Analysis of Engineering Structures Based on Statistical Theory

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Abstract. With the increasing need for construction engineering projects, the reliability problem of engineering structures has been the focus of public concern.Currently, there is a lack of a systematic summary of structural reliability theory. Therefore, this paper attempts to summarize some basic principles of structural reliability theory. This paper discusses various factors affecting reliability analysis and three typical analysis methods. It is pointed out that multiple factors affecting reliability analysis, such as the applying loads and resistance parameters such as material properties and dimension parameters, can be regarded as random variables and obey specific statistical distribution. First Order Reliability Method presents the most excellent convenience in calculating failure probability, especially for the structure with a relatively small, and the accuracy of this method is acceptable when the nonlinear degree of structural performance function is not significant.The Second Order Reliability Method could provide a relatively accurate solution with a more complicated calculation process. Monte Carlo Simulation is general enough to be applied to all the cases despite the considerable number of simulations resulting from a minuscule failure probability. Researchers are expected to implement improvement measures to simplify analysis methods by combining them with mathematics and computer science. The results of this paper can provide a theoretical reference for the researchers and designers of structural reliability theory.

Keywords: Reliability analysis; probabilistic model; loads; structural resistance parameters; probability of failure.

1. Introduction

Legendary spectacles are often associated with magnificent engineering projects, such as Confederation Bridge connecting Prince Edward Island and New Brunswick, the underground water supply and drainage system that protects London from cholera, the Sydney Opera House, and so forth. Moreover, the engineering project also involves the infrastructure that people rely on daily, including roads, railways and bridges, energy and water facilities, and waste networks. However, some terrifying accidents, including the collapse of a building caused by failure of structures, occur frequently. Consequently, these safety accidents may lead to property losses and even casualties. With the increasing need for construction engineering projects, the reliability problem of engineering structures has been the focus of public concern. Therefore, engineers must consider the failure risk of engineering structures at the project's design stage and conduct reliability analysis of the structures in advance, ensuring that the projects would keep intact and well-preserved to endure the test of the natural surroundings.

The emergence of structural reliability theory is marked by applying probability theory and mathematical statistics to structural safety analysis in the early 20th century. A.M.Freudenthal in the United States established the theoretical basis for structural reliability in a paper on the statistical aspects of structural safety in 1947 [1]. The American Concrete Institute (ACI) established the structural safety committee (ACI348) in 1964 to conduct systematic research. C.A. Cornell in the United States put forward the reliability index related to structural failure probability as a quantitative index to measure structural safety in late 1968 [2]. N.C. Lind in Canada revised the definition of reliability index and established a more reasonable form of reliability index in 1971 [3]. Later, the introduction of linearizing the nonlinear limit state equation with the Taylor Expansion and normalizing the non-Gauss distributed random variables at design points allows the structural reliability theory to deal with more general problems. In 1971, the structural safety committee (JCSS) was established, and the international system of uniform structural standards and specifications was compiled.

The primary purpose of this article is to summarize and analyze the current research results on structural reliability. First, several key factors affecting structural reliability are briefly introduced, such as various typical loads, properties of materials and geometry, and probability models used to represent their variability. Then, the theoretical background of reliability analysis and the concept, principle, advantages, and disadvantages of three commonly used analysis methods are briefly introduced. Finally, some improvement suggestions are put forward according to their respective advantages and disadvantages.

2. Variability of Quantities in Structural Analysis

In structural analysis, uncertainties can be found both in the demand (e.g., variability in the load) and the capacity of a structure (e.g., material properties and dimension parameters of the components). It is of vital importance to get to know these essential elements and find a suitable way to model their uncertainties.

2.1. Loads

The variability of loads is a crucial factor in structural reliability analysis. Many factors such as the deviation in structural size and the change of material unit weight, have some influences in determining the value of load affecting the structure, and all kinds of loads present random characteristic [4]. Therefore, probability theory and mathematical statistics are introduced into the study of load to determine its value and the prime aim is to establish a probability model that summarizes the randomness of the load. As the load changes with time, its value is random, and the action position is also random and has a certain dynamic effect. Therefore, the load can be regarded as a random process [5]. However, in the reliability analysis, regarding the load as a random process is inconvenient for analysis and calculation. Therefore, the random process is usually transformed into a random variable [5]. This is because the structure designer usually only focuses on the maximum load and its probability distribution in the design reference period.

