Study on optimal Forest Management system based on Logistic derivative method

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Abstract. This paper takes the forest as the research object, selects coniferous forest and broad-leaved forest as the representative, firstly analyzes the forest value in many aspects to form a comprehensive forest evaluation system, and makes an empirical analysis with an example. A carbon sequestration model was developed, the Logistic derivative model of trees was introduced, and the Charlize curve fitted the growth function of tree volume. The effects of tree species, topography, climate, and the development of forest products on carbon sequestration were studied. Then the relation function between forest carbon sequestration and time is obtained (for example, the carbon sequestration of the broad-leaved forest is). Finally, through mathematical analysis and calculation, it is found that different types of forests are affected by different environments. The relevant parameters in the carbon sequestration model need to be adjusted separately to find the best tree harvesting period and estimate the carbon sequestration.

Keywords: Carbon Sequestration Model; Charlize curve; Logistic model.

1. Introduction

At present, the greenhouse effect is a significant challenge facing humanity. Human emission of many carbon dioxide and other heat-absorbing greenhouse gases into the atmosphere makes the greenhouse effect continuously strengthened, thus causing some severe problems such as global warming. Forests are an essential part of absorbing carbon dioxide [1]. Establishing a better forest management system is crucial for reducing carbon dioxide in the atmosphere. So we decided to build a mathematical model to improve the existing forest management system by considering the balance of forests in carbon sequestration, conservation of ecological diversity, economic value, humanism, and other aspects.

2. Forest carbon sequestration model

Carbon sequestration is when we sequester more carbon dioxide reserves out of the atmosphere through the biosphere, etc. [2]. Forests are necessary "carbon sinks" in the biosphere, sequestering carbon dioxide in living plants and tree products. According to the forest appearance classification, the world's forests are divided into coniferous forest, broad-leaved forest, and mixed coniferous broad-leaved forest. Therefore, we introduced broadleaf forest, coniferous forest, and mixed forest as the research object, comprehensively consider the factors such as tree age, tree species, topography, climate, and development status of forest products in various regions, and deeply studied the relationship between the amount of fixed carbon dioxide independent variable forest and the independent variable tree age. Carbon sequestration is modeled by balancing the benefits gained from harvesting forest products with the value of allowing forests to continue to grow and absorb carbon as living trees [3].

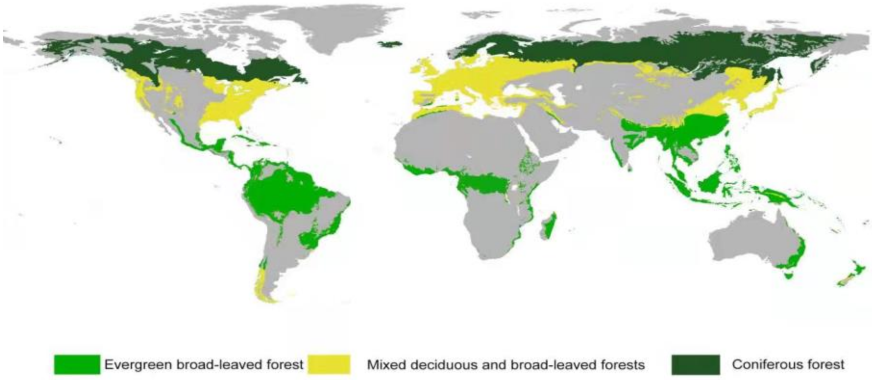


Figure. 1 Distribution of forest types.

To calculate the sequestration effect of forest on CO2, we need to consider the relationship between the increase of forest stock and other biological components of forest to solve the change of forest carbon pool within a period [4]. Relevant data show that it is common to use forest biomass to calculate forest carbon benefits. Therefore, based on Yang Yonghui and other methods, we comprehensively consider various factors and establish a carbon sequestration model.

