Shape adjustment of "FAST" active reflector

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Abstract. In this paper, the relevant working principle of "FAST" Chinese Eye is studied, and a mathematical model is established to solve the equation of the ideal paraboloid. The ideal paraboloid model is obtained by rotating the paraboloid around the axis in the two-dimensional plane. On this basis, the specific solutions of each question are discussed, and the parabolic equation, the receiving ratio of the feed cabin to the reflected signal, the numbering information and coordinates of the main cable node and other parameters are obtained. This paper for solving directly above the benchmark of spherical observation of celestial bodies when ideal parabolic equation, according to the geometrical optics to knowledge should be clear all the signals of the incoming signal after the ideal parabolic will converge to the focal point of basic rules, then through converting ideal parabolic model of ideal parabolic equation in a two-dimensional plane, An optimization model was established to minimize the absolute value of the difference between the arc length and the arc length of the parabola in the diameter of 300 meters. The known conditions were substituted into Matlab to solve the equation of the ideal parabola by rotating the parabola around the axis: $z = 1.7847 \times 10^{-3}(x^2 + y^2) - 300.6049$. In order to determine the ideal paraboloid of the celestial body, a new spatial cartesian coordinate system is first established with the line direction between the celestial body and the spherical center as the axis, so that the observed object is located directly above the new coordinate system. The same model in question 1 is established to obtain the vertex coordinates of the ideal paraboloid at this time. Then the vertex coordinates are converted to the coordinates in the original space cartesian coordinate system by rotation transformation between space cartesian coordinate systems. The solution of its vertex coordinates$(-49.5287, -37.0203, -294.1763)$.

Keywords: Ideal parabolic model; Optimization model; Search algorithm; Ergodic solution; Geometric optical system.

1. Introduction

1.1 The background of the question

FAST, also known as China's Celestial Eye -- five-hundred-meter Aperture Spherical Radio Telescope, is the world's largest and most sensitive radio telescope. China has its own intellectual property rights to the telescope, which is of great significance for China to achieve breakthroughs in the frontiers of science and accelerate innovation-driven development.

1.2 The known conditions

The basic structure of FAST is known, which is mainly composed of active reflector, signal receiving system (feed cabin) and related control, measurement and support system. The active reflector is an adjustable sphere, and the adjustable sphere is composed of main cable network, reflection panel, pull cable, actuator and support structure. The main cable network (used to support reflective panels) is composed of flexible main cables in a triangular grid of geodetic lines, each of which is fitted with a reflective panel, and finally the entire cable network is fixed to the surrounding supporting structure.

The active reflector has two states, one is the reference state and the other is the working state. The reflection surface of the reference state is a sphere with a radius of about 300 meters and a diameter of 500 meters, which is also called the reference sphere. The reflection surface of the working state is an approximately rotating paraboloid with a diameter of 300 meters, and its shape is also called the working paraboloid. By analyzing the figure 1 of FAST during observation, C is the spherical center of the reference sphere, and the moving region of the plane center received by the feed cabin is on a sphere (focal plane) concentric with the reference sphere, where the radius difference between the two concentric spheres is $F = 0.466R$, where R is the radius of the reference sphere, and F/R is the focal diameter ratio. The feed module can receive a signal only if it is sent to the effective receiving disk area with a center diameter of 1 meter, where the signal is received by the feed module.

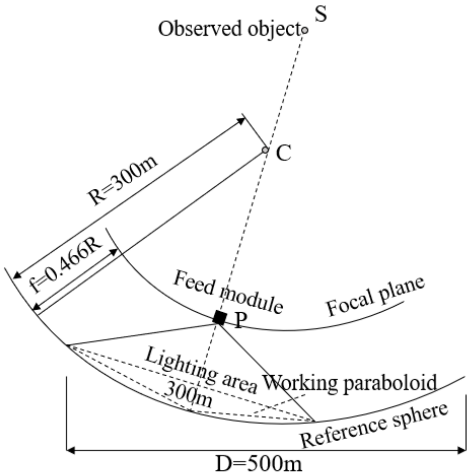

Figure 1. FAST profile diagram

When FAST observes a celestial object S in a certain direction, the center of the receiving area of the feed module will first move to the intersection of the straight line SC and the focal plane P, and then adjust some of the reflection panels to form an approximate rotating paraboloid with the straight line SC as the symmetrical axis and the focus P, thus converging more reflections of parallel electromagnetic waves from the target celestial body to the effective area of the feed module.