2.1.1. Dead Loads

Dead loads, also known as permanent or static loads, refer to loads that are relatively constant over time. The dead loads include the weight of all materials of construction incorporated into the building, such as the weight of the structure itself, the weight of components that do not bear loads, and other additional decorative components.

The variability of dead load in the whole life cycle of the building is relatively slight because the weight of the structure could usually maintain stability. Nevertheless, due to wear and erosion damage, the dead load will still change to a certain extent. Generally,

when the value of a random variable is affected by too many different factors, and the influence of every single factor is insignificant, the random variable obeys or approximately obeys the normal distribution. Similarly, the structure’s weight is affected by the weights of all components in the structure, which meet the conditions mentioned above. Consequently, the variability of dead loads obeys the normal distribution [6].

2.1.2. Live loads

Live loads, also known as imposed loads may vary over time. Typically, the live loads are caused by the daily activities of the people who use the structure. During the use of the structure, the impermanent weight’s load caused by these activities on the floor can be regarded as the live loads, such as the weight of people, furniture, portable equipment, and other movable things. Live loads can be divided into two types: sustained live loads and extraordinary live loads [4]. As for sustained live loads, it is caused by furniture, equipment, and people’s ordinary activities. Variability of this type of live load tends to be stable in a certain period as long as the structure users have not presented a considerable change. However, there are some exceptions, such as the sudden gathering of a large number of people or equipment on the floor, which makes the loads far exceed the normal size. The live loads at this time belong to extraordinary live loads. This load has no regularity, often has an enormous value when it appears, and the applied time is short, making it difficult to study. The conventional method cannot reasonably determine its value. Based on previous research, the live load effect follows the extreme value type I distribution [7].

2.1.3. Wind loads

When the wind blows against a structure, the resulting forces applying on the surface of the structure are defined as wind loads. Generally, in structure calculation, wind pressure can be used to measure the force of the wind acting on the structure. Wind pressure is a pressure air cushion formed when a surface stops the airflow. Wind pressure is related to wind speed, which means the same wind speed results in the same force of the wind on the structure. Therefore, the wind load is usually calculated according to the average wind speed, equivalent to a static load.

According to previous research, the annual maximum value shall be adopted for the statistical sample of wind speed, which obeys the extreme value type distribution as well [7].

2.1.4. Snow loads

Snow loads represent the additional forces pressing down on a structure's roof caused by the fallen snow. Like wind loads, snow pressure is used to measure snow loads. It also obeys the extreme value type distribution, using the annual maximum value as its statistical sample [7].

2.1.5. Earthquake

In addition to the above-mentioned common loads, there are still some events that do not occur frequently, but they will have a significant impact on the structure once they occur. Specifically, their load effects on the structure are also worth studying. Earthquake is one of the representatives of such events.

The occurrence of earthquakes with different intensities can also be characterized by probability models. It is assumed that the magnitude-frequency distribution follows the G-R model. Regarding the occurrence of earthquakes as a random event, it obeys uniform Poisson distribution [2]. Besides, according to the statistical analysis in North, Northwest, and southwest China, the seismic intensity obeys the extreme value III distribution [8].

2.2. Resistance Properties

The variability of resistance properties is another vital factor in structural reliability analysis. Structural resistance is the structure's ability to resist the effects caused by loads and the environment. Similar to the loads, the structural resistance properties can also be regarded as random variables. The variability in the resistance properties will determine the fluctuations in the capacity of a structure. Generally, the resistance properties obey the normal distribution or lognormal distribution.

2.2.1. Variability in material properties

Due to the differences in material quality, manufacturing process, and other possible factors, Due to the differences in material quality and manufacturing process, the properties of the same material are random to some extent. $K_{M.}$, the ratio of actual value to the standard value of material properties is usually used to characterize the variability of material properties of components [4]. Since the strength of the materials used in the structural components is a major consideration in the structural design, strength is the main object in the statistical analysis of material properties. The statistics of strength variability of some common building materials, such as concrete and reinforcing bars are listed in Table 1.