Where $\varepsilon$the increase in the amount of CO2 is absorbed by forests, φ is the increase of forest organic matter, μ is the carbon content in forest/forest organic matter content, and v is the relative molecular weight ratio of CO_2 and C.

$$\varepsilon = \varphi \cdot \mu \cdot v \tag{1}$$

Among them, by considering the storage mode of photosynthetic organic matter, we determined the increase of organic matter （φ） in the forest with a standing area of S, which was composed of the rise in the volume of forest trees and living plants （φ_1）, the increase of forest litter (φ_2) and the increase of soil organic matter (φ_3). In addition, γ_2 is the annual amount of leaf litter per unit area γ_3 is the yearly increase of soil organic matter per unit area.

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 \tag{2}$$

$$\varphi_2 = s \cdot \gamma_2 \tag{3}$$

$$\varphi_3 = s \cdot \gamma_3 \tag{4}$$

Meanwhile, we use the Logistic derived model of trees to analyze the growth curve of trees[5] and find that the natural growth law of most trees is to accelerate growth first, then slow down growth, then stop growth, and finally slowly decay. Specific can be quantified as the volume of the increase gradually expand first. After t1, it gradually shrinks, slowly reducing to zero, and after t2 becomes a negative number.

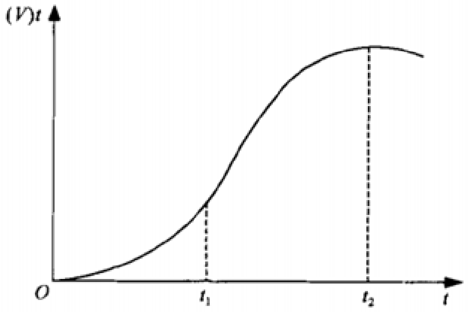


Figure. 2 Logistic derived models of trees.

We estimate the individual growth through mathematical analysis and introduce the forest expansion coefficient to assess the population growth and the tree density factor related to the terrain

[6-8]. Where V is the annual volume growth of a single tree, κ is the forest expansion coefficient, and ρ is the tree planting density under different terrain conditions.

$$\varphi_1 = V \cdot S \cdot \kappa \cdot \rho \tag{5}$$

3. Application of carbon sequestration model

Schima Superba forest in Jiangxi province is a typical broad-leaved forest variety. Therefore, we selected the Schima Superba forest as the research object to analyze the growth characteristics of the broad-leaved forest [9-11]. By consulting relevant data and using SPSS software for Charlize curve fitting, we finally obtained the change function of the growing amount of the height, DBH, and volume of Schima Superba with tree age a.

The height growth equation of Schima Superba is

$H_1 = 23.6267 \times (1 - e^{(-0.0422 \times a)})^{1.0476}$, and the correlation coefficient is 0.9993.

The DBH growth equation of Schima Superba is

$D_1 = 42.8232 \times (1 - e^{(-0.03593 \times a)})^{1.8277}$, and the correlation coefficient is 0.9980.

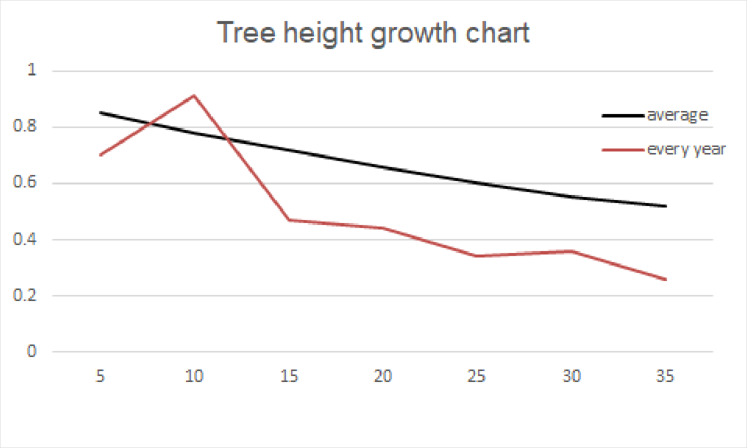


Figure. 3 Height growth curve of Schima Superba.