1.3 Questions that need to be solved

Question 1: When the celestial body S to be observed is directly above the reference sphere, that is $\alpha = 0°, \beta = 90°$, the ideal paraboloid is determined considering the adjustment of the reflection panel.

Question 2: When the celestial body S to be observed is located in $\alpha = 36.795°, \beta = 78.169°$, determine the ideal paraboloid at this time.

The azimuth α and elevation β of celestial body S are shown in Figure 2 below:

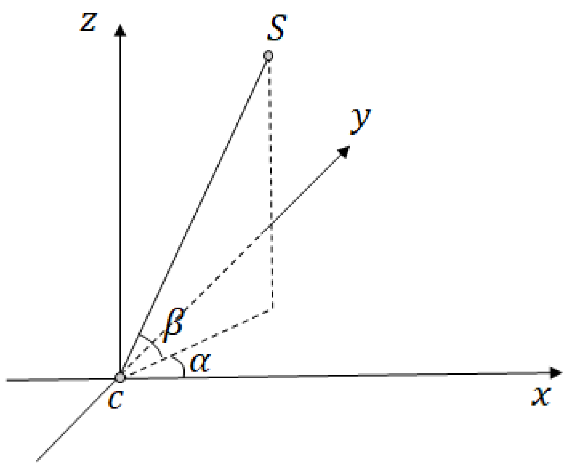

Figure 2. Schematic diagram of azimuth and elevation of celestial body S

2. Analysis of the question

2.1 Analysis of the question 1

According to the analysis of question 1, when the celestial body S to be observed is located directly above the reference sphere, the space rectangular coordinate system is established with the straight line SC as the axis z and the center of the reference sphere as the origin. To find an ideal paraboloid, a parabolic section can be made across the axis z. The ideal paraboloid is to make the signals transmitted to the paraboloid reflect to the effective area where the signal is received by the feed cabin through reflection, so that the reception rate of the signal is as large as possible, and at the same time, the absolute value of the difference between the arc length and the arc length of the paraboloid in the diameter of 300 meters should be minimized.

It is known that the aperture length of the reflection surface in the working state is 300 meters. In order to ensure that all signals are received exactly, the right end of the parabolic plane is taken as the zero boundary point. Calculate the coordinate value of the exact signal can be received at the point of zero boundary, and establish the optimization model by combining some known information given by the question. The unique parabolic equation can be obtained, and then the expression of the parabola can be obtained according to the parabolic equation.

2.2 Analysis of the question 2

In the second question, the position of the object S to be observed has changed, and its azimuth and elevation are known. First, the original space cartesian coordinate system can be rotated around the origin, first, the axis y is rotated to SC in the projection line of xoy, and then the axis z is rotated to the line SC to form a new space cartesian coordinate system. After rotation, the coordinates of the principal cable nodes associated with the reflector and the ideal paraboloid consistent with the equation in question 1 in the new coordinate system can be determined.

2.3 Assumptions of the model

1). Since the object to be observed is infinitely far away from the reflector, it is assumed that both the electromagnetic wave signal and the reflected signal can be regarded as straight line propagation.

2). It is assumed that when the main cable node is adjusted, the distance change between adjacent nodes is ignored.

3). It is assumed that the influence of natural factors such as atmospheric turbulence and the interference between electromagnetic wave signals are not considered when receiving information.

4). It is assumed that the reflector panel is approximately a smooth plane without holes and will not affect the reflection of celestial electromagnetic waves.

3. Preparation before the model

3.1 Numerical preparation

Before establishing the model, we need to make further calculations on some known information in the question. It is mentioned in the question that the length of the lower cable will not change, but because the FAST is built in the uneven mountains, the length of each lower cable is different, in order to change the shape of the reflector, it is necessary to rely on the expansion of the actuator. The length of each actuator can be expressed d_i. To obtain the size d_i, substitute the upper and lower endpoints of the actuator into the following equation:

$$d_i = \sqrt{\left(x_i^t - x_i^b\right)^2 + \left(y_i^t - y_i^b\right)^2 + \left(z_i^t - z_i^b\right)^2} \quad \left(i = 1,2,3,\cdots,2226\right)$$

It is calculated that:

$$d_1 = d_2 = d_3 = \cdots = d_{2226}$$

That is, the initial length of all actuators (the reflector is not adjusted) is consistent.