Table 1. Statistic of K. M. [9].

Material	Property	Mean	Coefficient of variation
Fe360D (ISO) Steel	Tensile Strength	1.02	0.08
RB400W (ISO) Steel	Tensile Strength	1.14	0.07
C20 Concrete	Compressive Strength	1.66	0.23
C30 Concrete	Compressive Strength	1.41	0.19
C40 Concrete	Compressive Strength	1.35	0.16

2.2.2. Variability in dimension properties

Some random errors will also occur during the manufacturing and installation of structural components, which makes the dimension parameters of components also random variables. Similarly, K.A., the ratio of actual value to the standard value of dimension parameters, is usually used to characterize the variability of dimension parameters of components [4]. The statistics of dimension variability of some common dimension parameters are listed in Table 2.

Table 2. Statistic of K.A. [9].

Dimension Parameter	Mean	Coefficient of variation
Height and width of the section	1.00	0.02
effective height of the section	1.00	0.03
Section Area of the Vertically-Pulled Steel	1.00	0.03
The thickness of the Concrete Cover	0.85	0.30
Spacing of stirrups	0.99	0.07
Anchorage length of longitudinal reinforcement	1.02	0.09

3. Reliability of Engineering Structure

3.1. Basic Concept of Structural Reliability

In the statistical reliability theory, structural reliability is defined as the probability that the structure will manage to achieve the excepted function under the specified period and conditions. Otherwise, the corresponding probability is called failure probability if the structure fails to achieve the excepted function. The failure and reliability of the structure are a pair of complementary events. So, it is also common to use failure probability to define structural reliability.

The critical state of structural failure is defined as the limit state. Generally, supposing the whole or part of structure reaches a specific state such as excessive applied loads, instability, deformation, and crack width exceeding the specified limit, it will fail to meet an expected design requirement [4]. This specific state is the limit state. In structural reliability analysis, the limit state is defined by the structural performance function $g(x)$, As shown in the formula (1).

$$g(x) = g(\overline{X}) = g(x_1, x_2, \dots, x_n). \tag{1}$$

As shown in the formula (1), which$\overline{X} = (x_1, x_2, \cdots, x_n)$ is composed of basic random variables x which represent the loads and structural resistance parameters. By definition, when $g(x) > 0$, the loads applying to the structure is within the capacity of carrying loads, so the structure is reliable enough; when $g(x) < 0$, loads exceed the capacity, and the structure fails. Specifically, $g(x) = 0$, which is called the limit state equation, indicates the limit state of a structure. In terms of geometric interpretation,$g(x) = 0$defines what is called the failure surface [10]. The failure domain (where $g(x) < 0$) is located below the failure surface.

The failure probability P_f can be calculated by the integral of joint probability density function of the basic variables in the spatial region F (the failure domain), as shown in the formula (2):

$$P_f = P[g(\overline{X}) \leq 0] = \int_F f_{\overline{X}}(\overline{x})d\overline{x} = \int_F f_{X_1X_2 \dots X_n}(x_1, x_2, \cdots, x_n)dx_1dx_2 \cdots dx_n. \quad (2)$$

As shown in the formula (2), where $f_{\overline{X}}(\overline{x}) = f_{X_1X_2 \dots X_n}(x_1, x_2, \cdots, x_n)$ is the joint probability density of the basic variables.

3.2. Methods in Reliability Analysis

3.2.1. First Order Reliability Method (FORM)

Normally, the failure probability obtained by directly solving the equation (2) is accurate, but due to the difficulty in calculation, it is usually necessary to simplify the limit state equation and random variables. For example, the computation is convenient for standard normally distributed basic variables and the linear limit state equation. Therefore, linearizing the nonlinear equation with the Taylor Expansion at the point of mean value, the failure probability can be solved approximately by assuming that the random variables obey normal distribution. According to this, First Order Reliability Method (FORM) is devised.