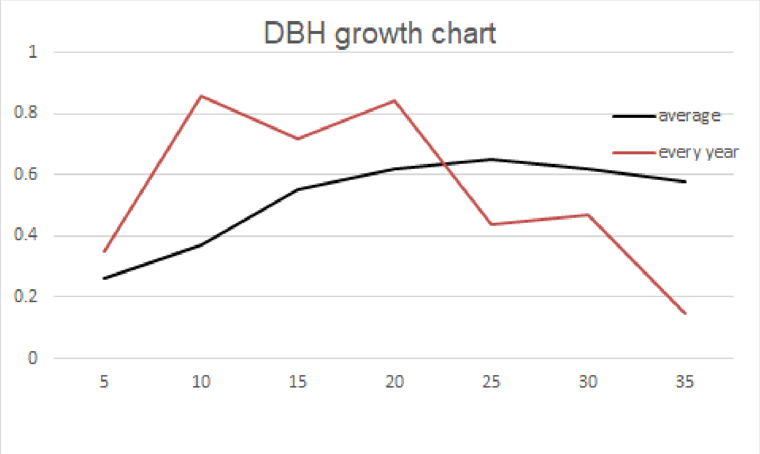


Figure. 4 DBH growth curve of Schima Superba.

According to the data of H1andD1, the annual volume growth data of Schima Superba was calculated through geometric analysis. Finally, the wood volume growth equation of Schima Superba $V_1 = 1.0148 \times (1 - e^{(-0.4401 \times a)})^{4.2244}$ was obtained through the fitting, and the correlation coefficient was 0.9988, which was taken as the volume growth equation of broadleaf forest. The annual growth trend of broadleaved forest volume fluctuated wildly, generally rising slowly at first, then gradually accelerating, reaching the maximum value at 30a, and decreasing progressively at last.

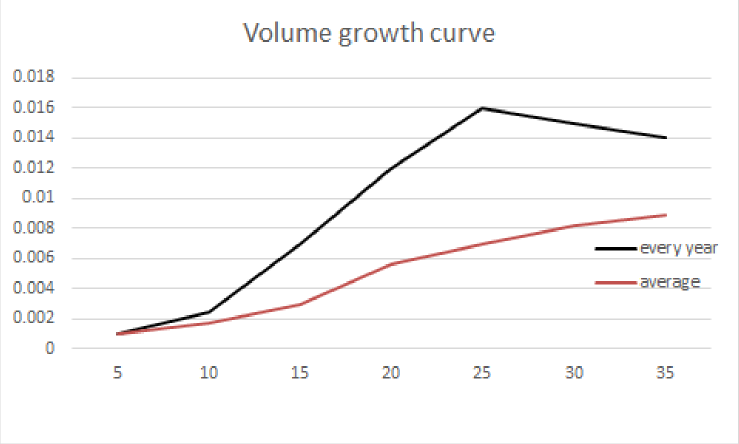


Figure. 5 Growth curve of Schima Superba volume.

Spruce is a representative coniferous forest in the subalpine region of China. Therefore, we selected spruce forest as the research object to analyze the growth characteristics of coniferous forest. By consulting relevant data and using SPSS software for Charlize curve fitting, we finally obtained the change function of spruce height, DBH, and volume growth with tree age A.

Spruce tree height growth equation: $H_2 = 21.6735 \times (1 - e^{(-0.0387 \times a)})^{1.0285}$ and the correlation coefficient is 0.9987.

The DBH growth equation of spruce is: $D_2 = 39.7632 \times (1 - e^{(-0.02707 \times a)})^{1.9106}$ and the correlation coefficient is 0.9996.