To sum up, when adjusting the actuator, it is not necessary to consider the impact of the included Angle between the actuator and the cable, but only need to adjust the stretching length of the actuator to achieve the adjustment of the reflecting panel.

3.2 Rotation of Spatial Cartesian coordinate system

Step 1:The xoy plane rotates around the z axis

As can be seen from figure 2, the azimuth α is the angle between the xoy plane and the x-axis projected by the straight line SC. In order to rotate the y-axis to the projection line, the z-axis should be used as the rotation axis. Rotate $90°-\alpha$ in the direction of $y \to x$, and the z-axis remains the same, and the following formula is obtained:

$$\begin{cases} x_0 = x\cos(90°-\alpha) - y\sin(90°-\alpha) = x\sin\alpha - y\cos\alpha \\ y_0 = x\sin(90°-\alpha) + y\cos(90°-\alpha) = x\cos\alpha + y\sin\alpha \\ z_0 = z \end{cases}$$

Step 2:The y_0oz_0 plane rotates around the x_0 axis

As can be seen from figure 2, the elevation β is the angle between the straight line SC and the z_0 axis. In order to rotate the z_0 axis to the straight line SC, the x_0 axis should be used as the rotation axis and rotate β in the direction of $z_0 \to y_0$. When x_0 is constant, the following formula can be obtained:

$$\begin{cases} x' = x_0 = x\sin\alpha - y\cos\alpha \\ y' = y_0\sin\beta - z_0\cos\beta \\ z' = y_0\cos\beta + z_0\sin\beta \end{cases}$$

The following formula can be obtained from the above two steps:

$$\begin{cases} x' = x\sin\alpha - y\cos\alpha \\ y' = x\cos\alpha\sin\beta + y\sin\alpha\sin\beta - z\cos\beta \\ z' = x\cos\alpha\cos\beta + y\sin\alpha\cos\beta + z\sin\beta \end{cases} \tag{1}$$

According to the above formula, the coordinate transformation results of each point in space cartesian coordinate system after rotation around different rotation axes can be obtained.

3.3 Reflection law of parabolic concave mirror and spherical mirror to parallel light

For a parabolic concave mirror, when a parallel light shines parallel into the concave mirror, the parallel light will converge to the focus of the paraboloid after the mirror is reflected; for the spherical mirror, the parallel light will eventually converge to a straight line that passes through the center of the circle and is perpendicular to the chord, and the distance from the intersection between the line and the sphere is half of the radius of the sphere, that is, the converging point is the focus of the circular reflection.

In addition, the relevant properties of the signal and the parallel light are consistent, so when the signal reflection question is analyzed and considered, it can be classified according to the reflection law of the parallel light.

4. Ideal Paraboloid Model for question 1

4.1 The model establishment of question 1

By analyzing the relevant knowledge of the rotating paraboloid, the question can be simplified first, because the parabolic axis section passing through the axis of rotation is the same everywhere, that is the size of the parabola on each axis section is also the same. The reference sphere is also cut on the axis of SC, and the arc segment cut in the area of the reference sphere is of the same size and shape. Considering that the axis of rotation (straight line SC) in question 1 is the z axis of the spatial Cartesian coordinate system, the parabola and arc segments cut from all planes can be made. Because the relative position relationship between the parabola and the arc segment is uncertain, the tangent diagram may have the following four cases:

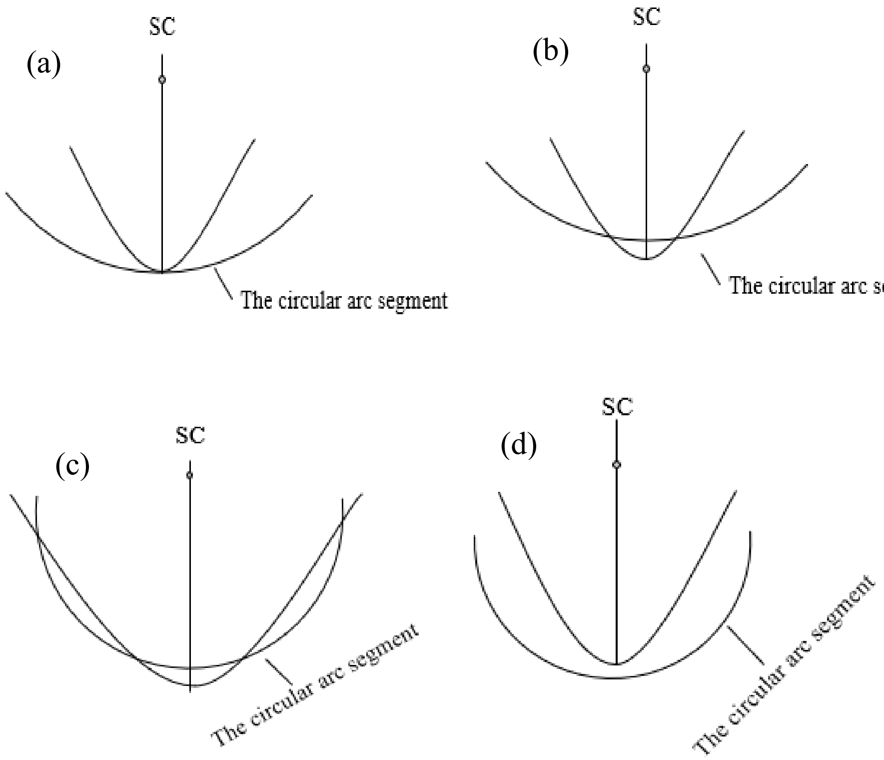

Figure 3. (a) Two arcs have only one intersection point (b) Two arcs have two intersection points (c) Two arcs have four intersection points (d) Two arcs have no intersection point

In order to solve the expression of the ideal paraboloid in Question 1, the general expression of the expression can be set as:

$$z = \frac{1}{4F'}(x^2 + y^2) + b$$

To determine the equation of the ideal paraboloid, the demand solution yields the values of F' and b, whose vertex coordinates are uncertain and are $(0,0,b)$. In order to solve the ideal paraboloid equation, an optimization model is established to minimize the difference between the arc surface and each point on the paraboloid in a two-dimensional plane with a diameter of 300 meters. With the constraints given in the question, the values of F' and b can be obtained.

$$\min \quad u = \left| S_{\text{round}} - 100\pi \right|$$

Where S_{round} represents the arc length of the parabola and 100π represents the arc length.

At the same time, it should be noted that the critical case where the paraboloid needs to be adjusted to the minimum and can receive the most signals should be selected for calculation. In order to simplify the calculation process, an ideal parabola model was established, and the critical value was calculated after converting the three-dimensional space surface into a two-dimensional space curve.

Taking the zox plane as the tangent plane of the rotating parabolic surface, let the parabolic equation in two-dimensional coordinates be:

$$z = \frac{1}{4F'} x^2 + b \tag{2}$$

The right end point of the ideal parabola is point Q, the tangent l_1 of the ideal parabola at that point is l_1, the slope of tangent l_1 is k_1, and then the perpendicular l_2 of tangent l_1 is k_2.

It is known that the incident signals are parallel to the parabola of the z-axis, and a beam of signal α that happens to be at point Q is taken, then the angle between the signal α and the vertical line l_2 is set to θ_1, and the reflection angle of the signal is θ_1, and finally the signal happens to be reflected to the leftmost end P of the receiving line segment.

The angle between the reflected signal and the positive direction of the x-axis is set to the γ angle, and the angle between the reflected normal and the positive direction of the x-axis is set to the φ angle. The specific parameter expressions are shown in figure 4 below:

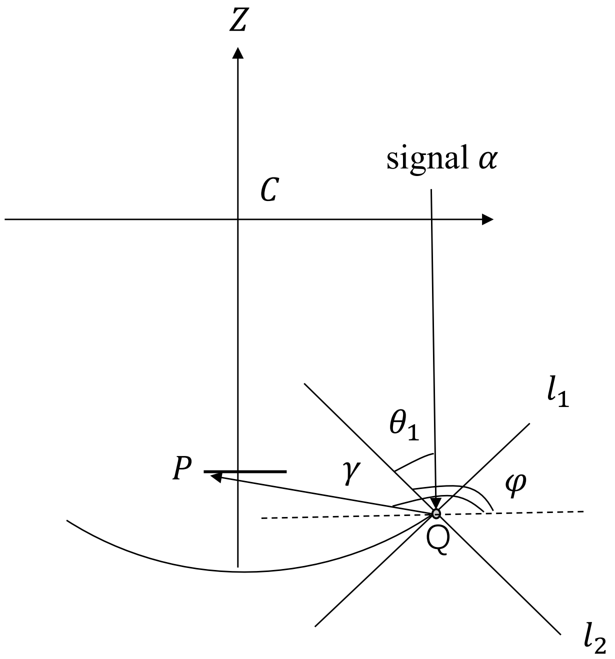

Figure 4. Schematic diagram of single signal reception

The following relationship exists between the above parameters:

$$\begin{cases} k_1 = \left.\dfrac{dz}{dx}\right|_{x=x_Q} = -\dfrac{1}{k_2} \\ \varphi = \arctan k_2 \\ \gamma = \varphi + \theta_1 = 2\arctan k_2 - \dfrac{\pi}{2} \end{cases} \tag{3}$$

The slope of line PQ can be expressed by Equation (2). The slope of line PQ can be expressed as:

$$k_{PQ} = \tan\gamma = \tan(2\arctan k_2 - \frac{\pi}{2}) \tag{4}$$

If the x-axis coordinate of the point Q is known, then the coordinate of the Q point can be expressed as $\left(150\,, \dfrac{5625}{F'} + b\right)$. It is given that the diameter of the receptive region is 1 meter, that is, the coordinate of the P point can also be expressed as $\left(-0.5\,, \left(R - 0.466R\right)\right)$. That is, another expression of k_{PQ} can be obtained by calculating the ratio of the difference between the ordinate and Abscissa of P and Q points:

$$k_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} \tag{5}$$

The ideal parabolic model can be established through the equality relation of equations (3) and (4). This model can solve the equation of two-dimensional parabola in zox plane by substituting relevant known information to solve the value of F'. Then the parabolic equation obtained can be rotated around the z-axis to obtain the rotating parabolic equation required by the first question. The ideal parabola model is sorted out as follows:

$$\min \quad u = \left|S_{\text{round}} - 100\pi\right|$$

$$\begin{cases} z = \dfrac{1}{4F'}x^2 - b \\ k_1 = \left.\dfrac{dz}{dx}\right|_{x=x_Q} = -\dfrac{1}{k_2} \\ \varphi = \arctan k_2 \\ \gamma = \varphi + \theta_1 = 2\arctan k_2 - \dfrac{\pi}{2} \\ k_{PQ} = \tan(2\arctan k_2 - \dfrac{\pi}{2}) = \dfrac{y_Q - y_P}{x_Q - x_P} \\ b > 300.4 \end{cases}$$

4.2 The model solution of question 1

We know that the abscissa of Q point is 150. by substituting it into equation (3), we can find that the value of k_1 is $\frac{75}{F'}$. Because $k_1 * k_2 = -1$, that is the value of k_2 is expressed as $-\frac{F'}{75}$, that is, the slope of the straight line PQ can be expressed as:

$$k_{PQ} = \tan(2\arctan(-\frac{F'}{75}) - \frac{\pi}{2})$$

The value of R in the question is also known, which is 300m. By substituting it into the coordinate expression of P, we can find that the coordinate of P is $(-0.5, -160.2)$ and the coordinate of Q point is $(150, \frac{5625}{F'} + b)$. Another expression of the PQ slope of the straight line can be obtained by substituting the coordinates of these two points into the equation (5):

$$k_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{\frac{5625}{F'} + b + 160.2}{150.5}$$

The optimization model and constraints are given, as shown in the following formula:

$$\begin{cases} \min u = \left|S_{\text{round}} - 100\pi\right| \\ \tan(2\arctan(-\dfrac{F'}{75}) - \dfrac{\pi}{2}) = \dfrac{\frac{5625}{F'} + b + 160.2}{150.5} \end{cases} \tag{6}$$

When the equation (6) is solved by Matlab, it can be obtained that the value of F' is 140.0819, and the value of b is -300.6049. The coordinate of Q point is (150, -260.45). The value of F' is substituted into the equation of tangent parabola (2). The equation of ideal parabola is as follows:

$$z = 1.7847\times10^{-3}x^2 - 300.6049$$

By rotating the ideal parabolic equation around the z -axis, the ideal parabolic equation required by question 1 is obtained:

$$z = 1.7847\times10^{-3}(x^2 + y^2) - 300.6049$$

In order to draw the spatial schematic diagram of the ideal parabola, the ideal parabolic equation and the reference spherical equation are substituted into Matlab for image drawing. The reference figure is shown in Fig 5 below:

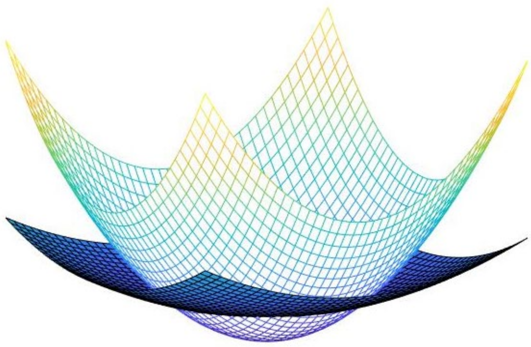

Figure 5. spatial diagram of ideal paraboloid

It is observed in Fig 5 that there is an intersecting ring between the ideal paraboloid and the reference sphere, and that part of the ideal paraboloid in the ring extends downward beyond the reference circle, while the ideal paraboloid in the outer part of the ring extends upward beyond the reference source surface.

5. Ideal Paraboloid Model for question 2

In question 2, the position of the observed celestial body S is not directly above the reference sphere, but has an angular offset. In order to use the ideal parabola model in question 1 to solve the ideal parabola of question 2, first of all, it is necessary to establish a new spatial Cartesian coordinate system to make the observed celestial bodies on the axis of the new spatial Cartesian coordinate system. At this time, the equation of the ideal paraboloid of question 2 in the new spatial Cartesian coordinate system will be consistent with the ideal paraboloid equation in question 1. In order to obtain the equation of the ideal paraboloid of question 2 in the original spatial Cartesian coordinate system, we only need to find the law of rotation between the two spatial Cartesian coordinate systems.

5.1 The model establishment of question 2

In order to find the ideal model paraboloid of the celestial body S located at $\alpha = 36.795°, \beta = 78.169°$, the ideal parabola model in question 1 can be used to solve it, because the model of question 1 is established in the spatial Cartesian coordinate system when the observed celestial body S lies directly above the reference sphere. Therefore, if we want to use the model of question 1 to solve the ideal parabolic equation of question 2, we need to rotate the spatial Cartesian coordinate system, and the axes after rotation are represented by z'、y'、x' respectively.

According to the relationship between the coordinate system transformation in the preparation part of the previous model, we know that in the new spatial Cartesian coordinate system, the equation of the ideal paraboloid can be set as:

$$z'=\frac{1}{4F'}(x'^2+y'^2)+b$$

According to the analysis, as long as the celestial body S is located on the positive half axis of the z-axis of the new spatial Cartesian coordinate system, the equation of its ideal paraboloid in the new spatial Cartesian coordinate system is determined, that is, the value of F' is the same as that of question 1. with the size of 140.0819 and the value of b is -300.6049.

5.2 The model solution of question 2

According to the solution of the formula of the ideal paraboloid in question 2, the equation of the ideal paraboloid in the new spatial rectangular coordinate system is $z'=1.7847\times10^{-3}(x'^2+y'^2)-300.6049$, and the coordinates of the vertex of the ideal paraboloid $(x',y',z')=(0,0,-300.6049)$. The coordinates of this point are substituted into Equation (1), where $\alpha=36.795°,\beta=78.169°$, and the following formula is obtained:

$$\begin{cases}0=x\sin\alpha-y\cos\alpha\\0=x\cos\alpha\sin\beta+y\sin\alpha\sin\beta-z\cos\beta\\-300.6049=x\cos\alpha\cos\beta+y\sin\alpha\cos\beta+z\sin\beta\end{cases}$$

The solution of the above equation results in $\begin{cases}x=-49.5287\\y=-37.0203\\z=-294.1763\end{cases}$, so the vertex coordinates of the ideal paraboloid are $\left(-49.5287,-37.0203,-294.1763\right)$.

6. Conclusion

This paper designs an ideal parabolic model for The Eye FAST, analyzes the three-dimensional surface axis after cutting into two-dimensional curves, making the analysis process of the model more simple and clear, at the same time, using the transformation of the space coordinate system, provides a very clever way to solve the problem, for everyone to learn from, At the same time, this paper also has far-reaching practical significance to solve the adjustment of the actual FAST reflector panel, and provides a valuable reference for solving the efficiency reception problem.

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