Whereas, there are still many drawbacks in the early First Order Reliability Method. For instance, it is unreasonable to expand at the mean of random variables. It is because the mean of random variables is not on the failure surface, so the approx failure plane may deviate from the actual failure surface to a large extent, and for those limit state functions which are essentially equivalent but only in different expressions, the calculated reliability indexes differ significantly [11]. What is more, the idealization that regards all the random variables as Gauss distributed random variablesis also unreasonable for ignoring their actual distribution type.

Fortunately, Hasofer and Lind came up with an improvement [3]. They defined the reliability index as the shortest distance between the origin of the space of basic variables and the failure surface, taking the point corresponding to the shortest distance as the design point. In this way, the reliability index is only related to the failure surface instead of the limit state function, which solves the problem that the calculation result of the reliability index depends on the expression of the limit state function. Besides, some methods are used to normalize uncorrelated non-normal variables [11, 12].

At present, various improved first-order reliability methods are widely used in the reliability analysis of a structure. However, this method still has some limitations. First of all, approximating the failure function to a linear function is not always accurate, especially when the nonlinear degree of the failure function is high. In addition, the limit state function needs to be uniquely determined during calculation, which requires the experiments to figure out the function.

3.2.2. Second Order Reliability Method (SORM)

In the FORM, the limit state equation is expanded into a linear equation, which means only the first-order term and the constant term are retained. To solve the problem that the failure probability calculated by the First Order Reliability Method is not accurate enough when the nonlinear degree of the limit state function is high near the design point, structural design researchers manage to use the second-order function to approximate the limit state function, which is the Second Order Reliability Method (SORM).

In terms of fundamental principle, the principles of SORM and form are similar, except that the influence of the quadratic term on the fitting accuracy is additionally considered based on the FORM. As a result, some drawbacks of FORM also exist in SORM, such as the requirement of a determined limit state function. Moreover, although the fitting accuracy of SORM is somewhat higher than that of FROM, the additional quadratic term brings a considerable increase in the amount of computation.

3.2.3. Monte Carlo Simulation (MCS)

Monte Carlo Simulation uses repeated random sampling instead of solving integral problems to settle out structural reliability problems.

Generally, when the number of independent repeated tests is big enough, the frequency of the event will converge to the probability of its occurrence. Therefore, to figure out the failure probability of the structure, the possible value of the variable can be randomly generated by the computer according to the known variable probability distribution and substituted into the limit state function, after which whether the structure fails can be recorded. If the number of tests is huge enough, the ratio of the number of failures of structure in these tests to the total sampling times can be approximately taken as the obtained probability.

Using MCS for reliability analysis can avoid the mathematical difficulties in structural reliability analysis. Even for a complex limit state equation with high dimensions, the calculation of MCS is not tricky. Besides, it is not necessary to linearize the equation and normalize the random variables [11]. Therefore, MCS can be applied to almost all the cases, and the error is only related to the sampling times and variance. However, although the MCS is the most general, for the structure with an extreme small failure probability, it is necessary to carry out a tremendous number of simulation sampling, which leads to less efficiency than using FORM or SORM in the case that failure probability is too small [11].

4. Conclusion

The article discusses various factors that should be considered in structural reliability analysis, including variability of applying loads, material and geometric properties, and how to perform a reliability analysis to figure out the probability of failure by using three commonly used methods such as First Order Reliability Method, Second Order Reliability Method, Monte Carlo Simulation. It is pointed out that First Order Reliability Method presents the greatest convenience in the calculation of failure probability especially when the failure probability is quite small, and the accuracy of this method is acceptable when the nonlinear degree of structural performance function is not significant; Second Order Reliability Method could provide a more accurate solution with a more complicated calculation process; Monte Carlo Simulation is general enough to be applied to all the cases despite the considerable number of the simulation as a result of a quiet small failure probability. In further research, researchers are expected to make an effort to improve the First Order Reliability Method and Second Order Reliability Method to simplify their calculation process and come up with improvement measures that can not only reduce the huge number of the simulation but also ensure the accuracy of analysis by combining with newly developed mathematics and computer science.

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