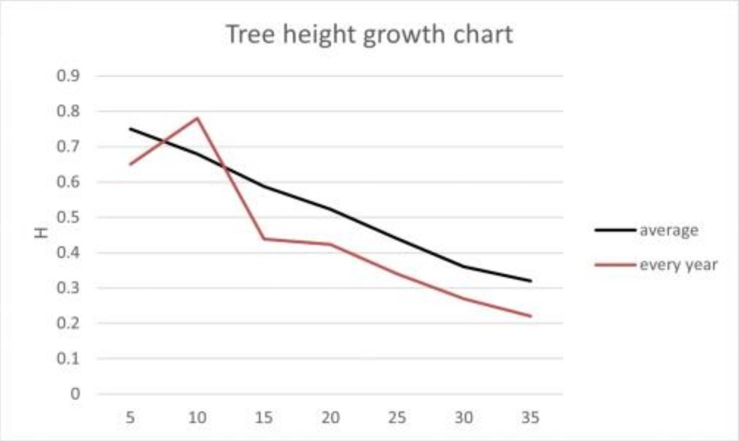


Figure. 6 Spruce height growth curve.

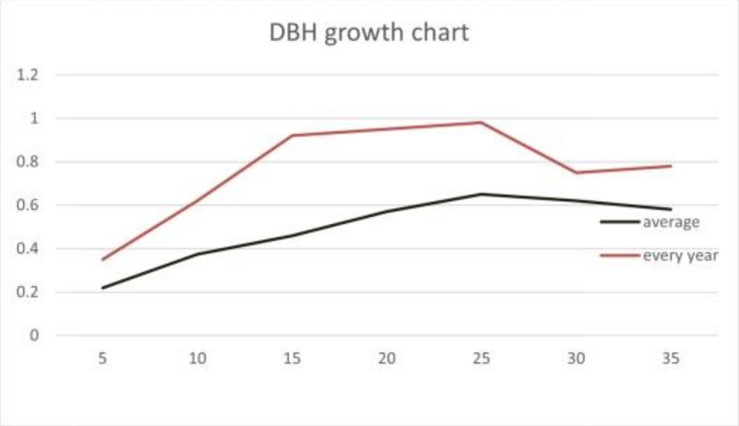


Figure. 7 Spruce DBH growth curve.

According to the data of H2 and D2, spruce's annual volume growth data was calculated through geometric analysis. Finally, the volume growth equation of spruce was obtained through the fitting, and the correlation coefficient was 0.9976. $V_2 = 1.003 \times (1 - e^{(-0.02707 \times a)})^{3.9822}$ It is regarded as the volume growth equation of coniferous forest. The annual growth trend of coniferous forest volume was relatively stable, generally rising slowly at first, then gradually accelerating, reaching the maximum at 30A, and decreasing progressively at last.

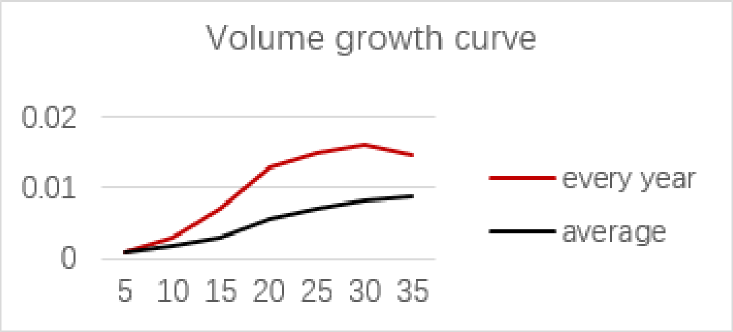
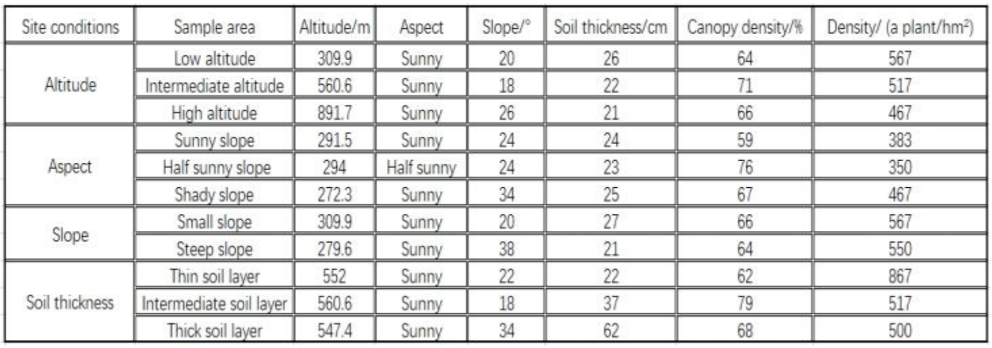


Figure. 8 Volume growth curve of coniferous forest

We obtained the influence of other topographic conditions and soil thickness on tree height and DBH of the coniferous broad-leaved forest through literature review and used Excel statistical data and visualization.



Site conditions	Sample area	Altitude/m	Aspect	Slope/°	Soil thickness/cm	Canopy density/%	Density/ (a plant/hm²)
Altitude	Low altitude	309.9	Sunny	20	26	64	567
	Intermediate altitude	560.6	Sunny	18	22	71	517
	High altitude	891.7	Sunny	26	21	66	467
Aspect	Sunny slope	291.5	Sunny	24	24	59	383
	Half sunny slope	294	Half sunny	24	23	76	350
	Shady slope	272.3	Sunny	34	25	67	467
Slope	Small slope	309.9	Sunny	20	27	66	567
	Steep slope	279.6	Sunny	38	21	64	550
Soil thickness	Thin soil layer	552	Sunny	22	22	62	867
	Intermediate soil layer	560.6	Sunny	18	37	79	517
	Thick soil layer	547.4	Sunny	34	62	68	500

Figure.9 Effects of topographic conditions and soil thickness on tree growth.

Among them, altitude has little effect on tree height, while semi-sunny slope or shady slope, small slope, and thick soil layer are conducive to the increase of tree height. Similarly, altitude and slope had little effect on DBH, but thick soil promoted the rise of DBH on semi-sunny or shady slopes.

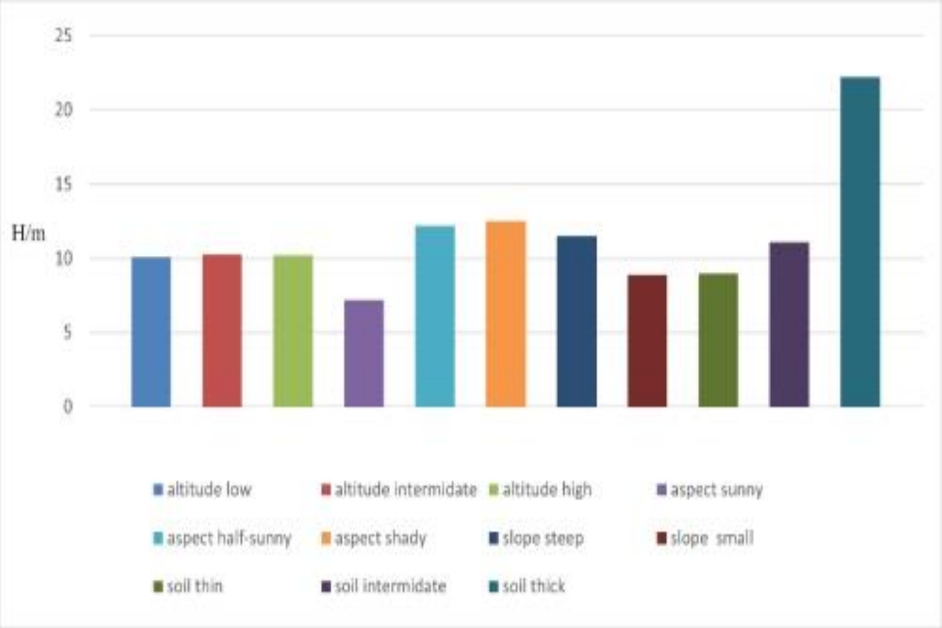


Figure. 10 Influence of different terrain conditions on tree height.

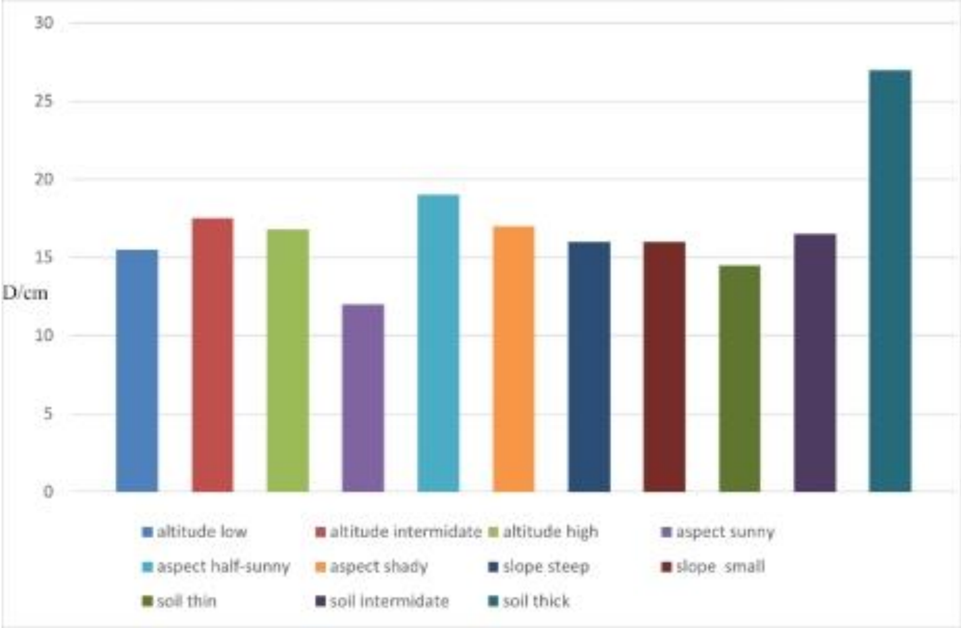


Figure. 11. Influence of varying terrain conditions on DBH.

We obtained the following results through consulting relevant data and data: Among them, the carbon content in the forest/forest organic matter and the forest expansion coefficient are based on the measurement results of Yang Yonghui and others on the mixed coniferous and broad-leaved forest in Hebei Province. The ratio of Co2/C was calculated according to the periodic table of chemical elements. The average value of soil organic matter was calculated according to Han Qingmin et al. The soil organic matter is averaged according to the influence of Han Qingmin et al. on the soil organic matter after the mixed forest of Pinus tabulaeformis

The amount of CO_2 absorbed in the forest: $\varepsilon = \mu \cdot v \cdot s\left(V \cdot \kappa \cdot \rho + \gamma_2 + \gamma_3\right)$

The annual increase in the amount of CO_2 absorbed in per unit area of the broadleaf forest:

$$\varepsilon_1 = 1.6428\left(1 - e^{(-0.4401a)}\right)^{4.2244} \cdot \rho + 17.4075 \tag{6}$$

The annual increase in the amount of CO_2 absorbed in per unit area of the coniferous forest:

$$\varepsilon_2 = 1.6237\left(1 - e^{(-0.02707a)}\right)^{3.9822} \cdot \rho + 17.4075 \tag{7}$$

The annual increase in the amount of CO_2 absorbed in per unit area of the mixed forest:

$$\varepsilon = c_1 \cdot \varepsilon_1 + c_2 \cdot \varepsilon_2 \tag{8}$$

The proportion of broadleaf and coniferous forest accounted for the mixed forest is c1 and c2.

4. Conclusions of carbon sequestration model

To determine which forest management plans are most effective at sequestering CO2, we first need to balance the value of the amount of carbon dioxide that living trees can absorb over the rest of their lives with the average benefit from local wood products. Assuming that trees are cut down for carbon sequestration when they are n years old, the average life of trees themselves is n', and the annual increase of fixed carbon dioxide in the forest in the current year is ε_n. Therefore, the fixed total amount of CO2 of forest during the period $n \sim n'$ is $\sum_{i=n}^{i=n'}\left(\sum_{i=0}^{i=t}\varepsilon_i\right)$

Currently, there are two common methods of carbon dioxide sequestration that benefit from forests in the world: One is to calculate the afforestation cost price of forest fixed carbon dioxide; the other is the reduction of industrial output value for every 1 ton of carbon dioxide released by the industry. We can calculate the benefit coefficient according to the average value of local afforestation cost and industrial output value. At the same time, we need the local market price of wood and the commercial profit of related products ω. Finally, we can compare the two values to get the best years n of cutting trees for carbon sequestration.

$$\sum_{i=n}^{i=n'}\left(\sum_{i=0}^{i=t}\varepsilon_i\right)\cdot\lambda=\omega \tag{9}$$

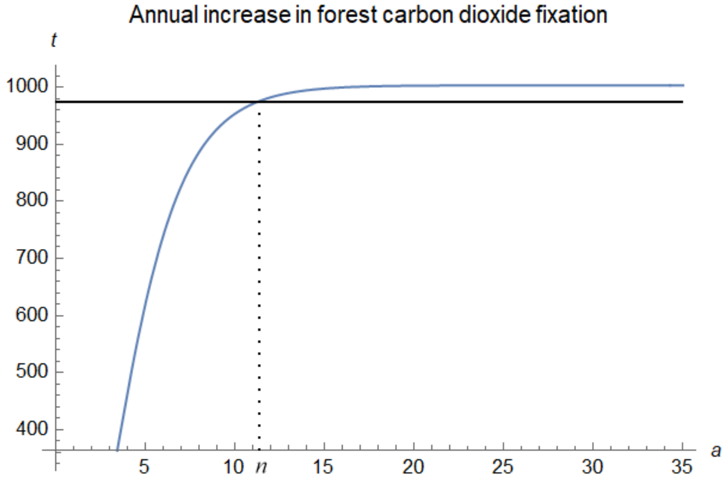


Figure. 12 Annual growth curve of forest fixed carbon dioxide with tree age.

In addition, to maintain the stability of the ecosystem, the optimal forest management plan should carry out the cutting of trees in batches so that the average age of the whole forest is maintained at about n/2. The annual fixed amount of forest is about $\sum_{i=0}^{i=\frac{n}{2}}\varepsilon_i$, and according to the relevant requirements of Finland, the first country to specify the forest law, the cutting cannot exceed 30%. Therefore, for the sustainable development of the forest, our follow-up calculation was based on 1/5 of trees reaching tree age n. At the same time, the products produced by the trees will also permanently sequester CO_2, so the total amount ($\overset{*}{\varepsilon}$)of the forest and its products expected to sequester CO_2 after χ years is equal to the sum of the number of fixed CO_2 by the products made of trees from $0\sim n$ the year and the amount of fixed CO_2 by the whole forest each year. Taking the average of each term, it is estimated that the annual absorption of CO_2 each hectare forest is 225.6402 tons.

5. conclusions

As an important part of the biosphere carbon cycle, forests play a vital role in mitigating climate change. Therefore, this paper first introduces the Logistic derivative model of trees, uses the Charlize curve to fit the growth function of tree volume, and studies the effects of tree species, topography, climate, and the development of forest products in different regions on carbon sequestration based on single-factor analysis. Then the relation function between forest carbon sequestration and time is obtained (for example, the relationship between carbon sequestration of broad-leaved forest and time is $\varepsilon_1=1.6248(1-e^{(-0.4401\times a)^{4.2244}}\cdot\rho+17.4075)$ Finally, through mathematical analysis and calculation, it is found that different types of forests are affected by different environments. The relevant parameters in the carbon sequestration model need to be adjusted separately to find the best time for individual tree felling and to estimate carbon sequestration (about 225.6402 tons per hectare of carbon absorbed by broad-leaved forests per year).

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